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Delay Preserving Substructures in Wireless Networks Using Edge Difference between a Graph and its Square Graph

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***Abstract*—**In practice, wireless networks has the property that the signal strength attenuates with respect to the distance from the base station, it could be better if the nodes at two hop away are considered for better quality of service. In this paper, we propose a procedure to identify delay preserving substructures for a given wireless ad-hoc network using a new graph operation $G^2 - E(G) = G^*$ (Edge difference of square graph of a given graph and the original graph). This operation helps to analyze some induced substructures, which preserve delay in communication among them. This operation G* on a given graph will induce a graph, in which 1-hop neighbors of any node are at 2-hop distance in the original network. In this paper, we also identify some delay preserving substructures in G*, which are (i) set of all nodes, which are mutually at 2-hop distance in G that will form a clique in G*, (ii) set of nodes which forms an odd cycle C_{2k+1} in G, will form an odd cycle in G* and the set of nodes which form a even cycle C_{2k} in G that will form two disjoint companion cycles (of same parity odd/even) of length k in G*, (iii) every path of length 2k+1 or 2k in G will induce two disjoint paths of length k in G*, and (iv) set of nodes in G*, which induces a maximal connected sub graph with radius 1 (which identifies a substructure with radius equal 2 and diameter at most 4 in G). The above delay preserving sub structures will behave as good clusters in the original network.

***Keywords*—**Clique, cycles, delay preserving substructures, maximal connected sub graph.

I. Introduction

An ad hoc network is a multi hop wireless communication, supporting mobile nodes without any fixed infrastructure. For a technology to be commercially successful, it must support more mobile nodes with faster delivery rate and less latency. Though, the physical capabilities of network devices, node locations and peripheral settings provide a potential topology of an ad hoc network, it is desirable to identify some simple delay preserving substructures in the original network. In large and dynamic ad hoc networks, it is very hard to construct an efficient network topology. Hence, by clustering the entire network, one can decrease the size of the problem into small sized clusters. Clustering has many advantages in ad hoc networks. Clustering makes the network maintenance easier. Also, by clustering the network, one can build a virtual backbone, which can make routing faster.

In the literature, various graph theoretic variants are used to cluster the network. They are, different graph dominating parameters which are due to concepts such as independent dominating set, weakly connected dominating set, connected dominating set, K-distance independent dominating set, K – distance independent dominating set, paired dominating set and also due to minimum spanning tree. All these methods, though, partition the given network into efficient clusters, butr delay factor is not considered by the above methods. This motivates us to introduce a graph operations denoted by $G^* = G^2 - E(G)$ operation, which identifies substructures, preserving delay in communication as in original network. The delay (distance) preserving property of substructures improves the performance of delivery rate, reduction in latency and the efficient transmissions in the network. In section III, we explain various necessary definitions and terminology that are needed in the subsequent sections. In section IV, some of the structural properties of G* graph related to G are discussed. In section V, hereditary delay (distance) preserving substructures are identified. In section VI, evaluation criteria for the identified substructures are given. In section VII, identification of substructures in G* graph is shown with an illustrative example.

II. Related Work

Clustering based on graph theoretic techniques has been explored extensively. Clustering is the problem of building a hierarchy among nodes [20]. The substructures that are collapsed in higher levels are called as clusters. Clustering usually entails the computation of a dominating set (DS) of the network. The domination problem seeks to determine a minimum number of nodes D (called dominating nodes or cluster-heads), such that any node $x \in G$ not in D is adjacent to at least one node in D. The computation of a DS of minimum cardinality for arbitrary graphs is known to be NP-Complete [1, 2]. Baker and Ephremides [3] proposed an independent dominating set algorithm called highest vertex ID. In this algorithm, each node traverse its closed neighbor

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set and chooses the higher id neighbor as a clusterhead. Gerla and Tsai [4] proposed an algorithm similar to highest ID algorithm called lowest ID algorithm, in which each node with the lowest ID within its closed neighborhood is selected as a clusterhead. In the work [5] Chen et al. proposed that these algorithms do not work correctly for some graphs, and hence he developed the k-distance independent dominating set algorithm. In this algorithm, Chen et al. adds one more rule to the above algorithm such that in a k-distance dominating set, every clusterhead must be at least k+1 distance from each other.

In the work [6, 7], Han and Jia proposed a weakly connected dominating set (WCDS) based clustering algorithm. In that work they construct a maximal independent set, using degrees of the nodes as the decision heuristic. Chen and Liestman [19] and Alzoubi et al. [8] have constructed well known WCDS algorithms.

Clustering using connected dominating set (CDS) has many advantages in network applications such as ease of broadcasting and constructing virtual backbone [9]. In constructing CDS, our major problem is to find the minimum connected dominating set. Ghua and Khuller in [10] proposed two centralized greedy algorithms for finding suboptimal connected dominating set. Chatterjee and Das in [11] proposed an on-demand weighted clustering algorithm, in which, four parameters are used to determine the weight of each node. This algorithm chooses mobile nodes with minimum weight in the local area to be the cluster heads. Dai and Wu [12] presented a distributed solution for computing a k-connected k-dominating set as a backbone of wireless networks. Their approach combines multiple domination and k-vertex connected property, which guarantees that a CDS remains vertex connected even when removing upto k-1 nodes from graph.

Shi and Srimani [13] introduce a distributed algorithm for computing d-hop connected d-hop dominating set. For any graph G, let G_d denote the d-closure of G, which means the graph built on vertices of G such that any pair of vertices in G are connected by an edge in G_d, if the vertices are within distance d of each other in G. A d-hop DS is d-hop connected if it is connected in G_d. Janakiraman et. al. [14] introduced a graph variant called ranking for clustering the mobile ad hoc networks. In this proposed algorithm, each node is ranked based on its signal strength. This algorithm elects the higher ranked node as the cluster head. All the above mentioned algorithms will produce star shaped clusters.

Janakiraman in [15] introduced another graph domination called paired domination which is used for clustering the mobile ad hoc networks. Finding the minimum connected paired dominating set is known to be NP-Complete [2]. In this work [15], each cluster has a master and a proxy master as cluster heads. Cluster heads are elected based on the closeness index of each node with respect to all the other nodes in the network.

Gallagher et. al. [16] proposed a distributed algorithm which determines a minimum weight spanning tree for an undirected graph that has distinct weights for every edge. Gentile et.al. [17] proposed a distributed routing and clustering algorithm to minimize the overhead messages for multi-hop routes with minimum power in a MANET. Ya-Feng et.al. [18] focused on the construction of the optimal virtual multicast backbone with the fewest forwarding nodes to decrease overhead cost, as there is only limited resources are available.

III. Definitions and Terminologies

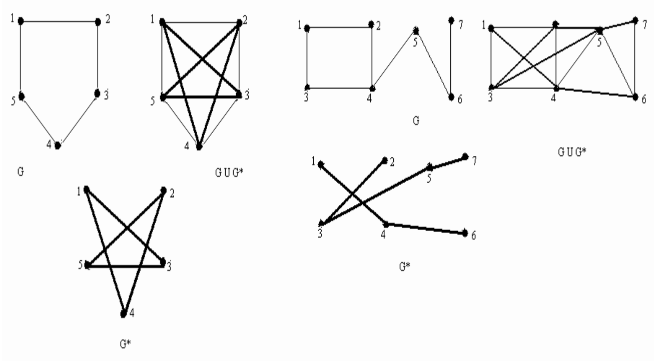
A network can be represented as a graph G = (V, E) consisting of the set V of n nodes (vertices), the set $E \subseteq V \times V$ of edges (arcs). A *Path (chain)* joining i and j with $i \neq j$ is the set of different vertices $i_0, i_1, \ldots, i_k$ with $i = i_0$, $i_k = j$ and all intermediate vertices i_j, $j > 0$, any two consecutive vertices are adjacent in G. A *closed path (closed chain)*, with initial and terminal vertices are same is called a cycle. The length of any shortest path joining i and j is called d (i, j) the *distance* between i and j in G and is denoted by d (u, v|G) and in short, if there is no confusion d (u,vj) simply. The maximum distance between a point v and its farthest point is called as *eccentricity* of the point v in that graph. The minimum and maximum eccentricities are defined as *radius r* and *diameter d* respectively of the graph in discussion.

A shortest path connecting any two nodes will be also termed as *Geodesic*. A cycle is said to be a *Geodesic cycle* if every pair of nodes in that cycle is connected by a shortest path.

Let V' is a nonempty subset of V (G). The sub graph of G whose vertex set is V' and whose edge set is the set of edges of G that have both ends in V' is called sub graph of G *induced* by V' and is denoted by G[V']. A graph G is said to be *isomorphic* to H if the number of vertices and edges in H is same as in G and if $\psi : V(G) \to V(H)$ such that uv $\in$ E (G) if and only if $\psi(u)\psi(v) \in E(H)$. A maximum complete sub graph is called as a *Clique.* A substructure $H \subseteq G$ is said to be *distance* (*delay) preserving substructure* if for every pair of vertices (u, v) $\in$ V (G), d (u, v|G) = d (u, v|H) i.e. the distance between any two vertices in the original network is maintained in the substructure. The n^{th} *power of a graph* G, denoted by G^n, is obtained from G by joining every pair of non-adjacent vertices (u, v) with d (u, v|G) $\leq$ n by an edge. A subset S of V is called an *independent set* of G if no two vertices of S are adjacent in G.

A graph G* = (V*, E*), where the vertex set V* = V (G) and any two vertices u and v are adjacent in G* if and only if d (u, v|G) = 2. Two cycles $C_k^{(1)}, C_k^{(2)}$ are termed as companion cycles in G*, if there exists an even cycle C_{2k} in G say $(v_1, v_2, \ldots, v_{2k}, v_1)$ such that the odd and even labeled vertices of C_{2k} appear in $C_k^{(1)}, C_k^{(2)}$ respectively.

Construction of G* graph with an example:



(a) (b)

Fig. 1(a) shows G* graph is connected (b) shows G* graph is disconnected

IV. STRUCTURAL PROPERTIES OF G* GRAPH RELATED TO G

Here, we see some of the structural properties of G* graph related to G.

Theorem 1: A*ny set of vertices, which induces an odd cycle in an original graph, will induce an odd cycle in G*.*

Proof: Let $v_1, v_2, \ldots, v_{2n+1}$ is an odd cycle in G, which has a structure in G* in the form $<v_1\text{-}v_3\text{-}v_5\text{-}\ldots\text{-}\ v_{2n+1}\text{-}v_2\text{-}v_4\ldots.\text{-}v_{2n}\text{-}v_{1}>$, which results in another odd cycle whose distance (delay) preserving limit will be $\leq n$.

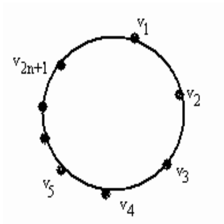


Fig. 2 Odd cycle

Theorem 2: *Any set of vertices which induces an even cycle in a original graph will induce either two even/ two odd companion cycles in G*.*

Proof: Let $v_1, v_2,.., v_{2n}$ is an even cycle in G, which result in a structure in G*, in the form of two disjoint companion cycles $<v_2,\ v_4\text{-}v_6\text{-}\ldots\text{-}\ v_{2n}\ \text{-}v_2>$ and $<v_1\text{-}v_3\text{-}v_5\text{-}\ldots\text{-}\ v_{2n-1}\ \text{-}v_{1}>$ of same parity in length, whose distance (delay) preserving limit will be $\leq n/2$ in G*.

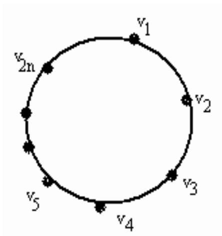


Fig. 3 even cycle

The following theorem is trivial from the definition of G*.

Theorem 3: *Every clique in G* operation denotes the set of vertices, which are mutually at two hop distance in the original network.*

V. IDENTIFICATION OF HEREDITARY DELAY (DISTANCE) PRESERVING SUBSTRUCTURES

The hereditary delay preserving substructures are identified by applying E (G^2) – E (G) operation over a given network, which results in G* graph, as a virtual network. This virtual network obtained due to the above operation can be either connected or disconnected. From the definition, it is clear that $d(u, v|G) = 2 \Rightarrow d(u, v|G^*) = 1$. The substructures which are identified from G* graph are embedded with added intermediate vertices to preserve the delay in G.

A. Procedure for Obtaining G Graph from G*

Procedure for obtaining virtual G* graph from G is given below.

1. FIND second power graph G^2 for the given graph G.
2. FIND the edge difference between second power of a given graph and the original graph G.
   i.e. $G^* = G^2 - E(G)$

B. Procedure for Identifying Cycle Substructure in G Graph*

If G* graph is disconnected run DFS (Depth-First Search) algorithm for each components. Cycles obtained through this procedure can be either odd/even cycle. Any cycle can be of a single cluster in the case of odd cycle or two independent clusters in case of even cycles.

1. IDENTIFY the cycles in G* graph using DFS algorithm.

The clusters, which are formed, are of good delay preserving with respect to the original network. The time complexity to run the above procedure is O (|V|+|E|), where V represents the vertex set and E the edge set.

Remark-1: In case of a cycle of length k identified in G*, which is partially embedded as a two hop chain into a cycle of length 2k, then the cycle C_{2k} is the required cluster in G with the help of a companion cycles of length k in G*. It is clear that the companion cycle vertices of length k in G* will form another two hop chain in the cycle of length C_{2k} in G.

Now we prove the following propositions relating cycle structures in G and G*.

Proposition 1: *If $d_G(u, v) = 2k$ then $d_{G^*}(u, v) = k$.*

Proof: Clearly, from the definition of G* graph, if the shortest distance between any two vertices in Graph G is at the distance two then those two vertices are made adjacent in

G* graph. Hence, if the distance between any two vertices are 2k in G, then, they will be at distance k in the G* graph.

Remark-2: If C_n is a cycle identified in G*, then it is either a set of vertices which will induce an odd cycle in G or it forms a 2-hop chain embedded in some cycle C_{2n} in G.

C. Procedure for Identifying Clique Substructure in G Graph*

Though, finding a maximum clique is a NP-complete problem, the proposed procedure identifies a complete (which need not be a maximum) sub structure in the G* graph in polynomial time.

1. FIND an independent set I of a given graph, where each pair in the set I is at 2 –hop distance in the original network.

D. Procedure for identifying a path substructure in G graph*

This procedure identifies a delay (distance) preserving substructure in G*.

1. IDENTIFY a path of length diam/2 in G*.
2. EMBED as a 2-hop chain in a path of length at most the diameter in G.

Now we prove the following proposition relating to the path structure in G and G*

Proposition 2: *Any path of length k in G will induce two vertex disjoint path of length at most k/2 in G**

Proof: Consider a path of length k, which are labeled as v_0, v_1, …, v_k. The odd and even labeled vertices in the above path separately form two induced paths in G* of the required length.

E. Procedure to identify an induced maximal substructure of radius 1 in G graph*

This procedure identifies an induced maximal substructure in G*, which is of radius 1.

1. IDENTIFY a set of nodes in G*, which induces a maximal connected sub graph of radius 1.

Now we prove the following proposition.

Proposition 3: Any set of vertices, which induces a maximal connected sub graph of radius 1, will identify a substructure with radius equal to 2 and diameter at most 4.

Proof: Let {v_1, v_2, v_3, v_4 } be the set of vertices, which induces a maximal connected sub graph of radius 1 in G*. Let v_1 be the vertex whose eccentricity is 1 in G*. Then the neighbors of vertex v_1 in G* and the first neighbors of v_1 in G will form a delay (distance) preserving substructure with radius equal to 2 and diameter at most 4 in G.

VI. Evaluation Criteria

The number of metrics used to evaluate the performance of the network depends on many factors. Here, we discuss some of the related factors to evaluate the performance of the given network using the above procedure.

A. Delivery rate

The most important factor that is considered in the communication network is the data delivery rate. The delay preserving substructures, which are identified in G* will maintain the delivery rate through the intermediate nodes in original network.

B. Latency

A second metric is the latency, the response time for a message generated at node i and sent to node j. The latency of communication from node i to node j is inversely proportional to the distance. The substructures which are identified in G* could be embedded in G, as a partial or full 2-hop sets in G, which among themselves maintain the latency as they are delay (distant) preserving. This metric has applications, in WI-Fi, where the signal strength varies with respect to distance.

C. Transmission

Transmission efficiency is directly proportional to the distance between the pair of nodes in discussion. The hereditary delay (distance) preserving substructure which are identified in G* graph will maintain the same transmission efficiency as in G.

VII. An Illustrative Example

We demonstrate here, how $G^2 - E(G)$ operation produces a G* virtual graph and the identification of hereditary delay (distance) preserving substructures in G* graph.

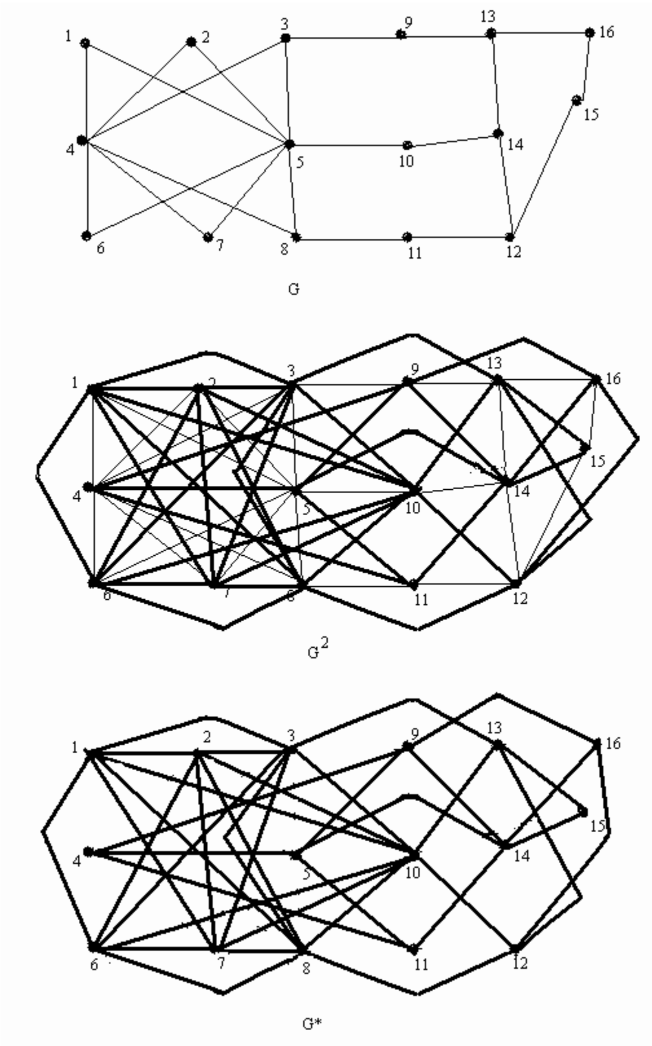


Fig. 4 Constructing G^2 and G*Graphs

In Fig. 4, from the graph G, G^2 graph is constructed, in which $G^2 - E(G)$ operation is performed to produce G* graph. In G* graph, nodes 1, 2, 3,6,7,8 and 10 are mutually at 2-hop distance, hence according to Theorem-3, they form a clique substructure in G* graph. The two disjoint odd cycle substructures 5-14-11-5, 10-12-8-10 of length 3, when embedded in G, gives an even cycle of length 6 as explained Proposition 2, similarly, this two disjoint odd cycles 3-13-10-3 and 5-9-14-9 when embedded in G, gives another even cycle. In case of odd cycle, 15-13-12-16-14-15, when it is embedded in G, it induces another odd cycle as in Theorem 1. Nodes 9, 5, 11 and 14 induces a maximal connected graph of radius 1, nodes 14 and 5 has radius 1 in this induced graph, hence 14, 9, 5,11,10,12,13 will be a delay preserving substructure whose radius is equal to 2 and diameter is 3.

VIII. CONCLUSION

The proposed delay (distance) preserving substructures find its application in wireless networks, which has high power base stations. In this paper cycle, clique, path and maximal induced sub graph of radius 1 substructures are identified in G* graph. In future, we intend to identify star and double star substructures.

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