Cost and Profit Analysis of Markovian Queuing System with Two Priority Classes: A Computational Approach

S. S. Mishra and D. K. Yadav

Abstract**—**This paper focuses on cost and profit analysis of single-server Markovian queuing system with two priority classes. In this paper, functions of total expected cost, revenue and profit of the system are constructed and subjected to optimization with respect to its service rates of lower and higher priority classes. A computing algorithm has been developed on the basis of fast converging numerical method to solve the system of non linear equations formed out of the mathematical analysis. A novel performance measure of cost and profit analysis in view of its economic interpretation for the system with priority classes is attempted to discuss in this paper. On the basis of computed tables observations are also drawn to enlighten the variational-effect of the model on the parameters involved therein.

Keywords**—**Cost and Profit, Computing, Expected Revenue, Priority classes.

I. INTRODUCTION

ANALYSIS of priority preemptive resume has taken a mature place in queuing investigations. It is studied under controlling of queuing discipline, where the discipline FCFS may be not fallowed in certain situations and one may serve in accordance with a priority scheme than others. Thus, high-priority customers preempt over low-priority customers for providing service in the system.

Service priority is one of the main criteria which is used by various companies to allocate limited production capacity among customers with different necessities and willingness. Hence, notion of priority queuing has many important applications in commercial information systems, production systems, networking, computer systems and in military management.

Another very practical application of priority based queue may be seen in case of hospitalization of patient in a hospital. If emergency patients come to admit, admission of a general patient is resumed and given a low priority than emergency patient. Thus, such systems are often encountered in practice, particularly in service-oriented operations.

Such queuing models under preemptive resume priority have been studied by many researchers so for. Reference [16] analyzed an exponential single server priority queue with two classes of customers and find out recursive formulae for steady state distributions using theory of matrix-geometric invariant probability vectors, as in [18]. It also studied the problem of preemptive queuing system with two priorities classes whereas [3] analyzed the same with time - dependent problem. Also, various authors presented their works regarding various aspects of priority queuing systems such as [20], [5], [11]. Here, it assumed a cut off priority queue discipline and derived several results of performance measures, as in [15]. Further, it dealt with an approximate analysis for heterogeneous multiserver systems with priority jobs, as in [27]. Moreover, a two-class single server preemptive priority queue is analyzed with balking assuming Poisson arrival and exponential service time, vide [26]. Again, it discussed a two-class priority queueing system with state dependent arrivals and computed the average equilibrium queue length for both classes, as in [6].

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Moreover, it considered a multi-class priority queues with no preemptive services controlled by an exponential time and multiple vacations, vide [28]. Again, [25] discussed about a dynamic priority queue. They provided an algorithmic analysis with a single server and two independent Poisson streams of customers with general service time distributions. A performance analysis of a discrete-time priority queuing system with correlated arrivals is presented, as in [17]. They also demonstrated the impact of the correlated arrival process in the considered system. Several other researchers such as [10], [9], [2] have made their important contributions in the analysis of various aspect of priority queuing systems.

In this series of investigation, very recently [1] presented the analysis of M/M/1 queuing system with two priority classes; where first class customers are served under FCFS preemptive resume discipline having finite room of capacity and of high priority. The second class customers have infinite waiting space and of low priority. Service of low-priority customers interrupted due to entrance of high-priority customers in the system. Author also derived expected queue lengths of customers of both classes.

Cost and profit analysis is an important aspect of queuing system that leads to the economic interpretation of the system, which is useful in the application of real-life situations arising out of industrial and technical situations. In the literature, we observe that most of investigations related to queuing theory are devoted to analyze priority queuing system, but no more attempts have been made to discuss about cost and profit analysis of priority queuing models whereas fact is that without analyzing cost and profit structure of the system, we are not sufficient to critically examine the effect of prioritization on the system.

We also consult some of interesting researches related to cost analysis of queueing systems, vide for example [8], [7], [19], [13], [31], [21], [22], [29], [14], and [23]. Further, as in [30] it discussed the profit analysis of $M/E_K/1$ machine repair problem with a non reliable service station. Recently, [25] developed a computational approach to profit optimization of a loss-queueing system. The cost analyses of machine interference model by applying an algorithmic approach are also discussed as in [24], [26].

Related literature revealed that [1] made no attempt to consider an economic interpretation of the queueing model with priority classes. With this motivation in mind, in this paper, we propose to investigate the cost and profit analysis of a single server Markovian queuing system with two priority classes as a novel performance measure of the system with priority classes. First, we define total expected cost (TEC) of the system with both priority classes and subject it to optimize with respect to service rates of the classes. Consequently, we find out the optimal service rates for high and low priority classes and total optimal cost (TOC) of the system with respect to most efficient service of the system. Further, profit analysis of the system is carried out and total expected profit (TEP) of the system is also evaluated. In order to compute the total optimal cost and total expected profit of the queueing system, a system of non-linear equations is constructed and its computing algorithm has been developed based on a fast converging Newton's method for solving the above system (vide [4], [12]). We prefer this technique for it requires least computing time and lesser memory space as compared to other methods (computer program is developed in C++). Numerical demonstration with its observations based on tables and graphs have also been added to gain a significant insight into the problem.

The paper has been organized in various important sections such as introduction, notations, description of the model, cost analysis of the system, algorithm for computation, profit analysis of the system, observations, conclusion and graphics.

II. NOTATIONS

There are following notations and their descriptions used in the analysis of this model.

λ_1 = arrival rate of high-priority class.
λ_2 = arrival rate of low-priority class.
μ_1 = service rate of high-priority class.
μ_2 = service rate of low-priority class.
ρ_1 = traffic intensity relative to high-priority class.
ρ_2 = traffic intensity relative to low-priority class.
L_H = expected queue length of high-priority class.
L_L = expected queue length of low-priority class.
C_1 = cost per service per unit time associated with high-priority class.
C_2 = cost per service per unit time associated with low-priority class.
C_h = holding cost per customer per unit time of high-priority class.
C_l = holding cost per customer per unit time of low-priority class.
T(C) =total expected cost (TEC) of the system.
R = earned revenue for providing service per customer.
$T(P_H)$= total expected profit of high-priority class customers.
$T(P_L)$= total expected profit of low-priority class customers.
$T(P_S)$ = total expected profit of the system.

III. DESCRIPTION OF THE SYSTEM AND ITS COST ANALYSIS

Here, it is considered a single server queueing system with Poisson arrival and exponential service pattern having two classes of customers. First class has high priority and second class has low priority for getting services. Assume that service discipline within each class is as FCFS and priority system is preemptive resumed that means during the service of a low-priority customer, if a high-priority customer enters the system, the service of a low-priority customer is interrupted, and will be resumed again when there is no high-priority customer in the he established the expected queue length for high and low priority classes as following:

L_H (mean queue length of high priority class)

$$= \frac{\rho_1[1-(N+1)\rho_1^N + N\rho_1^{N+1}]}{(1-\rho_1)(1-\rho_1^{N+1})}$$

The number of the first class customer is restricted to a finite number N including the one being served, if any, and the number of the second class customers is infinite also suppose that λ_1 and λ_2 are the arrival rates for two classes and let μ_1 and μ_2 be the service rates for two classes in the system. Thus, traffic intensities are given as $\rho_1 = \lambda_1/\mu_1$ and $\rho_2 = \lambda_2/\mu_2$ .Thus, [1] discussed the system with steady state probabilities and find out the marginal distributions of both classes. Further, L_L (mean queue length of low priority class)

$$= \frac{(1-\rho_1)}{[1-\rho_1-\rho_2(1-\rho_1^{N+1})]}\Big[\frac{\rho_2(1-\rho_1^{N+1})}{(1-\rho_1)} + \frac{\lambda_2}{\mu_1}\frac{\rho_1}{(1-\rho_1)^2(1-\rho_1^{N+1})}(1-\rho_1^{2N+1}-(2N+1)(1-\rho_1)\rho_1^N)\Big]$$

We can further write the above expressions as

$$L_H = \frac{\rho_1[1-(N+1)\rho_1^N + N\rho_1^{N+1}]}{(1-\rho_1)(1-\rho_1^{N+1})} = \frac{\rho_1-(N+1)\rho_1^{N+1}+N\rho_1^{N+2}}{1-\rho_1-\rho_1^{N+1}+\rho_1^{N+2}} \quad (1)$$

This implies that

$$L_H = \frac{\alpha}{\beta}, \text{ (say)}$$

where

$$\alpha = \rho_1 - (N+1)\rho_1^{N+1} + N\rho_1^{N+2}$$
$$\beta = 1-\rho_1-\rho_1^{N+1}+\rho_1^{N+2} \quad (2)$$

$$L_L = \frac{\rho_2(1-\rho_1^{N+1})}{1-\rho_1-\rho_2(1-\rho_1^{N+1})} + \frac{\lambda_2}{\mu_1}\frac{\rho_1-\rho_1^{2(N+1)}-(2N+1)(\rho_1^{N+1}-\rho_1^{N+2})}{(1-\rho_1-\rho_1^{N+1}+\rho_1^{N+2})(1-\rho_1-\rho_2(1-\rho_1^{N+1}))}$$

which can be expressed as

$$L_L = \frac{\theta}{\varphi} + \frac{\lambda_2}{\mu_1}\frac{\psi}{\beta\varphi} \quad \text{(say)} \quad (3)$$

where,

$$\theta=\rho_2(1-\rho_1^{N+1}),\quad \varphi=1-\rho_1-\rho_2(1-\rho_1^{N+1}),\quad \psi=\rho_1-\rho_1^{2(N+1)} -(2N+1)(\rho_1^{N+1}-\rho_1^{N+2}) \tag{4}$$

Now, the total expected cost of the system is defined as
$T(C) = C_1\mu_1 + C_2\,\mu_2 + C_h\,L_H + C_l\,L_L$

In the light of equations (1) and (3), above expression can obviously be rewritten as

$$T(C)=C_1\mu_1+C_2\mu_2+C_h\left(\frac{\alpha}{\beta}\right)+C_l\left(\frac{\theta}{\varphi}+\frac{\lambda_2}{\mu_1}\frac{\psi}{\beta\varphi}\right) \tag{5}$$

Further, from equations (2), and (4), we get the following expressions:

$$\frac{\partial\alpha}{\partial\mu_1}=\frac{1}{\mu_1}\left[-\rho_1+(N+1)^2\rho_1^{N+1}-N(N+2)\rho_1^{N+2}\right] = \alpha_1'$$

$$\frac{\partial^2\alpha}{\partial\mu_1^2}=\frac{1}{\mu_1^2}\begin{bmatrix}2\rho_1-(N+1)^2(N+2)\rho_1^{N+1}+\\ N(N+2)(N+3)\rho_1^{N+2}\end{bmatrix}=\alpha_1''$$

$$\frac{\partial\alpha}{\partial\mu_2}=0=\alpha_2',\quad \frac{\partial^2\alpha}{\partial\mu_2^2}=0=\alpha_2'',\quad \frac{\partial\beta}{\partial\mu_2}=0=\beta_2',$$

$$\frac{\partial^2\beta}{\partial\mu_2^2}=0=\beta_2''$$

$$\frac{\partial\beta}{\partial\mu_1}=\frac{1}{\mu_1}\left[\rho_1+(N+1)\rho_1^{N+1}-(N+2)\rho_1^{N+2}\right]=\beta_1'$$

$$\frac{\partial^2\beta}{\partial\mu_1^2}=\frac{1}{\mu_1^2}\left[-2\rho_1-(N+1)(N+2)\rho_1^{N+1}+(N+2)(N+3)\rho_1^{N+2}\right]=\beta_1''$$

$$\frac{\partial\theta}{\partial\mu_1}=\frac{1}{\mu_1}\left[(N+1)\rho_2\rho_1^{N+1}\right]=\theta_1',$$

$$\frac{\partial^2\theta}{\partial\mu_1^2}=\frac{1}{\mu_1^2}\left[-(N+1)(N+2)\rho_2\rho_1^{N+1}\right]=\theta_1''$$

$$\frac{\partial\varphi}{\partial\mu_1}=\frac{1}{\mu_1}\left[\rho_1-\rho_2(N+1)\rho_1^{N+1}\right]=\varphi_1',$$

$$\frac{\partial^2\varphi}{\partial\mu_1^2}=\frac{1}{\mu_1^2}\left[-2\rho_1+(N+1)(N+2)\rho_1^{N+1}\right]=\varphi_1''$$

$$\frac{\partial\varphi}{\partial\mu_2}=\frac{1}{\mu_2}\left[\rho_2(1-\rho_1^{N+1})\right]=\varphi_2',$$

$$\frac{\partial^2\varphi}{\partial\mu_2^2}=\frac{1}{\mu_2^2}\left[-2\rho_2(1-\rho_1^{N+1})\right]=\varphi_2''$$

Further, in view of equation (4), we have

$$\frac{\partial\psi}{\partial\mu_1}=\frac{1}{\mu_1}\begin{bmatrix}-\rho_1+2(N+1)\rho_1^{2(N+1)}+\\ (2N+1)(N+1)\rho_1^{N+1}\\ -(2N+1)(N+2)\rho_1^{N+2}\end{bmatrix}=\psi_1'$$

$$\frac{\partial^2\psi}{\partial\mu_1^2}=\frac{1}{\mu_1^2}\begin{bmatrix}2\rho_1-2(N+1)(2N+3)\rho_1^{2(N+1)}-\\ (2N+1)(N+1)(N+2)\rho_1^{N+1}\\ +(2N+1)(N+2)(N+3)\rho_1^{N+2}\end{bmatrix}=\psi_1''$$

$$\frac{\partial\psi}{\partial\mu_2}=0=\psi_2'\quad(\text{say});\quad \frac{\partial^2\psi}{\partial\mu_2^2}=0=\psi_2''$$

Also,

$$\frac{\partial}{\partial\mu_2}(\theta_1')=-\frac{1}{\mu_1}\frac{1}{\mu_2}(N+1)\rho_1^{N+1}\rho_2=\theta_3',$$

$$\frac{\partial}{\partial\mu_2}(\varphi_1')=\frac{1}{\mu_1}\rho_1+\frac{1}{\mu 2}(N+1)\rho_1^{N+1}\rho_2=\varphi_3'$$

$$\frac{\partial}{\partial\mu_1}(\theta_2')=\frac{1}{\mu_1}\frac{1}{\mu_2}(N+1)(-\rho_1^{N+1}\rho_2)=\theta_4',$$

$$\frac{\partial}{\partial\mu_1}(\varphi_2')=\frac{1}{\mu_1}\frac{1}{\mu_2}(N+1)\rho_1^{N+1}\rho_2=\varphi_4'$$

Now differentiation of equation (5) with respect to μ_1 and μ_2 gives us

$$\frac{\partial T(C)}{\partial\mu_1}=C_1+C_h\left(\frac{\beta\alpha_1'-\alpha\beta_1'}{\beta^2}\right)+ C_l\begin{bmatrix}\dfrac{\varphi\theta_1'-\theta\varphi_1'}{\varphi^2}+\\ \dfrac{\lambda_2}{\mu_1}\left(\dfrac{\psi_1'}{\beta\varphi}-\dfrac{\psi\varphi_1'}{\beta\varphi^2}-\dfrac{\psi\beta_1'}{\beta^2\varphi}\right)-\dfrac{\lambda_2}{\mu_1^2}\dfrac{\psi}{\beta\varphi}\end{bmatrix}=U\ (\text{say}) \tag{6}$$

$$\frac{\partial T(C)}{\partial\mu_2}=C_2+C_l\left[\frac{\varphi\theta_2'-\theta\varphi_2'}{\varphi^2}+\frac{\lambda_2}{\mu_1}\frac{\psi}{\beta}\left(-\frac{\varphi_2'}{\varphi^2}\right)\right] = V\quad(\text{say}) \tag{7}$$

For optimal values of μ_1 and μ_2, we have

$$C_1+C_h\left(\frac{\beta\alpha_1'-\alpha\beta_1'}{\beta^2}\right)+ C_l\begin{bmatrix}\dfrac{\varphi\theta_1'-\theta\varphi_1'}{\varphi^2}+\dfrac{\lambda_2}{\mu_1}\left(\dfrac{\psi_1'}{\beta\varphi}-\dfrac{\psi\varphi_1'}{\beta\varphi^2}-\dfrac{\psi\beta_1'}{\beta^2\varphi}\right)\\ -\dfrac{\lambda_2}{\mu_1^2}\dfrac{\psi}{\beta\varphi}\end{bmatrix}=0 \tag{8}$$

This can easily be expressed as

$$U(\mu_1,\mu_2)=0 \tag{9}$$

$$C_2+C_l\left[\frac{\varphi\theta_2'-\theta\varphi_2'}{\varphi^2}+\frac{\lambda_2}{\mu_1}\frac{\psi}{\beta}\left(-\frac{\varphi_2'}{\varphi^2}\right)\right]=0$$

This is also rewritten as

$$V(\mu_1,\mu_2)=0 \tag{10}$$

Here, equations (9) and (10) construct a system of nonlinear equations which can be solved for μ_1 and μ_2 by developing a computer algorithm and using computer programming in C++. The values of μ_1 and μ_2 obtained in such a way will be optimal. These optimal values of μ_1 and μ_2 after substituting in the expression of total expected cost, will give us total optimal cost of the system. For the solution of above nonlinear system following expressions are required, therefore from equations (6) and (7), we have

$$\frac{\partial U}{\partial\mu_1}=C_h\left[\frac{\beta^2\alpha_1''-2\beta\beta_1'\beta_1'-\beta\alpha\beta_1''+2\alpha\alpha(_1')^2}{\beta^3}\right] +C_l\left[\frac{\varphi^2\theta_1''-2\varphi\varphi_1'\varphi_1'-\varphi\theta\varphi_1''+2\theta\theta(_1')^2}{\varphi^3}\right]+ \begin{pmatrix}\dfrac{\lambda_2}{\mu_1}C_l\dfrac{\beta\varphi\psi_1''-\psi_1'(\beta\beta_1'+\varphi\beta_1')}{(\beta\beta\varphi^2}\\ -\dfrac{\lambda_2}{\mu_1}C_l\dfrac{\beta\varphi^2(\psi\psi_1''+\varphi_1'\psi_1')-\psi\varphi_1'(2\beta2\beta_1'+\varphi^2\beta_1')}{(\beta\beta^2)^2}\\ -\dfrac{\lambda_2}{\mu_1}C_l\dfrac{\beta^2\varphi(\psi\beta_1''+\psi_1'\beta_1')-\psi\beta_1'(\beta^2\varphi_1'+2\beta\beta_1'\varphi)}{(\beta^2\varphi)^2}\end{pmatrix}$$

$$-\frac{\lambda_2}{\mu_1^2}\; C_1\left(\frac{\psi_1'}{\beta\varphi}-\frac{\psi\varphi_1'}{\beta\varphi^2}-\frac{\psi\beta_1'}{\beta^2\varphi}\right)- \qquad (11)$$

$$\frac{\lambda_2}{\mu_1^2}C_1\left(\frac{\beta\varphi\psi_1'-\psi(\beta\varphi_1'+\varphi\beta_1')}{(\beta\beta\varphi^2}\right)+C_1\frac{2\lambda_2}{\mu_1^3}\left(\frac{\psi}{\beta\varphi}\right)$$

$$\frac{\partial U}{\partial \mu_2}=C_1\left[\frac{\varphi^2\theta_3'-\varphi\theta_2'\varphi_2'-\varphi\theta_1'\varphi_2'-\varphi\theta\varphi_3'-\varphi\varphi_1'\theta_2'+2\theta\theta_1'\varphi_2'}{\varphi^3}\right]+$$

$$C_1\left[\begin{matrix}\frac{\lambda_2}{\mu_1}\left(\begin{matrix}\frac{\psi_1'}{\beta}\left(-\frac{\varphi_2'}{\varphi^2}\right)-\frac{\psi}{\beta}\left(\frac{\varphi^2\varphi_3'-2\varphi\varphi_2'\varphi_1'}{\varphi^4}\right)+\\ \frac{\psi\beta_1'}{\beta^2}\left(\frac{\varphi_2'}{\varphi^2}\right)\end{matrix}\right)\\ +\frac{\lambda_2\psi}{\mu_1^2\beta}\left(\frac{\varphi_2'}{\varphi^2}\right)\end{matrix}\right] \qquad (12)$$

$$\frac{\partial V}{\partial \mu_1}=C_1\left[\frac{\varphi^2\theta_4'-\varphi\varphi_1'\theta_2'-\varphi\theta\varphi_4'-\varphi\varphi_2'\theta_1'+2\theta\theta_1'\varphi_2'}{\varphi^3}\right]$$

$$-C_1\frac{\lambda_2}{\mu_1}\frac{\psi}{\beta}\left[\frac{\varphi^2\varphi_4'-2\varphi\varphi_1'\varphi_2'}{\varphi^4}\right] \qquad (13)$$

$$\frac{\partial V}{\partial \mu_2}=C_1\left[\frac{\varphi^2\theta_2''-2\varphi\varphi_2'\varphi_2'-\varphi\theta\varphi_2''-\varphi\theta\varphi_2''+2\theta\theta({}_2')^2}{\varphi^3}\right]$$

$$-C_1\frac{\lambda_2}{\mu_1}\frac{\psi}{\beta}\left[\frac{\varphi^2\varphi_2''-2\varphi\varphi({}_2')^2}{\varphi^4}\right] \qquad (14)$$

According to Newton's method, for the total optimum cost, we obtain

$$\begin{pmatrix}(\mu_1)_{k+1}\\(\mu_2)_{k+1}\end{pmatrix}=\begin{pmatrix}(\mu_1)_k\\(\mu_2)_k\end{pmatrix}-\begin{pmatrix}\frac{\partial^2 T}{\partial\mu_1^2} & \frac{\partial^2 T}{\partial\mu_1\partial\mu_2}\\ \frac{\partial^2 T}{\partial\mu_2\partial\mu_1} & \frac{\partial^2 T}{\partial\mu_2^2}\end{pmatrix}^{-1}\begin{pmatrix}\frac{\partial T}{\partial\mu_1}\\ \frac{\partial T}{\partial\mu_2}\end{pmatrix}$$

where Hessian matrix and gradient on the right-hand side are evaluated at $(\mu_1,\mu_2)=((\mu_1)_k,(\mu_2)_k)$.

This further gives us as
$$\begin{pmatrix}(\mu_1)_{k+1}\\(\mu_2)_{k+1}\end{pmatrix}=\begin{pmatrix}(\mu_1)_k\\(\mu_2)_k\end{pmatrix}-\begin{pmatrix}\frac{\partial U}{\partial\mu_1} & \frac{\partial V}{\partial\mu_1}\\ \frac{\partial U}{\partial\mu_2} & \frac{\partial V}{\partial\mu_2}\end{pmatrix}^{-1}\begin{pmatrix}U\\V\end{pmatrix}$$

which in turn, on solving yields the followings:

$$(\mu_1)_{k+1}=(\mu_1)_k-\frac{U\frac{\partial V}{\partial\mu_2}-V\frac{\partial V}{\partial\mu_1}}{\frac{\partial U}{\partial\mu_1}\frac{\partial V}{\partial\mu_2}-\frac{\partial V}{\partial\mu_1}\frac{\partial U}{\partial\mu_2}},$$

$$(\mu_2)_{k+1}=(\mu_2)_k-\frac{V\frac{\partial U}{\partial\mu_1}-U\frac{\partial U}{\partial\mu_2}}{\frac{\partial U}{\partial\mu_1}\frac{\partial V}{\partial\mu_2}-\frac{\partial V}{\partial\mu_1}\frac{\partial U}{\partial\mu_2}}$$

where terms on the right-hand side are evaluated at $(\mu_1,\mu_2)=((\mu_1)_k,(\mu_2)_k)$.

These quantities are used in computing algorithm to numerically compute the optimal service rates and total optimal cost, which are given in following tables.

IV. COMPUTING ALGORITHM

The following computing algorithm has been developed to compute the optimal service rates and total optimal cost and profit of the system with two priority classes.

Step 1: begin
Step 2: input all variables
Step 3: compute derived variables
Step 4: compute derivatives
Step 5: compute functions
Step 6: t_1 ← initial service rate of HPC
Step 7: t_2 ← initial service rate of LPC
Step 8: iterating initial service rates
Step 9: while (error=0.0000000001)
Step 10: compute optimal service rates
Step 11: compute total optimal cost
Step 12: compute total optimal profit
Step 13: end

V. NUMERICAL DEMONSTRATION OF THE RESULTS

Numerical demonstration is very useful to exhibit the results (performance measures) of the model. It focuses on the sensitivity analysis of one parameter relative to other parameters for determining the direction of future data-input. Parameters for which the model is relatively sensitive would require more attention of researchers engaged in this field, as compared to the parameters for which the model is relatively insensitive or less sensitive. Respective numerical demonstrations are given in the forth coming tables.

VI. PROFIT ANALYSIS OF THE SYSTEM

A. *Profit Evaluation for High Priority Class Customers*

Let R_1 be the earned revenue for providing service to each high priority customer, then total expected revenue (TER) is given as

$$\mathrm{TER}=R_1L_H \qquad (15)$$

Also, corresponding to high priority class customer, total expected cost (TEC) is given as

$$\mathrm{TEC}=C_1\mu_1+C_hL_H \qquad (16)$$

Now, total expected profit (TEP) for high priority class customer is expressed as

$$TEP=TER-TER$$

Thus, from above expressions we have

$$\mathrm{TEP}=R_1L_H-(C_1\mu_1+C_hL_H)=R_1L_H-C_1\mu_1-C_hL_H$$
$$=(R_1-C_h)L_H-C_1\mu_1$$

$$\mathrm{TEP}=(R_1-C_h)\left(\frac{\alpha}{\beta}\right)-C_1\mu_1=T(P_H)\quad \text{(say)} \qquad (17)$$

B. *Profit Evaluation for Low Priority Class Customers*

Let R_2 be the earned revenue for providing service to each low priority customer, then total expected revenue (TER) is given as-

$$\mathrm{TER}=R_2L_L \qquad (18)$$

Also, corresponding to low priority class customer total expected cost (TEC) is given as

$$TEC = C_2\mu_2 + C_l L_L \quad (19)$$

Now, total expected profit (TEP) for high-priority class customer is expressed as $TEP = TER - TER$

Thus, from above expressions we have

$$TEP = R_2 L_L - (C_2\mu_2 + C_l L_L) = R_2 L_L - C_2\mu_2 - C_l L_L = (R_2 - C_l)L_L - C_2\mu_2 \quad (20)$$

$$TEP = (R_2 - C_l)\left(\frac{\theta}{\varphi} + \frac{\lambda_2}{\mu_1}\frac{\psi}{\beta\varphi}\right) - C_2\mu_2 = T(P_L) \quad \text{(say)}$$

VII. TOTAL EXPECTED PROFIT (TEP) OF THE SYSTEM

In the light of equations (17) & (20), we can easily find the total expected profit of the system,

$$T(P_S) = T(P_H) + T(P_L) = (R_1 - C_h)\left(\frac{\alpha}{\beta}\right) - C_1\mu_1 + (R_2 - C_l)\left(\frac{\theta}{\varphi} + \frac{\lambda_2}{\mu_1}\frac{\psi}{\beta\varphi}\right) - C_2\mu_2 = T(P_S) \quad \text{(say)} \quad (21)$$

VIII. OBSERVATIONS

We draw the following observations on the basis of the optimum values computed for the performance measures of the system.

- When service cost associated with high priority class customers increases, total optimal cost (TOC) of the system also increases.
- Increment in service cost associated with low priority class customers; amounts to increase the total optimal cost (TOC) of the system.
- When revenue per unit customer of both priority classes increases, total expected profit of high and low priorities customers also increase.
- A slight decrement in the total expected profit (TEP) of the system is shown when holding cost associated with low priority class (LPC) and high priority class (HPC) customers increase.
- Further it is indicated that when holding cost per unit customer of both priority classes increases, total expected profit of high and low priorities customers decrease.
- When holding cost associated with high priority class customers increases, a slight decrement in the total optimal cost (TOC) of the system is observed.
- A slight decrement in the total optimal cost (TOC) of the system is observed when holding cost associated with low priority class customers increases.
- We also observed that when revenue per unit customer of both priority classes increases, total expected profit of the system also increases.

IX. CONCLUSION

In this paper, an attempt has been made to present cost analysis along with profit analysis of a single server Markovian queuing system with two priority classes. We define total expected cost (TEC) of the system with both priority classes. Since, such systems are often encountered in practice, particularly in service-oriented operations, therefore we have attempted to optimize the total expected cost (TEC) of the system with respect to service rates of both classes and find out the optimal service rates for high and low priority classes and TOC of the system. Evaluation of TEP of both priority classes and of whole system has been carried out which brings the efficacy of the model closer to a realistic situation. The aim of the numerical demonstration is to study the variability of the model that is, to assess the effect of one parameter on the others especially such parameters which characterize the performance measures of the model. Numerical demonstration carried out with the help of search program is mainly based on the simulations or hypothetical data-input. In this paper, we have preferred the hypothetical data-input to run the search program developed in the paper, which at later stage can also be tested for any real case study. It has good deal of potential to the applications in various areas such as inventory management, production management, voting management of electoral system, computer system and Telecommunications etc.

TABLE I
COMPUTATION OF TOTAL OPTIMAL COST
(λ_1=7, λ_2=9 and N=100)

C_1	C_2	C_h	C_l	μ_1^*	μ_2^*	TOC
15	13	20	24	10.3	12.2	298.0
17	13	20	24	10.3	13.3	333.4
19	13	20	24	10.3	14.4	369.0
21	13	20	24	10.3	15.5	404.7
23	13	20	24	10.4	16.6	440.5
15	15	20	24	10.3	11.9	320.0
15	17	20	24	10.4	11.7	341.1
15	19	20	24	10.4	11.5	361.3
15	21	20	24	10.5	11.3	380.6
15	13	22	24	10.2	11.3	291.6
15	13	24	24	10.2	10.5	285.1
15	13	26	24	10.2	9.7	278.5
15	13	28	24	10.2	8.8	271.9
15	13	20	26	10.2	12.3	293.7
15	13	20	28	10.2	12.4	289.4
15	13	20	30	10.2	12.5	285.0
15	13	20	32	10.2	12.5	280.5
15	13	20	34	10.1	12.6	276.0

TABLE II
COMPUTATION OF TOTAL EXPECTED PROFIT FOR HPC
(μ_1=7, λ_1=5, N=100)

C_h	C_1	R_1	$T(P_H)$
15	25	200	17862.5
17	25	200	17667.5
19	25	200	17422.5
21	25	200	17277.5
15	27	200	17848.5
15	29	200	17834.5
15	31	200	17820.5
15	33	200	17806.5
15	25	250	22737.5
15	25	300	27612.5
15	25	350	32487.5
15	25	450	37362.5
15	25	500	42237.5

TABLE III
COMPUTATION OF TOTAL EXPECTED PROFIT FOR LPC
(μ_1=5, μ_2=4, λ_1=10, λ_2 =12, N=100)

C_2	C_l	R_2	$T(P_L)$
15	10	700	1596.00
17	10	700	1588.00
19	10	700	1580.00
21	10	700	1572.00
23	10	700	1564.00
15	12	700	1591.20
15	14	700	1586.40
15	16	700	1581.60
15	18	700	1576.80
15	10	750	1716.00
15	10	800	1836.00
15	10	850	1956.00

TABLE IV
COMPUTATION OF TOTAL EXPECTED PROFIT
OF THE SYSTEM
(μ_1=5, μ_2=4, λ_1=10, λ_2 =12, N=100)

C_1	C_2	C_h	C_l	R_1	R_2	$T(P_H)$	$T(P_L)$	$T(P_S)$
25	15	15	10	700	1000	560	2316	2876
28	15	15	10	700	1000	545	2316	2861
31	15	15	10	700	1000	530	2316	2846
34	15	15	10	700	1000	515	2316	2831
37	15	15	10	700	1000	500	2316	2816
25	18	15	10	700	1000	560	2304	2864
25	21	15	10	700	1000	560	2292	2852
25	24	15	10	700	1000	560	2280	2840
25	27	15	10	700	1000	560	2268	2828
25	15	18	10	700	1000	557	2316	2873
25	15	21	10	700	1000	554	2316	2870
25	15	24	10	700	1000	551	2316	2867
25	15	27	10	700	1000	548	2316	2864
25	15	15	13	700	1000	560	2308	2868
25	15	15	16	700	1000	560	2301	2861
25	15	15	19	700	1000	560	2294	2854
25	15	15	21	700	1000	560	2287	2847
25	15	15	10	750	1000	610	2316	2926
25	15	15	10	800	1000	660	2316	2976
25	15	15	10	850	1000	710	2316	3026
25	15	15	10	700	1050	560	2436	2996
25	15	15	10	700	1100	560	2556	3116
25	15	15	10	700	1150	560	2676	3236

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