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Chaos Theory and Application in Foreign Exchange Rates vs. IRR (Iranian Rial)

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***Abstract*—**Daily production of information and importance of the sequence of produced data in forecasting future performance of market causes analysis of data behavior to become a problem of analyzing time series. But time series that are very complicated, usually are random and as a result their changes considered being unpredictable. While these series might be products of a deterministic dynamical and nonlinear process (chaotic) and as a result be predictable. Point of Chaotic theory view, complicated systems have only chaotically face and as a result they seem to be unregulated and random, but it is possible that they abide by a specified math formula.

In this article, with regard to test of strange attractor and biggest Lyapunov exponent probability of chaos on several foreign exchange rates vs. IRR (Iranian Rial) has been investigated. Results show that data in this market have complex chaotic behavior with big degree of freedom.

Keywords—Chaos, Exchange Rate, Nonlinear Models.

I. INTRODUCTION

CHAOS theory was introduced and applied by Edward Lorenz in 1965 for the first time in meteorology [1]. Nowadays this science is the milestone for fundamental changes in sciences especially meteorology, astrology, mechanics, physics, mathematics, biology, economics, statistic and management. Even though chaos theory in recent decades has been part of survey in miscellaneous scientific fields, but its basic concept has its root in primitive humans understanding of the universe. The Greek word of chaos that is translated to disorder and lawlessness shows ancient Greek understands of the universe. According to this view of point, although world's entities seem to be chaotic and random and as a result unpredictable, but at the same time they are in order and deterministic.

If we cut one chaotic system in a specific time, we will be faced with chaos and unpredictability. While we have a process view point and look the development of chaotic system in an adequate time period, then after we can discern a fixed degree of order in chaos. Chaotic process is a product of a dynamic non linear system. Such systems have been noticed in nature and also in human behaviors. For example heart pulses, clock's oscillating movement like a pendulum and economic fluctuations all show a dynamic non linear behavior. Based on this theory, happening in world are such dynamic and complicated that they sound to be chaotic, but in effect, chaotic systems have fundamental and fixed order. Although identifying this covert order is not impossible but it is difficult. Because many parameters and factors in dynamic and unpredictable dealing, form phenomena's behavior and make typical future behavior [2,3].

In this article, we have tried to do a quick review of basis concept and mathematical, furthermore different methods of chaotic test to be introduced and assessed. In second part we will introduce. Parts III, has been allocated to henon chaotic model. Parts IV, discusses chaotic tests. In part V foreign exchange rate has been discussed. In section VI we have done a case study of Iran exchange rate and finally in part VII we have culminated above mentioned discussion.

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II. ECONOMIC DYNAMICS

Historically, economists have, whenever possible, used linear equations to model economic phenomena, because they are easy to manipulate and usually yield unique solutions. However, as the mathematical and statistical tools available to economists have become more sophisticated, it has become impossible to ignore the fact that many important and interesting phenomena are not amenable to such treatment.

There is a strong support in economics for both the significance of linear models, and the advantages of nonlinear models. But nonlinear models clearly outperform linear models. Clearly the economic world is nonlinear, so it would appear that focusing on linear dynamics is of limited interest. However economists have typically found nonlinear models to be so difficult and intractable that they have adopted the technique of linearization to deal with them.

Important phenomena for which linear models are not appropriate include depressions and recessionary periods, stock market price bubbles and corresponding crashes, persistent exchange rate movements and the occurrence of regular and irregular business cycles. Therefore, economic theorists are turning to the study of non-linear dynamics and chaos theory as possible tools to model these and other phenomena [4].

The most exciting feature of nonlinear systems is their ability to display chaotic dynamics. Much economic data has this random-like behavior, but it comes from agents and markets that are presumably rational and deterministic. Random-like data that economists often encounter might not be coming from a random system. The generating system could be deterministic and perhaps the economy can be explained by a relatively simple nonlinear system.

Chaos is widely found in the fields of physics and other natural sciences. However, the existence of chaos in economic data is still an open question. Since the mid eighties several economists have tried to test for nonlinearity and in particular for chaos in economic and financial time series [5]. One route toward finding a nonlinear underlying system in the economy would be to show that the data itself demonstrates nonlinear or chaotic properties Researchers developed tests for chaos and nonlinearity in data. There are two major classes of tests for chaos within data. The first is ways to look at the paths or trajectories of the data when the system's initial conditions are adjusted slightly. This can be done by estimating a Lyaponuv exponent.

The Lyaponuv exponents are a measure of the average divergence (or convergence) between experimental data trajectories generated by systems with infinitesimally small changes in their initial conditions. If the data points deviate exponentially when there is a very small tweak on a deterministic model, it will have a positive Lyaponuv exponent. If the paths converge back to a steady state, then the Lyaponuv exponent will be negative. A positive exponent signals that the system must have sensitive dependence to initial conditions and therefore it is chaotic [6]. The second type of test for chaos examines the dimensionality of the system. It may seem easy to explain that a square has two dimensions, and a line has one, but it is significantly more complicated for chaotic systems since they have non-integer dimensionality. The "fractional" dimensionality is what coined the term "fractal" for shapes generated by chaotic data. Dimensionality analysis becomes extremely complicated with realistic chaotic systems. Chaos exists in many different fractal dimensions. Unfortunately, the analysis gets more and more complex the larger the fractal dimension being searched in, and the tests for chaos become weaker. Unless data displays low-dimensional chaos, it may be undetectable to current tests. This is a significant obstacle to chaotic-economic theory, and one of the main reasons the literature has not reached a consensus on the existence of chaotic dynamics in data [7].

III. Henon Chaotic Model

Chaotic series can be considered as a complex of non-linear processes that make irregular and very complicated results. One of chaotic map is Henon model. This model is 2 variable, non linear and quadratic. Relations of this model are as follows:

$$X_{n+1} = 1 - aX_n^2 + Y_n \qquad (1)$$
$$Y_{n+1} = bX_n$$

That in chaotic region a=1.4 and b=0.3. Primary conditions of is x_0=0.18, x_1=0.63. In Figs. 1 and 2, relation of variable in one dimension and two dimension spaces has been shown [2].

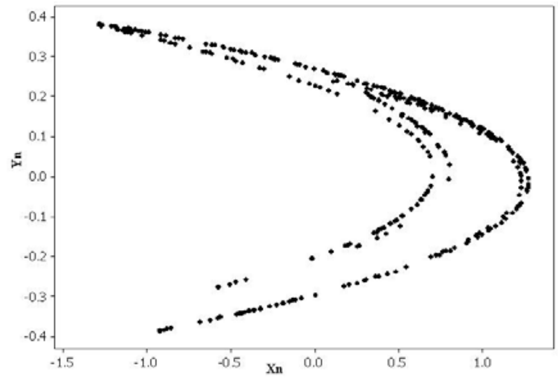

Fig. 1 One dimensional Henon map

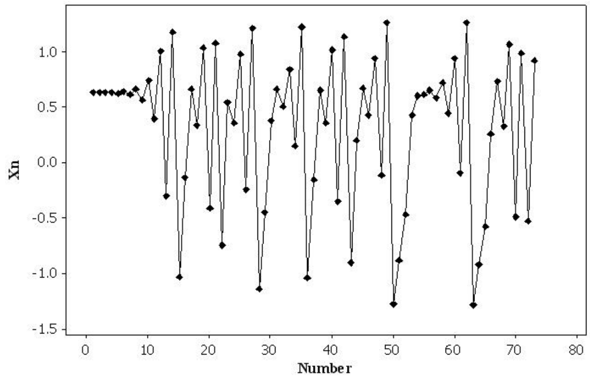

Fig. 2 Two dimensional Henon map

IV. Choatic Tests

Generally, 2 view points have been introduced for evaluation of situation of complicated time series. In first standpoint, this issue is examined whether the time series in question have been made by a definite process or random. In 2nd viewpoint it is tried to recognize whether the time series is a result of a chaotic behavior or a non chaotic behavior. Methods that are utilized in 1st view point are based on analysis of system's correlation dimension. Methods pertinent to 2nd viewpoint are mostly including biggest lyapunov exponent.

A. Attractor Dimension Test

This test is based on one of the special specifications of random process in comparison with chaotic process. Random processes include unlimited dimensions. But a chaotic process has more limited dimensions. It means that it includes a complex of points that time series would result in them. Therefore by calculating dimensions of a series, it is possible to understand its making process. According to this method if series domain was high, it would show a random process, otherwise, it would be a chaotic process. Attractor dimension by using a variable called integration correlation that was introduced by procaccia and grassberger in 1983 is calculated as follows [7].

Dimension is nominated as low bound of necessary independent variable's quantity for describing the model. Attractor is the developed concept of all equilibrium paths in phase space like equilibrium points and limit circles in stable systems that have got correct dimension. In contrast, chaotic systems attractor has fractal dimension and are called strange attractor. In most primitive method of designating of fractal dimension, M(L) are considered as quantity of ultra cubes with the dimension of M and length of the line "L" that covers attractor, based on this:

$$M(l) \sim l^{-D} \tag{2}$$

And "D" that is fractal dimension is obtained:

$$D = \lim_{l \to 0} \frac{-\log[M(l)]}{\log l} \tag{3}$$

According to definition, point, line and plate in 2 dimension space have dimensions of 0,1 and 2 respectively. This definition of fractal has practical limits and solely attractor's geometrical structure has been considered in it.

Correlation dimension is the most common estimation of attractor dimension that is simply calculated by procaccia and grassberger method. Based on this method, vectors "m" are part of X_i condition of X(t) time series that are made with the length of "N".

$$X_i = [x(t_i), \quad x(t_i + 1) \quad \cdots \quad x(t_i + m)] \tag{4}$$

Correlation integral for "N" vector with the distance less than "r" from each other is calculated like this:

$$C(r,m) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \left\| X_i - X_j \right\|\right) \tag{5}$$

In which C(r,m) is an estimation of a probability that 2 vectors of time series with length of N, have a distance less than "r" from each other. "I" also is a function of heavy side and is defined as below:

$$I(x,y) = \begin{cases} 0 & \|x-y\| > r \\ 1 & \|x-y\| < r \end{cases} \tag{6}$$

In case quantities of “N” points are big enough, distribution is Exponential function

$$C(r;m) \sim r^{\upsilon} \tag{7}$$

That υ is correlation dimension

$$\upsilon = \lim_{r \to 0} \frac{\log C(r,m)}{\log r} \tag{8}$$

This test is based on this fact that chaotic maps do not fill the space in big dimensions, but random data are not like this. When the chaotic process is more complicated, it is necessary that data should be considered in bigger and higher dimensions. A chaotic process can fill a space with "n" dimensions but it leaves big holes in "n+1" dimension. It is clear that this method is not practical in big dimension graphically.

B. Test of Lyapanuv's Biggest Exponent

lyapanuv's exponent has been recognized as one the most suitable methods for recognition of dynamic processes nowadays. Lyopanuv's exponent is an average exponent of dynamic processes and shows rate of divergence and convergence of condition routes in phase space. Divergence of condition route shows that a system with slight differences in primary conditions, with elapsing of time has very different condition routes from each other and predictability capability in these kinds of processes disappears quickly. According to definition, each system by having at least a positive lyapanuv's exponent is a chaotic system. Conversely the size of related exponent is common fitted with a time that after it, dynamical process would become unpredictable [8,9].

Level of being chaotic of time series is measured based on lyapanuv's exponent. This measure states that with alterations in primary conditions or model parameters, produced series up to which level differs from original series. Lyapanuv's exponent shows certainty of short term time series and therefore predictability of series. In other words this exponent shows level of being chaotic of a series. High quantities of this exponent show high sensitivity of series to primary quantities. If difference of primary quantities is a known quantity, difference of series quantity after certain quantity of stage, is equal to exponential function these quantities. The less the quantity of this exponent, the less is the growth of this exponential. Lyapanuv's biggest exponent test is one of the most important methods for recognizing chaos in time series. In order to calculate lyapanuv's exponent, the vectors contain “m” components are used.

$$X_i = [x(t_i), \quad x(t_i + 1) \quad \cdots \quad x(t_i + m)] \tag{9}$$

From vectors with distance less than "r" it is calculated as per follows:

$$r_0(m;i,j) = \left\| X_i^m - X_j^m \right\| \le r \tag{10}$$

The below term is calculated:

$$d_n(m;i,j) = \frac{\left\| X_{i+n}^m - X_{j+n}^m \right\|}{r_0(m;i,j)} \tag{11}$$

Than after, lyapanuv's biggest exponent is calculated:

$$L_e(m,n) = \sum_{i \ne j} \frac{\log[d_n(m;i,j)]}{N(N-1)} \tag{12}$$

"Le" sign shows entity of time series in question. Positive amount of "Le" shows chaotic ness of process and difficulty of predictability and when it is negative, it shows that process in the long run is non chaotic and predicable. If "Le" moves towards positive quantity near zero, chaotic system is weak and middle term predictability is possible [10].

C. Test Result on Henon Map

As it was mentioned, Henon map is one of the chaotic maps. This model complies with following certain behavior pattern.

With application of attractor's dimension and lyopanuv's biggest exponent tests on Henon map, following results were obtained. With the application of attractor's dimension test on Henon map this quantity proved to be equal to 1.21. In Table I, lyopanuv's test results have been calculated in restructured dynamics of 1 to 10 dimensional developed vectors.

TABLE I
LYOPANUV’S BIGGEST EXPONENT TEST RESULTS ON HENON MAP

	Max
D=1	4.93279
D=2	0.30935
D=3	0.25394
D=4	0.17964
D=5	0.09318
D=6	0.05379
D=7	0.03152
D=8	0.02073
D=9	0.01433
D=10	0.00888

V. Foreign Exchange Rate

An exchange rate represents the value of one currency in another. An exchange rate between two currencies fluctuates over time.

The foreign exchange market is the largest and most liquid of the financial markets. Foreign exchange rates are amongst the most important economic indices in the international monetary markets. The forecasting of them poses many theoretical and experimental challenges.

Foreign exchange rates are affected by many highly correlated economic, political and even psychological factors. The interaction of these factors is in a very complex fashion. Therefore, to forecast the changes of foreign exchange rates is generally very difficult. Researchers and practitioners have been striving for an explanation of the movement of exchange rates. Thus, various kinds of forecasting methods have been developed by many researchers and experts. Technical and fundamental analyses are the basic and major forecasting methodologies which are in popular use in financial forecasting. Like many other economic time series, forex has its own trend, cycle, season, and irregularity. Thus to identify, model, extrapolate and recombine these patterns and to give forex forecasting is the major challenge.

Foreign exchange rates were only determined by the balance of payments at the very beginning. The balance of payments was merely a way of listing receipts and payments in international transactions for a country. Payments involve a supply of the domestic currency and a demand for foreign currencies. Receipts involve a demand for the domestic currency and a supply of foreign currencies. The balance was determined mainly by the import and export of goods. Thus, the prediction of the exchange rates was not very difficult at that time. Unfortunately, interest rates and other demand} supply factors had become more relevant to each currency later on. On top of this the fixed foreign exchange rates was abandoned and a floating exchange rate system was implemented by industrialized countries in 1973. Recently, proposals towards further liberalization of trades are discussed in General Agreement on Trade and Tariffs. Increased Forex trading, and hence speculation due to liquidity and bonds, had also contributed to the difficulty of forecasting Forex [10].

Generally, there are three schools of thought in terms of the ability to profit from the financial market. The first school believes that no investor can achieve above average trading advantages based on the historical and present information. The major theory includes the Random Walk Hypothesis and Efficient Market Hypothesis. The second school's view is that of fundamental analysis. It looks in depth at the financial condition of each country and studies the effects of supply and demand on each currency.

Technical analysis belongs to the third school of thought who assumes that the exchange rates move in trends and these trends can be captured and used for forecasting. It uses such tools as charting patterns, technical indicators and specialized techniques like Gann lines, Elliot waves and Fibonacci series [11].

VI. Case Study

This method is applied on some foreign exchange rates data of day time slice sampled vs. 10RIAL, since March 25, 2002 to May 23, 2007. Results have shown in Table II.

TABLE II
STRANGE ATTRACTOR AND LYOPANUV'S BIGGEST EXPONENT TEST RESULTS

USD vs. 10RIAL

	D=1	D=2	D=3	D=4	D=5
Max	1.487592	9.671165	0.070811	0.105881	0.015348

	D=6	D=7	D=8	D=9	D=10
Max	0.015271	0.014686	0.014999	0.014799	0.013388

GBP vs. 10RIAL

	D=1	D=2	D=3	D=4	D=5
Max	11.90218	0.85443	0.172787	0.073399	0.037273

	D=6	D=7	D=8	D=9	D=10
Max	0.023838	0.014647	0.008689	0.005283	0.003755

EUR vs. 10RIAL

	D=1	D=2	D=3	D=4	D=5
Max	9.841619	0.729692	0.138067	0.053923	0.028326

	D=6	D=7	D=8	D=9	D=10
Max	0.016893	0.010379	0.008314	0.005841	0.004454

CAD vs. 10RIAL

	D=1	D=2	D=3	D=4	D=5
Max	11.66948	1.625273	0.190042	0.07603	0.039092

	D=6	D=7	D=8	D=9	D=10
Max	0.028777	0.014759	0.010711	0.009203	0.007213

CHF vs. 10RIAL

	D=1	D=2	D=3	D=4	D=5
Max	12.63066	0.866155	0.160869	0.076702	0.038524

	D=6	D=7	D=8	D=9	D=10
Max	0.024289	0.015484	0.01373	0.011291	0.010453

Strange Attractor

USD	GBP	EUR	CAD	CHF
8.3	9.84	9.56	9.28	8.84

VII. Conclusion

Progresses in calculation tools in recent decades have provided us with the possibility of utilizing theories based on existence of certain or chaotic non-linear patterns. Chaotic theory with more through study of specifications of complicated behavior and data that seem to be random, try to recognize order and pattern governing them and use them for predictability future trend in short term. Nowadays this knowledge with the help of data behavior analysis has provided the base of structural changes in future performance prediction.

In this article with reviewing the concepts of this theory and testing for knowing chaos existence, we have examined a case study. Results obtained from this study shows existence of complex chaotic behavior in foreign exchange rate market in Iran. We can say data have big degree freedom in their behavior and shows random like behavior.

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