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Convective Heat Transfer Enhancement in an Enclosure with Fin Utilizing Nano Fluids

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***Abstract*—**The objective of the present work is to conduct investigations leading to a more complete explanation of single phase natural convective heat transfer in an enclosure with fin utilizing nano fluids. The nano fluid used, which is composed of Aluminum oxide nano particles in suspension of Ethylene glycol, is provided at various volume fractions. The study is carried out numerically for a range of Rayleigh numbers, fin heights and aspect ratio. The flow and temperature distributions are taken to be two-dimensional. Regions with the same velocity and temperature distributions are identified as symmetry of sections. One half of such a rectangular region is chosen as the computational domain taking into account the symmetry about the fin. Transport equations are modeled by a stream function-vorticity formulation and are solved numerically by finite-difference schemes. Comparisons with previously published works on the basis of special cases are done. Results are presented in the form of streamline, vector and isotherm plots as well as the variation of local Nusselt number along the fin under different conditions.

***Keywords*—**Fin height, Nano fluid, natural convection, Rayleigh number.

I. INTRODUCTION

THE augmentation of heat transfer in the field of Nanotechnology is considered to be one of the most important inventions in this century. Convective heat transfer in a finned enclosure with adiabatic horizontal walls and isothermal vertical walls are a prototype of many industrial applications, such as electronic cooling, transportation, the environment and national security. Nano fluids exhibit superior heat transfer properties in comparison with conventional heat transfer fluids. In the literature, two main approaches have been adopted to find the heat transfer enhancement by small solid particles suspended in a fluid. The first approach is the two-phase model, which enables a better understanding of both the fluid and the solid phases role in the heat transfer process. The second approach is the single-phase model in which both the fluid phase and the particles are in thermal equilibrium state, and is used in the present study. It refers to a two phase mixture that is composed of saturated liquid and extremely fine metallic particles called nano particles. The suspended ultra fine particles change transport properties and heat transfer performance. The thermal conductivity is strongly dependent on the volume fraction and properties of nano fluids.

In the literature, various studies have been published on the mechanism of natural convection in differentially heated shallow enclosures with various wall conditions. Acharya and Goldstein [1] have studied numerically the natural convection heat transfer in an externally heated vertical or inclined square enclosure containing uniformly distributed internal energy sources. Results were obtained at lower Rayleigh numbers, and no flow separation in front of partial divider was noted. Acharya and Jetli [2] have numerically investigated the heat transfer and flow patterns in a partially divided, differentially heated square enclosure. Rayleigh numbers studied were in the range of 10^5- 10^6. The flow was weak in this stratified region and a tendency for flow separation behind the divider was noted**.** Scozia et al. [3] have studied the natural convection in a differentially heated slender rectangular cavity ($A_r = 20$) with multiple conducting fins on the cold wall for Rayleigh numbers 10^3 to 10^5. As the inter-fin aspect ratio was varied from 20 to 0.25, the flow patterns evolved considerably and the average Nusselt number exhibited maximum and minimum values whose locations depended on the value of Ra. Facas [4] have numerically studied the natural convection inside an air filled slender cavity with fins attached along both the heated and cooled sides of the cavity. Multi cellular flow structure was observed for the fin length of 0.1. However, for longer fin lengths, the flow broke down into secondary recirculation and resulted in higher heat transfer rate. Lakhal et al. [5] have numerically studied the natural convection heat transfer in an inclined rectangular enclosure with perfectly conducting fins attached to the heated wall. They found that the heat losses through the wall can be reduced considerably by using fins attached on the heated wall. Keblinsky et al. [6] have demonstrated that the thermal conductivity increases of nanofluids are due to Brownian motion of particles, molecular-level layering of the liquid at the liquid/ particle interface, the nature of heat transport in the nano particles, and the effect of nano particle clustering. Shi et al. [7] have investigated the laminar natural convection heat transfer in a differentially heated square cavity with fin on the hot wall for Rayleigh numbers 10^4 to 10^7. It showed that for high Rayleigh numbers the flow field was enhanced regardless of the fin's length and position. By placing fin on the lower half of the left wall significant energy can be routed through the fin. Khanafer et

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al. [8] was the first to investigate the problem of buoyancy-driven heat transfer enhancement of nanofluids in a two-dimensional enclosure. The results illustrated that the nanofluid heat transfer rate increases with an increase in the nano particle volume fraction. The authors used Khanafer's expression for the effective thermal conductivity of two component mixtures as a function of liquid and solid.

The present paper examines the effect of fin at different volume fractions on the enclosure heat transfer and flow patterns. From the literature review it is clear that the case of an enclosure with fin utilizing nano fluids has not been addressed. This problem may be encountered in a number of electronic cooling and MEMS applications. The second objective of the paper is to provide a more unifying picture of the various pertinent parameters on heat transfer characteristics of nano fluids within the enclosure.

II. Mathematical Model

A schematic of the two-dimensional system with geometrical and boundary conditions is shown in Fig. 1. In the present analysis, Cartesian coordinate system will be applied to the enclosure. The nano fluid in the enclosure is Newtonian, incompressible, and laminar and is assumed to have uniform shape and size. The nano fluid used, which is composed of Aluminum oxide nano particles in suspension of Ethylene glycol, has been provided at various particle concentrations ranging from 0 to 30% in volume. Moreover, it is assumed that both fluid phase and nano particles are in thermal equilibrium state and they flow at the same velocity.

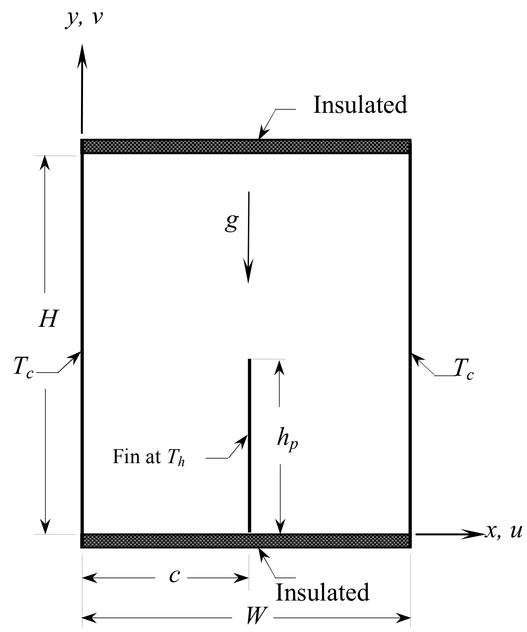


Fig. 1 Schematic for the physical model

The compressibility effects and viscous dissipation in the energy equation are neglected. The Boussinesq approximation is assumed to be valid. Fig. 2 depicts the computational domain indicating thereon the most appropriate boundary conditions relevant to the present study.

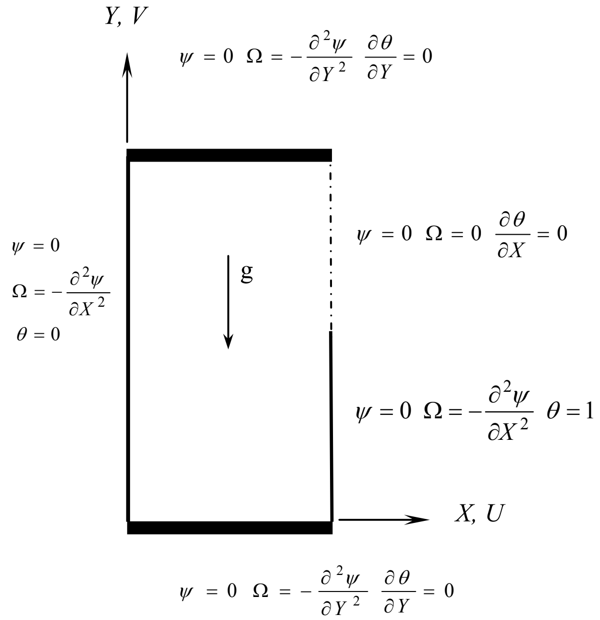


Fig. 2 Computational domain

Based upon the previous assumptions and introducing the following dimensionless variables,

$$X = \frac{x}{H} \qquad Y = \frac{y}{H} \qquad U = \frac{u}{u_c}$$

$$V = \frac{v}{u_c} \qquad \theta = \frac{T - T_c}{T_h - T_c} \qquad \tau = \frac{t\,u_c}{H}$$

$$A_r = \frac{H}{W} \qquad Gr = \frac{\beta g (T_h - T_c) H^3}{\nu^2}$$

$$\Pr = \frac{\mu Cp}{k_f} \qquad H_p = \frac{h_p}{H}$$

$$\alpha_{nf} = \frac{(k_{eff})_{stagnant}}{(\rho Cp)_{nf}}$$

The effective density of a fluid containing suspended particles at a reference temperature is given by:

$$\rho_{nf,o} = (1-\phi)\,\rho_{f,o} + \phi\,\rho_{s,o}$$

The effective viscosity of a fluid containing dilute a dilute suspension of small rigid spherical particles is given by Brinkman [9] as:

$$\mu_{eff} = \frac{\mu_f}{(1-\phi)^{2.5}}$$

The heat capacitance of the nano fluid can be calculated as

$$(\rho Cp)_{nf} = (1-\phi)(\rho Cp)_f + \phi(\rho Cp)_s$$
$$k_{eff} = (k_{eff})_{stagnant} + k_d$$

$$k_d = C(\rho Cp)_{nf}\left|\overline{V}\right|\phi d_p;\ d_p=10\text{nm}$$

$$\chi = \frac{\left[\frac{(k_{eff})_{stagnant}}{k_f}\right]}{(1-\phi)+\phi\frac{(\rho Cp)_s}{(\rho Cp)_f}} + C\phi\frac{d_p}{H}\Pr\sqrt{Gr}\sqrt{U^2+V^2}$$

Where $u_c = \sqrt{g\beta H(T_h - T_c)}$

The governing equations for the problem in dimensionless form are as follows:

Continuity:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \tag{1}$$

X – Momentum:

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = \frac{\partial P}{\partial X} + \frac{\mu_{eff}}{\rho_{nf,o}}\left(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}\right) \tag{2}$$

Y – Momentum:

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = \frac{\partial P}{\partial Y} + \frac{\mu_{eff}}{\rho_{nf,o}}\left(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}\right) + \frac{1}{\rho_{nf,o}}\left[\phi\rho_{s,o}\beta_s + (1-\phi)\rho_{f,o}\beta_f\right]g\theta \tag{3}$$

Energy:

$$U\frac{\partial\theta}{\partial X} + V\frac{\partial\theta}{\partial Y} = \frac{1}{\Pr\sqrt{Gr}}\left[\frac{\partial}{\partial X}\left(\chi\frac{\partial\theta}{\partial X}\right) + \frac{\partial}{\partial Y}\left(\chi\frac{\partial\theta}{\partial Y}\right)\right] \tag{4}$$

Stream function-Vorticity formulation:

Let $U = \frac{\partial\psi}{\partial Y}$, $V = -\frac{\partial\psi}{\partial X}$ and $\Omega = \frac{\partial V}{\partial X} - \frac{\partial U}{\partial Y}$

Stream function equation

$$\frac{\partial^2\psi}{\partial X^2} + \frac{\partial^2\psi}{\partial Y^2} + \Omega = 0 \tag{5}$$

Vorticity Transport equation

$$\frac{\partial\Omega}{\partial\tau} + U\frac{\partial\Omega}{\partial X} + V\frac{\partial\Omega}{\partial Y} = \frac{a1}{\sqrt{Gr}}\left(\frac{\partial^2\Omega}{\partial X^2} + \frac{\partial^2\Omega}{\partial Y^2}\right) + \lambda\frac{\partial\theta}{\partial X} \tag{6}$$

Where $$a1 = \frac{1}{(1-\phi)^{2.5}\left[\phi\frac{\rho_{s,o}}{\rho_{f,o}} + (1-\phi)\right]} \tag{7}$$

$$\lambda = \frac{1}{1+\frac{1-\phi}{\phi}\frac{\rho_{f,o}}{\rho_{s,o}}}\frac{\beta_s}{\beta_f} + \frac{1}{1+\frac{\phi}{1-\phi}\frac{\rho_{s,o}}{\rho_{f,o}}} \tag{8}$$

$$\because\ \frac{d}{dX}(3) - \frac{d}{dY}(2) \quad \Rightarrow (6)$$

III. Numerical Technique

The governing equations (4)-(6) were discretized using second order accurate finite difference schemes resulting in a system of linear algebraic equations. The coefficient matrix associated with these equations possesses pentadiagonal structure. However employing Gauss-Seidel line-by line method [10] of iterative solution procedure and using famous Thomas Algorithm can easily solve these equations. The numerical results presented and discussed in the succeeding section are obtained using a computer code developed exclusively for the present investigation. A converged steady state solution was obtained by iterating in time until variations in the variables between subsequent time steps were: $\left|\Lambda_{i,j}^{old} - \Lambda_{i,j}\right| \angle 10^{-6}$, where Λ stands for ψ, Ω and θ. Numerical experiments with different grid sizes, which correspond to 21x21, 41x41 and 81x81, are conducted to ascertain an optimum grid size. On the basis of this experiment, the grid size corresponding to 81x81 is found to be an optimum one. To test the computer code developed in this study, the problem of buoyancy-driven flow in a square enclosure that has different heated vertical walls and adiabatic upper and bottom walls is studied [11]. A good agreement is obtained between the present solution and the previous works as illustrated in Table I.

The Nusselt number [12] are averaged and evaluated along the fin which may be expressed as

$$Nu_{avg} = \int_2^{Np} -\frac{(k_{eff})_{stagnant}}{k_f}\left.\frac{\partial\theta}{\partial X}\right|_{X=\frac{1}{2A_r}} dY$$

TABLE I
COMPARISON OF VALUES WITH PREVIOUS WORKS FOR DIFFERENT RA VALUES

	Present study	Barakos and Mitsoulis [13]	G De Vahl Davis[11]	Khanafer et al.[8]
Ra = 10^3				
Nu_{avg}	1.152	1.114	1.118	1.118
U_{max} (at Y)	0.136 (0.825)	0.153 (0.806)	0.136 (0.813)	0.137 (0.812)
V_{max} (at X)	0.139 (0.175)	0.155 (0.181)	0.138 (0.178)	0.139 (0.173)
Ra = 10^4				
Nu_{avg}	2.304	2.245	2.243	2.245
U_{max} (at Y)	0.193 (0.833)	0.193 (0.818)	0.192 (0.823)	0.192 (0.827)
V_{max} (at X)	0.234 (0.116)	0.234 (0.119)	0.234 (0.119)	0.233 (0.123)
Ra = 10^5				
Nu_{avg}	4.636	4.51	4.519	4.522
U_{max} (at Y)	0.166 (0.887)	0.132 (0.859)	0.153 (0.855)	0.131 (0.854)
V_{max} (at X)	0.258 (0.062)	0.258 (0.066)	0.261 (0.066)	0.258 (0.065)
Ra = 10^6				
Nu_{avg}	9.241	8.806	8.799	8.826
U_{max} (at Y)	0.153 (0.937)	0.077 (0.859)	0.079 (0.850)	0.077 (0.854)
V_{max} (at X)	0.261 (0.037)	0.262 (0.039)	0.262 (0.038)	0.262 (0.039)

TABLE II
THERMO PHYSICAL PROPERTIES OF DIFFERENT PHASES

Property	Fluid phase (Ethylene glycol)	Solid phase (Aluminum oxide)
C_p (J/kg K)	2382	765
ρ (kg/m^3)	1116	3970
k (W/m K)	0.249	46
β (K^{-1})	0.65 x10^{-3}	7.4 x10^{-6}

IV. RESULTS AND DISCUSSION

The ranges of Rayleigh number for this investigation are $10^3 \le Ra \le 10^6$. The range of volume fraction ϕ used in this study varied between $0 \le \phi \le 30\%$. The thermo physical properties of fluid and solid phase are shown in Table II. Fig. 3 plots the streamlines and isotherms for different Rayleigh numbers of aspect ratio 1 to assess further the accuracy of the physical concept of this model. In Fig. 3, the flow rate exists with maximum value in the centre of the circulation. As the parameter Ra increases, the flow rates at the centers of circulation increase. The temperature drops gradually from the value at the fin to the value at the center. This behavior is clearly similar to the related flow pattern in all ranges of buoyancy parameter.

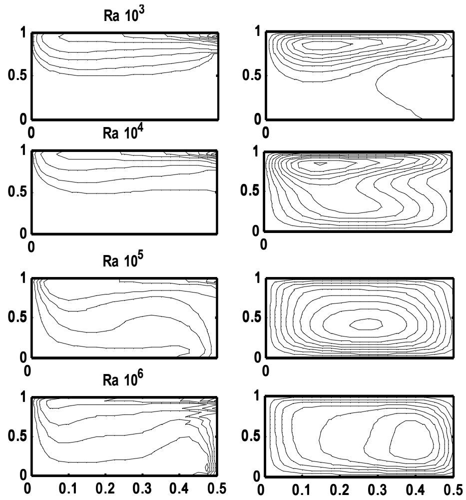


Fig. 3 Isotherms and streamlines of the enclosure with fin height 0.75, volume fraction 10%, Pr 204, Ar 1.0 for different Rayleigh numbers

The effect of the volume fraction on the streamlines and isotherms of nano fluids for various Rayleigh numbers is shown in Fig. 4. For a low Rayleigh number, a central vortex appears as a dominant characteristic of the fluid flow. As the volume fraction increases, the velocities at the center of the enclosure increase as a result of higher solid-fluid transportation of heat.

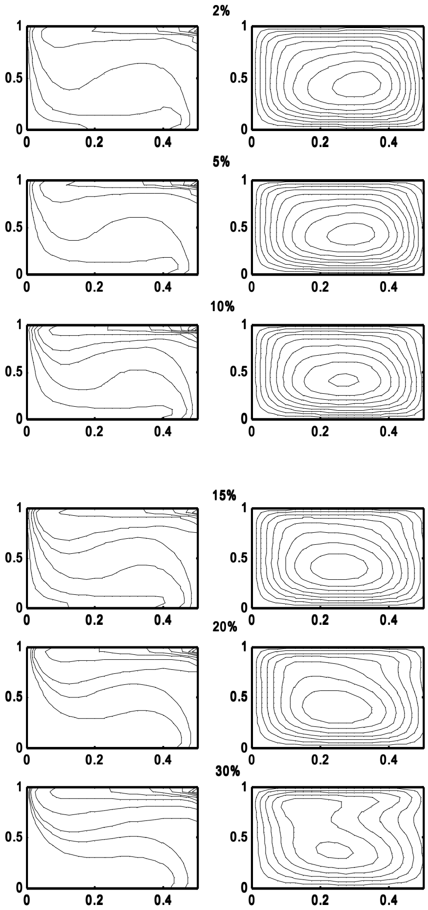


Fig. 4 Isotherms and streamlines of the enclosure with fin height 0.75, Pr 204, Rayleigh number 10^5 for different volume fractions

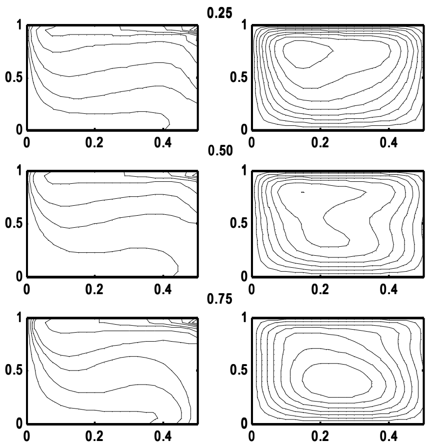
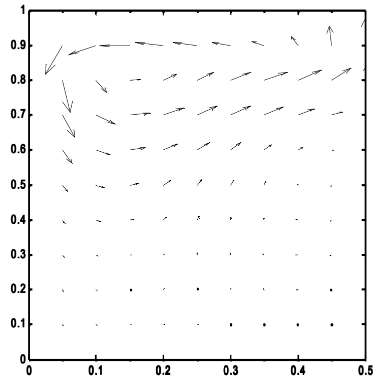
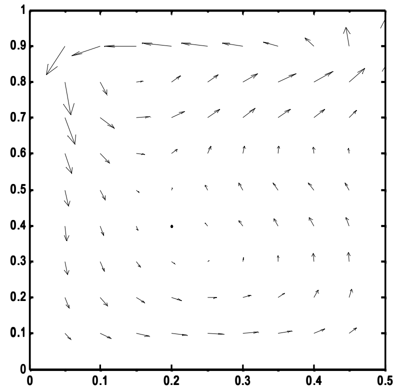


Fig. 5 Isotherms and streamlines of the enclosure with volume fraction 20% and Rayleigh number 10^5 for different fin heights

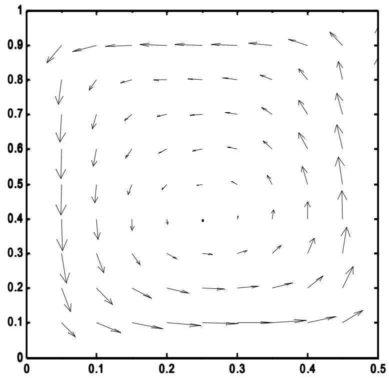
The effect of fin height on the isotherms and streamlines of the enclosure is shown in Fig. 5. The isotherms in Fig. 4 and 5 shows the vertical stratification breaks down with an increase in the volume fraction for higher Rayleigh numbers. This is due to a number of effects such as gravity, Brownian motion, ballistic phonon transport, layering at the solid/liquid interface, clustering of nano particles, and dispersion effect. In this study we considered only the effect of dispersion that may coexist in the main flow of a nano fluid. As Rayleigh number increases natural convection prevails, the temperature variation is restricted over a gradually diminishing region around the fin. It is also noticed that the heat affected zone becomes larger with the increasing fin height. The vector plots of the enclosure with fin height 0.75 for different Rayleigh numbers is depicted in Fig. 6.



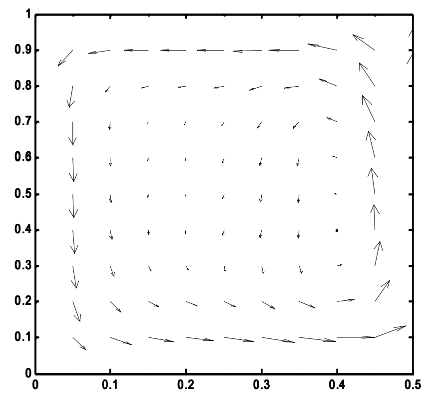
(a) Ra 10^3



(b) Ra 10^4

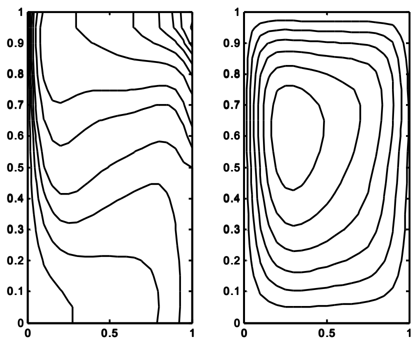


(c) Ra 10^5

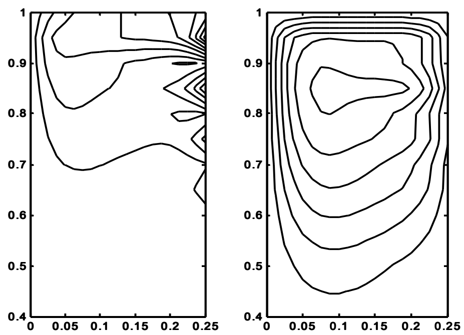


(d) Ra 10^6

Fig. 6 Vector plots of the enclosure with partition height 0.75, volume fraction 10% for different Rayleigh numbers



(a) $A_r = 0.50$



(b) $A_r = 2.00$

Fig. 7 Isotherms and streamlines of the enclosure with fin height 0.5, volume fraction 10%, Rayleigh number 10^4 and for different aspect ratios

The effect of aspect ratio of the enclosure upon streamlines and isotherms are demonstrated in Fig. 7. The effect of local Nusselt number along the fin for different aspect ratios is depicted in Fig. 8. The effect of local Nusselt number along the fin for different fin heights are depicted in Fig. 9. It is shown that the average Nusselt number at the fin is increased as the aspect ratio decreased. The effect of volume fraction on local Nusselt number is also depicted in Fig. 10.

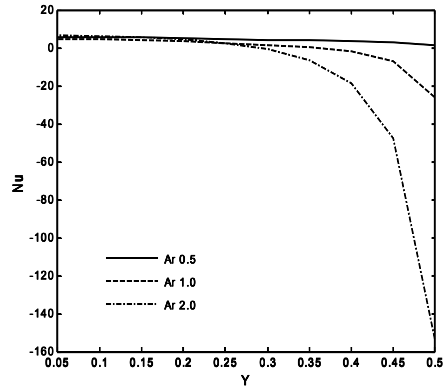


Fig. 8 Variation of local Nusselt number along the fin for different aspect ratios, of fixed values of Rayleigh number 10^4, volume fraction 10% and fin height 0.5

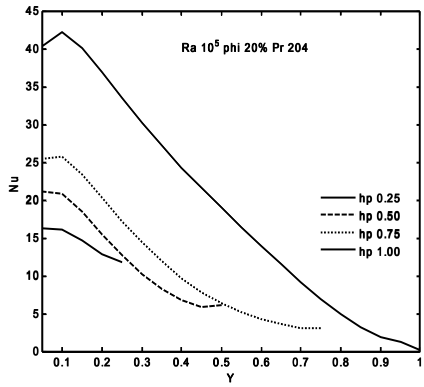


Fig. 9 Variation of local Nusselt number along the fin for different fin heights, for fixed values of Rayleigh number 10^5 and volume fraction 20%

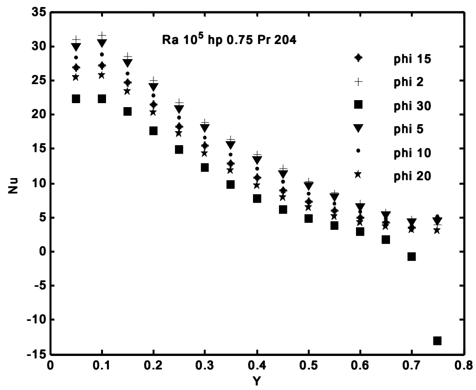


Fig. 10 Variation of local Nusselt number along the fin for different volume fractions, for fixed values of Rayleigh number 10^5 and fin height 0.75

V. Conclusion

In this study heat transfer enhancement of nano fluid in a finned enclosure is investigated for various pertinent parameters like volume fraction, fin height, Rayleigh numbers and aspect ratio of the enclosure. In addition, the results illustrate that the nano fluid heat transfer rate increases with an increase in the nano particles volume fraction. The presence of nano particles in the fluid is found to alter the structure of the fluid flow. The nano fluid used, which is composed of Aluminum oxide nano particles in suspension of Ethylene glycol, has been provided at various particle concentrations ranging from 0 to 30% in volume. As Rayleigh number increases natural convection prevails, the temperature variation is restricted over a gradually diminishing region around the fin. It is also noticed that the heat affected zone becomes larger with the increasing fin height.

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NOMENCLATURE

$a1$	constant defined in equ. (7)
A_r	aspect ratio of the enclosure
c	fin position
Cp	specific heat at constant pressure
d	diameter
g	acceleration due to gravity
Gr	Grashoff number
h	fin height
H	height of the enclosure
k	thermal conductivity
Nu	Nusselt number
P	dimensionless pressure
Pr	Prandtl number
Ra	Rayleigh number
t	time
T	temperature
u, v	velocity component in x- and y- directions
u_c	characteristic velocity
U, V	dimensionless velocity component in X- and Y- directions
W	width of the enclosure
x, y	dimensional Cartesian coordinates
X, Y	dimensionless Cartesian coordinates

Greek Letters

α	thermal diffusivity
β	coefficient of thermal expansion
ϕ	volume fraction
ν	kinematic viscosity
ρ	density
θ	dimensionless temperature diff.
ψ	dimensionless stream function
Ω	dimensionless vorticity
τ	dimensionless time
μ	dynamic viscosity
λ	constant defined in equ. (8)

Subscripts

avg	average
c	cold
eff	effective
f	fluid
h	hot
nf	nano fluid
o	reference value
p	particle/ partition
s	solid

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