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A Note on Characterization of Regular Γ-Semigroups in terms of $(\in, \in \vee q)$-Fuzzy Bi-ideal

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Abstract—The purpose of this note is to obtain some properties of $(\in, \in \vee q)$- fuzzy bi-ideals in a Γ-semigroup in order to characterize regular and intra-regular Γ-semigroups.

Keywords—Regular Γ-semigroup, belong to or quasi-coincident, $(\in, \in \vee q)$-fuzzy subsemigroup, $(\in, \in \vee q)$-fuzzy bi-ideals.

I. Introduction

THE concept of fuzzy set was introduced by Zadeh [26]. Many papers on fuzzy sets appeared showing the importance of the concept and its application to logic, set theory, group theory, semigroup theory, real analysis, measure theory, topology etc. It was first applied to the theory of groups by Rosenfeld [14]. Murali [12] proposed a definition of fuzzy point belonging to fuzzy subset. The idea of quasi coincidence of a fuzzy point with a fuzzy set, which is mentioned in [13], played a vital role to generate some different types of fuzzy subgroups. Bhakat and Das [1] gave the concept of (α, β)-fuzzy subgroups by using the belong to relation $(\in)$ and quasi-coincidence with relation (q) between a fuzzy point and a fuzzy subgroup, and introduced the concept of an $(\in, \in \vee q)$- fuzzy subgroup. In particular, $(\in, \in \vee q)$-fuzzy subgroup is an important and useful generalization of Rosenfeld's fuzzy subgroup. Yunqiang Yin and Dehua Xu [25] introduced the concepts of $(\in, \in \vee q)$- fuzzy subgroup and $(\in, \in \vee q)$-fuzzy ideals in semigroups. In [8], Jun and Song introduced the notion of generalized fuzzy interior ideals in semigroups. Sen and Saha in [23] defined the concepts of Γ-semigroups as a generalization of semigroups. Γ-semigroups have been analyzed by a lot of mathematicians, for instance Chattopadhay [2], Dutta and Adhikari [5], Hila [7], Chinram [3], Saha [15], Sen et. al [21], [22]. Seth [24]. Sardar and Majumder [16], [18] characterized subsemigroups, bi-ideals, interior ideals (along with Davvaz [19]), quasi ideals, ideals, prime (along with Mandal [17]) and semiprime ideals, ideal extensions (along with Dutta [6]) of a Γ-semigroup in terms of fuzzy subsets. They also studied their different properties directly and via operator semigroups of a Γ-semigroup. As a sequel to this study we introduced the concepts of $(\in, \in \vee q)$-fuzzy subsemigroups and $(\in, \in \vee q)$-fuzzy bi-ideals in a Γ-semigroup [20]. In this paper we obtain some important results of $(\in, \in \vee q)$- fuzzy bi-ideals in characterizing regular and intra-regular Γ-semigroups.

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II. Preliminaries

In this section we recall some elementary definitions which will be used in the sequel.

Let $S = \{x, y, z, \ldots\}$ and $\Gamma = \{\alpha, \beta, \gamma, \ldots\}$ be two non-empty sets. Then, S is called a Γ-*semigroup*[23] if there exists a mapping $S \times \Gamma \times S \to S$ (images to be denoted by $a\alpha b$) satisfying

(1) $x\gamma y \in S$,
(2) $(x\beta y)\gamma z = x\beta(y\gamma z)$, for all $x, y, z \in S$ and for all $\beta, \gamma \in \Gamma$.

A non-empty subset A of a Γ-semigroup S is called a *subsemigroup* of S if $A\Gamma A \subseteq A$. A subsemigroup A of a Γ-semigroup S is called a ***bi-ideal*** of S if $A\Gamma S\Gamma A \subseteq A$.

A Γ-semigroup S is called *regular* [4], if for each $a \in S$, there exist $x \in S$ and $\alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$.

A Γ-semigroup S is called *intra-regular* [4], if for each $a \in S$, there exists $x, y \in S$ and $\alpha, \beta, \gamma \in \Gamma$ such that $a = x\alpha a\beta a\gamma y$.

Let $\Gamma = H$, where H be a subgroup of a group G. For any $x, y \in G$ and $\gamma \in \Gamma$, define $x\gamma y = x \cdot \gamma \cdot y$, where $\cdot$ is the group operation on G. Then, G is a Γ-semigroup.

A function μ from a non-empty set X to the unit interval $[0, 1]$ is called a *fuzzy subset* [26] of X. A fuzzy subset μ of a set X of the form

$$\mu(y) = \begin{cases} t(\neq 0) & \text{if } y = x \\ 0 & \text{if } y \neq x \end{cases}$$

is said to be a fuzzy point with support x and value t and is denoted by x_t. A fuzzy point x_t is said to belong to (be quasi coincident with) a fuzzy subset μ, written as $x_t \in \mu$(resp. $x_t q\mu$) if $\mu(x) \geq t$(resp. $\mu(x) + t > 1$).

$x_t \in \mu$ or $x_t q\mu$ will be denoted by $x_t \in \vee q\mu$, $x_t \in \mu$ and $x_t q\mu$ will be denoted by $x_t \in \wedge q\mu$. $x_t \overline{\in} \mu$, $x_t \overline{\in \vee q} \mu$ and $x_t \overline{\in \wedge q} \mu$ will respectively mean $x_t \notin \mu, x_t \notin \vee q\mu$ and $x_t \notin \wedge q\mu$.

(S1) A non-empty fuzzy subset μ of a Γ-semigroup S is called a *fuzzy subsemigroup* of S if $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in S$ and for all $\gamma \in \Gamma$.

(B1) A fuzzy subsemigroup μ of a Γ-semigroup S is called a *fuzzy bi-ideal* of S if $\mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z)\}$, for all $x, y, z \in S$ and for all $\alpha, \beta \in \Gamma$.

Let X be a non-empty set and μ be a fuzzy subset of X. Then, for any $t \in (0, 1]$, the sets $\mu_t = \{x \in X : \mu(x) \geq t\}$

and $\text{Supp}(\mu) = \{x \in X : \mu(x) > 0\}$ are called t-level subset and supporting set of μ (simply support of μ), respectively.

Let S be a Γ-semigroup and λ, μ be two fuzzy subsets of S. Then, the product of λ and μ, denoted by, $\lambda \circ \mu$ is defined by

$$(\lambda \circ \mu)(x) = \begin{cases} \sup\limits_{x=y\gamma z} \min\{\lambda(y), \mu(z)\}, \ \forall y, z \in S, \forall \gamma \in \Gamma \\ 0 \qquad\qquad \text{otherwise} \end{cases}.$$

Also, for any fuzzy subsets λ, μ and ν of S, $(\lambda \circ \mu) \circ \nu = \lambda \circ (\mu \circ \nu)$.

III. Main Results

Definition III.1. (1) [20] A non-empty fuzzy subset μ of a Γ-semigroup S is said to be an $(\in, \in \vee q)$- fuzzy subsemigroup of S if $\forall x, y \in S$, $\forall \gamma \in \Gamma$ and $t, r \in (0, 1]$, $x_t, y_r \in \mu \Rightarrow (x\gamma y)_{\min(t,r)} \in \vee q\mu$.

(2) [20] An $(\in, \in \vee q)$- fuzzy subsemigroup μ of a Γ-semigroup S is said to be an $(\in, \in \vee q)$- fuzzy bi-ideal of S if $\forall x, y, z \in S, \forall \alpha, \beta \in \Gamma$ and $t, r \in (0, 1]$, $x_t, z_r \in \mu \Rightarrow (x\alpha y\beta z)_{\min(t,r)} \in \vee q\mu$.

Theorem III.2. *[20] Let μ be any non-empty fuzzy subset of a Γ-semigroup S. Then, the following statements are equivalent:*

(1) *μ is an $(\in, \in \vee q)$-fuzzy subsemigroup of S,*
(2) *for any $x, y \in Supp(\mu)$, $\gamma \in \Gamma$, $\mu(x\gamma y) \geq \min\{\mu(x), \mu(y), 0.5\}$,*
(3) *$\mu \circ \mu \subseteq \vee q\mu$,*
(4) *$\mu \circ \mu \cap 0.5_{Supp(\mu)} \subseteq \mu$,*
(5) *for any $r \in (0, 0.5]$, if μ_r is non-empty, then μ_r is a subsemigroup of S.*

Theorem III.3. *[20] Let μ be any $(\in, \in \vee q)$-fuzzy subsemigroup of a Γ-semigroup S. Then, the following statements are equivalent:*

(1) *μ is an $(\in, \in \vee q)$-fuzzy bi-ideal of S,*
(2) *for any $x, z \in Supp(\mu), y \in S$ and $\alpha, \beta \in \Gamma, \mu(x\alpha y\beta z) \geq \min\{\mu(x), \mu(z), 0.5\}$,*
(3) *$\mu \circ \chi_S \circ \mu \subseteq \vee q\mu$(where χ_S is the characteristic function of S),*
(4) *$\mu \circ \chi_S \circ \mu \cap 0.5_{Supp(\mu)} \subseteq \mu$(where χ_S is the characteristic function of S),*
(5) *for any $r \in (0, 0.5]$, if μ_r is non-empty, then μ_r is a bi-ideal of S.*

Theorem III.4. *[20] Let $\{\mu_i\}_{i \in I}$ be any family of $(\in, \in \vee q)$-fuzzy subsemigroups of S. Then, $\bigcap_{i \in I} \mu_i$ and $\bigcup_{i \in I} \mu_i$ is a $(\in, \in \vee q)$-fuzzy subsemigroup of S. If $\{\mu_i\}_{i \in I}$ is any family of $(\in, \in \vee q)$-fuzzy bi-ideals of S, then both $\bigcap_{i \in I} \mu_i$ and $\bigcup_{i \in I} \mu_i$ are $(\in, \in \vee q)$-fuzzy bi-ideals of S.*

Proposition III.5. *Let S be a Γ-semigroup and $A, B \subseteq S$. Then,*

(i) *$A \subseteq B$ if and only if $\mu_A \subseteq \mu_B$.*
(ii) *$\mu_A \cap \mu_B = \mu_{A \cap B}$.*
(iii) *$\mu_A \circ \mu_B = \mu_{A\Gamma B}$, where μ_A, μ_B denote the characteristic functions of A and B, respectively.*

Let S be a Γ-semigroup. Then for any $a \in S$, we define $A_a := \{(y, z) \in S \times S : a = y\gamma z$ for some $\gamma \in \Gamma\}$. For any two fuzzy subset μ_1, μ_2 of S, we define

$$(\mu_1 \circ \mu_2)(a) := \begin{cases} \sup\limits_{(y,z) \in A_a} \min\{\mu_1(y), \mu_2(z)\} & \text{if } A_a \neq \phi \\ 0 & \text{if } A_a = \phi. \end{cases}$$

Let S be a Γ-semigroup. For any two fuzzy subsets μ_1 and μ_2 of S, we define the 0.5-product of μ_1 and μ_2 by,
$(\mu_1 \circ_{0.5} \mu_2)(a) := t$, where

$$t = \begin{cases} \sup\limits_{(y,z) \in A_a} \min\{\mu_1(y), \mu_2(z), 0.5\} & \text{if } A_a \neq \phi \\ 0 & \text{if } A_a = \phi. \end{cases}$$

For a Γ-semigroup S the fuzzy subset 0(1) is defined by $0(x) = 0$, for all $x \in S$ (respectively $1(x) = 1$, for all $x \in S$).

For a Γ-semigroup S the fuzzy subset 0.5_S is defined by $0.5_S(x) = 0.5$, for all $x \in S$.

Let μ_1, μ_2 be any two fuzzy subsets of a Γ-semigroup S. Then $(\mu_1 \cap_{0.5} \mu_2)(x) = \min\{\mu_1(x), \mu_2(x), 0.5\}$, for all $x \in S$.

Proposition III.6. *Let S be a Γ-semigroup and $A, B \subseteq S$. Then*

(i) $\mu_A \cap_{0.5} \mu_B = \mu_{A \cap B} \cap 0.5_S$.
(ii) $\mu_A \circ_{0.5} \mu_B = \mu_{A\Gamma B} \cap 0.5_S$.

Proof: (i) Let $x \in S$. If $(\mu_A \cap_{0.5} \mu_B)(x) = 0 \Rightarrow \min\{\mu_A(x), \mu_B(x), 0.5\} = 0$. Then, $\mu_A(x) = 0$ or $\mu_B(x) = 0 \Rightarrow x \notin A \cap B$. Then, $\mu_{A \cap B}(x) = 0 \Rightarrow (\mu_{A \cap B} \cap 0.5_S)(x) = 0$.

If $(\mu_A \cap_{0.5} \mu_B)(x) \neq 0$, then $(\mu_A \cap_{0.5} \mu_B)(x) = \min\{\mu_A(x), \mu_B(x), 0.5\} \neq 0$. Then, $(\mu_A \cap_{0.5} \mu_B)(x) = 0.5$ and $\mu_A(x) = 1 = \mu_B(x) \Rightarrow x \in A \cap B \Rightarrow \mu_{A \cap B}(x) = 1$. Therefore, $(\mu_{A \cap B} \cap 0.5_S)(x) = \min\{\mu_{A \cap B}(x), 0.5\} = \min\{1, 0.5\} = 0.5$. Thus, $(\mu_A \cap_{0.5} \mu_B)(x) = (\mu_{A \cap B} \cap 0.5_S)(x), \forall x \in S$. Consequently, $\mu_A \cap_{0.5} \mu_B = \mu_{A \cap B} \cap 0.5_S$.

(ii) If $(\mu_{A\Gamma B} \cap 0.5_S)(x) = 0 \Rightarrow \min\{\mu_{A\Gamma B}(x), 0.5\} = 0$. Then $\mu_{A\Gamma B}(x) = 0 \Rightarrow x \notin A\Gamma B$. Then $x \neq a\alpha b$ $\forall a \in A, b \in B, \alpha \in \Gamma$. Then $(\mu_A \circ_{0.5} \mu_B)(x) = 0$.

If $(\mu_{A\Gamma B} \cap 0.5_S)(x) \neq 0$, then $(\mu_{A\Gamma B} \cap 0.5_S)(x) = \min\{\mu_{A\Gamma B}(x), 0.5\} \neq 0$. Then $\min\{\mu_{A\Gamma B}(x), 0.5\} = 0.5$ and $\mu_{A\Gamma B}(x) = 1 \Rightarrow x \in A\Gamma B$. Hence $(\mu_A \circ_{0.5} \mu_B)(x) = \sup\limits_{(u,v) \in A_x} \min\{\mu_A(u), \mu_B(v), 0.5\} = 0.5$. Thus, $(\mu_A \circ_{0.5} \mu_B)(x) = (\mu_{A\Gamma B} \cap 0.5_S)(x)$, for all $x \in S$. Consequently, $\mu_A \circ_{0.5} \mu_B = \mu_{A\Gamma B} \cap 0.5_S$. ■

Now we recall the following results for their use in the sequel.

Proposition III.7. *[20] Let μ and ν be $(\in, \in \vee q)$-fuzzy subsemigroups of a Γ-semigroup S. Then, $\mu \circ_{0.5} \nu$ is an $(\in, \in \vee q)$-fuzzy bi-ideal of S if any one of μ and ν is $(\in, \in \vee q)$-fuzzy bi-ideal of S.*

Proposition III.8. *[20] If μ_1, μ_2 are any two $(\in, \in \vee q)$-fuzzy subsemigroups (fuzzy bi-ideals) of a Γ-semigroup S, then $(\mu_1 \cap_{0.5} \mu_2)$ is an $(\in, \in \vee q)$-fuzzy subsemigroup (resp. fuzzy bi-ideal) of S.*

Let S be a Γ-semigroup. Then, a non-empty subset A of S is a subsemigroup(bi-ideal) of S if and only if the characteristic function μ_A of A is an $(\in, \in \vee q)$-fuzzy subsemigroup(resp. fuzzy bi-ideal) of S.

Theorem III.9. *[20] A fuzzy subset μ of a Γ-semigroup S is an $(\in, \in \vee q)$-fuzzy subsemigroup of S if and only if $\mu \circ_{0.5} \mu \subseteq \mu$.*

Theorem III.10. *[20] In a Γ-semigroup S the following are equivalent:*

(i) *μ is an $(\in, \in \vee q)$-fuzzy bi-ideal of S,*

(ii) *$\mu \circ_{0.5} \mu \subseteq \mu$ and $\mu \circ_{0.5} 1 \circ_{0.5} \mu \subseteq \mu$.*

EXAMPLE 1. Let $S = \{a, b, c\}$ and $\Gamma = \{\gamma, \delta\}$, where γ, δ is defined on S with the following Cayley tables:

γ	a	b	c
a	a	b	c
b	b	a	c
c	c	c	c

and

δ	a	b	c
a	b	a	c
b	a	b	c
c	c	c	c

Then, S is a Γ-semigroup. We define fuzzy subset $\mu : S \to [0, 1]$, by $\mu(a) = 0.6, \mu(b) = 0.5, \mu(c) = 0.4$. Clearly, μ is an $(\in, \in \vee q)$-fuzzy subsemigroup and $(\in, \in \vee q)$-fuzzy bi-ideal of S.

Now, $(\mu \circ_{0.5} \mu)(a)= \sup\limits_{(x,y)\in A_a} \min\{\mu(x), \mu(y), 0.5\} = 0.5 < 0.6 = \mu(a)$. Hence, $\mu \neq \mu \circ_{0.5} \mu$. Also $(\mu \circ_{0.5} 1 \circ_{0.5} \mu)(a) = 0.5 < 0.6 = \mu(a)$. Hence, $\mu \circ_{0.5} 1 \circ_{0.5} \mu \neq \mu$.

The above example shows that in the second relation of Theorem III.10 equality need not hold. But for a regular Γ-semigroup the situation is different. In this direction, we obtain the following characterization of a regular $\Gamma-$semigroup.

Theorem III.11. *A Γ-semigroup S is regular if and only if for every $(\in, \in \vee q)$-fuzzy bi-ideal μ of S, $\mu \circ_{0.5} 1 \circ_{0.5} \mu = \mu \cap 0.5_S$.*

Proof: Let S be a regular Γ-semigroup and μ be an $(\in, \in \vee q)$-fuzzy bi-ideal of S. We have $\mu \circ_{0.5} 1 \circ_{0.5} \mu \subseteq \mu$. Also for $a \in S$, $(\mu \circ_{0.5} 1 \circ_{0.5} \mu)(a)= \sup\limits_{(u,v)\in A_a} \min\{\mu(u), (1 \circ_{0.5} \mu)(v), 0.5\} \leq 0.5 = (0.5_S)(a)$. Then, $\mu \circ_{0.5} 1 \circ_{0.5} \mu \subseteq 0.5_S$. Hence, $\mu \circ_{0.5} 1 \circ_{0.5} \mu \subseteq \mu \cap 0.5_S$. Since S is regular for $a \in S$ there exists $x \in S, \alpha, \beta \in \Gamma$ such that $a = a\alpha x\beta a$. Then, $(\mu \circ_{0.5} 1 \circ_{0.5} \mu)(a) = \sup\limits_{(u,v)\in A_a} \min\{\mu(u), (1 \circ_{0.5} \mu)(v), 0.5\} \geq \min\{\mu(a), (1 \circ_{0.5} \mu)(x\beta a), 0.5\} \geq \min\{\mu(a), \min\{1(x), \mu(a), 0.5\}, 0.5\} = \min\{\mu(a), 0.5\} = (\mu \cap 0.5_S)(a)$. So, $\mu \circ_{0.5} 1 \circ_{0.5} \mu \supseteq \mu \cap 0.5_S$. Hence, $\mu \circ_{0.5} 1 \circ_{0.5} \mu = \mu \cap 0.5_S$.

Conversely, let for every $(\in, \in \vee q)$-fuzzy bi-ideal μ of S, $\mu \circ_{0.5} 1 \circ_{0.5} \mu = \mu \cap 0.5_S$. Let B be a bi-ideal of S. Then, μ_B is an $(\in, \in \vee q)$-fuzzy bi-ideal of S. Then, $\mu_B \circ_{0.5} 1 \circ_{0.5} \mu_B = \mu_B \cap 0.5_S$. Then, by Proposition 3.6, $\mu_{B\Gamma S\Gamma B} \cap 0.5_S = \mu_B \cap 0.5_S$. If $b \in B$, then $\mu(b) = 1$. Then, $(\mu_B \cap 0.5_S)(b)$=$\min\{\mu_B(b), 0.5\}$=$\min\{1, 0.5\}$ =0.5. Then, $(\mu_{B\Gamma S\Gamma B} \cap 0.5_S)(b) = 0.5$ which implies that $\min\{\mu_{B\Gamma S\Gamma B}(b), 0.5\} = 0.5$. Hence, $\mu_{B\Gamma S\Gamma B}(b) = 1$, and so $b \in B\Gamma S\Gamma B$. Then, $B \subseteq B\Gamma S\Gamma B$. Also, $B\Gamma S\Gamma B \subseteq B$. So, $B = B\Gamma S\Gamma B$. Hence, S is regular. ■

To conclude the paper we obtain the following characterization of a regular and intra-regular Γ-semigroup.

Theorem III.12. *Let S be a Γ-semigroup. Then the followings are equivalent:*

(i) *S is both regular and intra-regular,*

(ii) *$\mu \circ_{0.5} \mu = \mu \cap 0.5_S$, for every $(\in, \in \vee q)$-fuzzy bi-ideal μ of S,*

(iii) *$\mu \cap_{0.5} \nu = (\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)$ for all $(\in, \in \vee q)$-fuzzy bi-ideals μ and ν of S.*

Proof: (i⇒ii) Let S be both regular and intra-regular. Let μ be an $(\in, \in \vee q)$-fuzzy bi-ideal of S. Then, $\mu \circ_{0.5} \mu \subseteq \mu$. Also, $\mu \circ_{0.5} \mu \subseteq 0.5_S$. Then, $\mu \circ_{0.5} \mu \subseteq \mu \cap 0.5_S$. Let $a \in S$. Since S is regular and intra-regular, there exists $x, y, z \in S$ and $\alpha, \beta, \gamma, \delta, \eta \in \Gamma$ such that $a = a\alpha x\beta a$ and $a = y\gamma a\delta a\eta z$. Hence, $a = a\alpha x\beta a$ =$a\alpha x\beta a\alpha x\beta a = a\alpha x\beta(y\gamma a\delta a\eta z)\alpha x\beta a$ =$(a\alpha x\beta y\gamma a)\delta(a\eta z\alpha x\beta a)$. Then,

$(\mu \circ_{0.5} \mu)(a)$
$= \sup\limits_{(u,v)\in A_a} \min\{\mu(u), \mu(v), 0.5\}$
$\geq \min\{\mu(a\alpha x\beta y\gamma a), \mu(a\eta z\alpha x\beta a), 0.5\}$
$\geq \min\{\min\{\mu(a), \mu(a), 0.5\}, \min\{\mu(a), \mu(a), 0.5\}, 0.5\}$
$= \min\{\mu(a), 0.5\}$
$= (\mu \cap 0.5_S)(a)$. Then $\mu \circ_{0.5} \mu \supseteq \mu \cap 0.5_S$. Hence $\mu \circ_{0.5} \mu = \mu \cap 0.5_S$.

(ii⇒iii) Let μ and ν be two $(\in, \in \vee q)$-fuzzy bi-ideals of S. Then $\mu \cap_{0.5} \nu$ is also an $(\in, \in \vee q)$-fuzzy bi-ideals of S. Then, $(\mu \cap_{0.5} \nu) \circ_{0.5} (\mu \cap_{0.5} \nu) = (\mu \cap_{0.5} \nu) \cap 0.5_S = \mu \cap_{0.5} \nu$. Then, $\mu \cap_{0.5} \nu = (\mu \cap_{0.5} \nu) \circ_{0.5} (\mu \cap_{0.5} \nu) \subseteq \mu \circ_{0.5} \nu$. Similarly $\mu \cap_{0.5} \nu \subseteq \nu \circ_{0.5} \mu$. Hence $\mu \cap_{0.5} \nu \subseteq (\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)$. Also by Proposition 3.7, $\mu \circ_{0.5} \nu$ and $\nu \circ_{0.5} \mu$ are $(\in, \in \vee q)$-fuzzy bi-ideals of S. Then $(\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)$ is an $(\in, \in \vee q)$-fuzzy bi-ideal of S. Then

$(\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)$
$= ((\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)) \cap 0.5_S$
$= ((\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu)) \circ_{0.5} ((\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu))$
$\subseteq (\mu \circ_{0.5} \nu) \circ_{0.5} (\nu \circ_{0.5} \mu)$
$= \mu \circ_{0.5} (\nu \circ_{0.5} \nu) \circ_{0.5} \mu \subseteq \mu \circ_{0.5} 1 \circ_{0.5} \mu$
$\subseteq \mu \cap 0.5_S$.

Similarly, $(\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu) \subseteq \nu \cap 0.5_S$. Then $(\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu) \subseteq (\mu \cap 0.5_S) \cap (\nu \cap 0.5_S)$ =$\mu \cap_{0.5} \nu$. Hence

$$\mu \cap_{0.5} \nu = (\mu \circ_{0.5} \nu) \cap_{0.5} (\nu \circ_{0.5} \mu).$$

(iii⇒i) Let (iii) hold. In order to prove that S is regular and intra-regular we have to prove that $Q = Q\Gamma Q$ for every quasi-ideal Q of S. Let Q be a quasi-ideal of S. Then $Q\Gamma Q \subseteq S\Gamma Q \cap Q\Gamma S \subseteq Q$. Now Q is a bi-ideal of S. Hence χ_Q is $(\in, \in \vee q)$-fuzzy bi-ideal of S. By given condition (iii),

$\chi_Q \bigcap_{0.5} \chi_Q = (\chi_Q \circ_{0.5} \chi_Q) \bigcap_{0.5} (\chi_Q \circ_{0.5} \chi_Q)$
$\Rightarrow \chi_{Q\cap Q} \bigcap 0.5_S$= $(\chi_Q \circ_{0.5} \chi_Q) \bigcap 0.5_S$, by Proposition 3.6.
$\Rightarrow \chi_Q \bigcap 0.5_S$=$\chi_{Q\Gamma Q} \bigcap 0.5_S$

Let $q \in Q$. Then $(\chi_Q \bigcap 0.5_S)(q)$=$\min\{\chi_Q(q), 0.5\} = \min\{1, 0.5\} = 0.5$. Hence $(\chi_{Q\Gamma Q} \bigcap 0.5_S)(q) = 0.5 \Rightarrow \min\{\chi_{Q\Gamma Q}(q), 0.5\} = 0.5 \Rightarrow \chi_{Q\Gamma Q}(q) = 1 \Rightarrow q \in Q\Gamma Q$. Hence $Q \subseteq Q\Gamma Q$. So $Q = Q\Gamma Q$. Therefore, S is both regular and intra-regular. ■

IV. CONCLUSION

Operator semigroups of a $\Gamma-$semigroup have been found to be an effective tool in the study of $\Gamma-$semigroups which is evident from the work of Dutta et al[4], [5] and also from the work of Sardar et al[16], [17]. In this paper we have not used the technique of operator semigroups. Use of operator semigroups may give further insight in characterizing Γ-semigroups in terms of $(\in, \in \vee q)$-fuzzy bi-ideals.

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