

Manufacturing Dispersions Based Simulation and Synthesis of Design Tolerances

Nassima Cheikh, Abdelmadjid Cheikh, and Said Hamou

Abstract—The objective of this work which is based on the approach of simultaneous engineering is to contribute to the development of a CIM tool for the synthesis of functional design dimensions expressed by average values and tolerance intervals. In this paper, the dispersions method known as the ΔI method which proved reliable in the simulation of manufacturing dimensions is used to develop a methodology for the automation of the simulation. This methodology is constructed around three procedures. The first procedure executes the verification of the functional requirements by automatically extracting the functional dimension chains in the mechanical sub-assembly. Then a second procedure performs an optimization of the dispersions on the basis of unknown variables. The third procedure uses the optimized values of the dispersions to compute the optimized average values and tolerances of the functional dimensions in the chains. A statistical and cost based approach is integrated in the methodology in order to take account of the capabilities of the manufacturing processes and to distribute optimal values among the individual components of the chains.

Keywords—functional tolerances, manufacturing dispersions, simulation, CIM.

I. INTRODUCTION

NEW design processes of simultaneous engineering are under development. They are the concurrent engineering processes where all the engineering actors work simultaneously on a product. In these processes, the fact of integrating upstream the manufacturing analysis in the design stage of a product, should make it possible to optimise the tolerances and a better quality assurance of the finished products. This implies to have tools for tolerance analysis and synthesis which must integrate in their definition the functional aspects of design and the stochastic aspects of manufacture and inspection at the same time. The functional dimensioning and tolerancing tool naturally federates the technical data to ensure the functional requirements of the products. It becomes obvious that its integration in CAD/CAM systems is essential for the global definition of the product numerical model. Mastering the functional dimensioning and tolerancing tools with simultaneous engineering and co-operative work eliminates any source of

incompatibility between the design activities upstream and manufacturing activities downstream. Thus, the use of such tools avoids any loss of information and minimizes scrap parts as a consequence.

On the other hand, industrial experience shows that the tolerancing phase leads to significant choices which influence the manufacturing process and the cost of manufacturing the parts. Indeed, the decisions taken during this phase induce almost 70% of the total cost of producing the parts [1]. For that reason it is necessary to optimise the manufacturing means and to produce according to the functional requirements by checking the capabilities of the available means in the workshop. The objective of the optimisation of design tolerances is to minimize the total cost of manufacture of all the tolerances during production. Many researchers treat design tolerances [2] and manufacturing tolerances [3] separately. They often use assembly simulation by the Monte Carlo method which proved to be greedy in computing time. The synthesis of the tolerances is a more complex problem than the analysis of the tolerances. It aims at finding the values of the various tolerances taking part in the achievement of a functional requirement, by optimising the total cost of the production. The models used, are always empirical models that give only a rough idea of the production cost [4] — [5]. Some tolerance synthesis models integrate the capability parameters of the manufacturing processes in the optimisation problem in terms of statistical standard deviations [6]. But the majority of these models do not integrate these parameters in terms of machining dispersions and in any case do not permit the synthesis of optimised average values of design dimensions. The method of dispersions called ΔI method introduced by Bourdet [7] represents an effective method of integration of the capability parameters in term of dispersions in the process plan simulation [8]—[9] for the synthesis and optimisation of manufacturing dimensions and tolerances.

In the first part, this paper presents a methodology for the simulation of functional requirement conditions of a mechanical assembly and the synthesis of optimised design dimensions and tolerances by the ΔI method. In this work, the method of dispersions is combined with the method of the minimum transfer introduced by Duret [10] to automatically optimise the dispersions ΔI . The optimisation of dispersions is carried out starting from a matrix of unknown dispersions [1] in place of the matrix of minimal reference dispersions. The

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optimised dispersions are then used to compute the manufacturing tolerances and finally the design tolerances. In the second part of the paper, a statistical and cost based optimization approach is proposed to replace the equal-values tolerance synthesis procedure previously used to explain the simulation.

II. SIMULATION OF ASSEMBLY FUNCTIONAL REQUIREMENTS

A. Manufacturing Dispersions Model

During machining the length L of a part in batch production under the same conditions and a given tool adjustment, it is always noticed a dimensional variation of the parts. The scatter given by the successive values recorded for the batch of parts between the largest and the smallest dimensions is called total dispersion Δl and is given as :

$$\Delta l = L_{\max} - L_{\min} \quad (1)$$

The Δl model introduces the concept of simulation lengths L_i which makes it possible to locate, in each reference datum of the production, the reference surface and various machined surfaces. Δl_i is the allowed dispersion of a simulation length L_i representing the variation of the location of surface i in the fixed reference system of the production machine tool. Relations are then established between the machined dimensions and the simulation lengths. Since the functional dimensions are obtained by the machined dimensions of the finish operations, the simulation lengths are used to model the average dimension values where i and j are indices of the bound surfaces as follows:

$$(Cf_{ij})_{\text{moy}} = L_j - L_i \quad \text{with } j > i \quad (2)$$

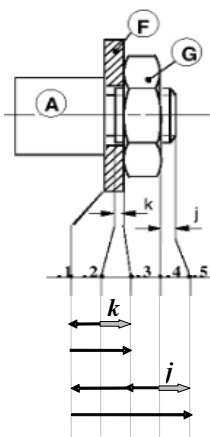


Fig. 1 Sub-assembly sample example [10]

In order to model the simulation of a mechanical assembly of parts, we consider the simplified sub-assembly example of Fig. 1 [9]. This assembly is constructed as a matrix of dispersions assigned to surfaces of the parts. After that, matrix algebra is carried out for each functional requirement

of the assembly by the minimal transfer method [10].

B. Chain Extraction Procedure

The assembly is constructed in the form of a matrix of I_s columns (surfaces) and I_p lines (parts). As the first table of Fig. 2 shows, element A_{I_s, I_p} of the matrix contains a value of dispersion only when surface I_s belongs to part I_p as a terminal or a contact surface, otherwise it is null. Then a verification procedure of the design functional requirements is carried out using the minimal transfer method as outlined in Fig. 3. When the condition of minimal transfer is satisfied, the design functional dimensions participating in the design functional requirement are those bounded by surfaces having the two dispersions stationed on the same line (same part). Thus, all the functional dimensions in the dimension chain are obtained for every functional requirement.

Design project in matrix format						
		Surfaces				
		1	2	3	4	5
Parts	A	$\Delta l_1^{(A)}$	$\Delta l_2^{(A)}$	0	0	$\Delta l_5^{(A)}$
	F	$\Delta l_1^{(F)}$	0	$\Delta l_3^{(F)}$	0	0
	G	0	0	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	0

Computation of functional requirement $k_{2,3}$ between surfaces $l=2$ and $m=3$:

$I^{(1)}$ - Set single element columns to zero except l and m :

		Surfaces				
		1	2	3	4	5
Parts	A	$\Delta l_1^{(A)}$	$\Delta l_2^{(A)}$	0	0	0
	F	$\Delta l_1^{(F)}$	0	$\Delta l_3^{(F)}$	0	0
	G	0	0	$\Delta l_3^{(G)}$	0	0

$I^{(2)}$ - Set single element lines to zero:

		Surfaces				
		1	2	3	4	5
Parts	A	$\Delta l_1^{(A)}$	$\Delta l_2^{(A)}$	0	0	0
	F	$\Delta l_1^{(F)}$	0	$\Delta l_3^{(F)}$	0	0
	G	0	0	0	0	0

Minimal transfer condition satisfied.
 Tolerance interval for functional requirement $k_{12,13}$ is :
 $\Delta k_{2,3} = \sum \Delta l_i = (\Delta l_1^{(A)} + \Delta l_2^{(A)}) + (\Delta l_1^{(F)} + \Delta l_3^{(F)})$
 Tolerance chain for $k_{2,3}$ is :
 $IT_{k_{2,3}} = IT_{Cf_{1,2}^{(A)}} + IT_{Cf_{1,3}^{(F)}}$
 Functional requirement $k_{2,3}$ is verified by :
 $IT_{k_{2,3}} \geq (\Delta l_1^{(A)} + \Delta l_2^{(A)}) + (\Delta l_1^{(F)} + \Delta l_3^{(F)})$

Fig. 2 Example of chain extraction for $CC = k_{2,3}$

Minimal transfer method

As Fig. 2 and Fig. 3 illustrate, the principle of the method is to recognize first the two surfaces which delimit a functional requirement. They are noted l and m . The verification procedure is carried out for each functional requirement (CC) by successive elimination of the dispersions from single element columns and lines, except for columns l and m . The elimination process is repeated until the minimal transfer condition given by zero or two Δl per column is reached. The functional dimensions of the parts participating in the chain are then extracted from the surfaces (columns) containing the dispersions present on the corresponding line of the matrix.

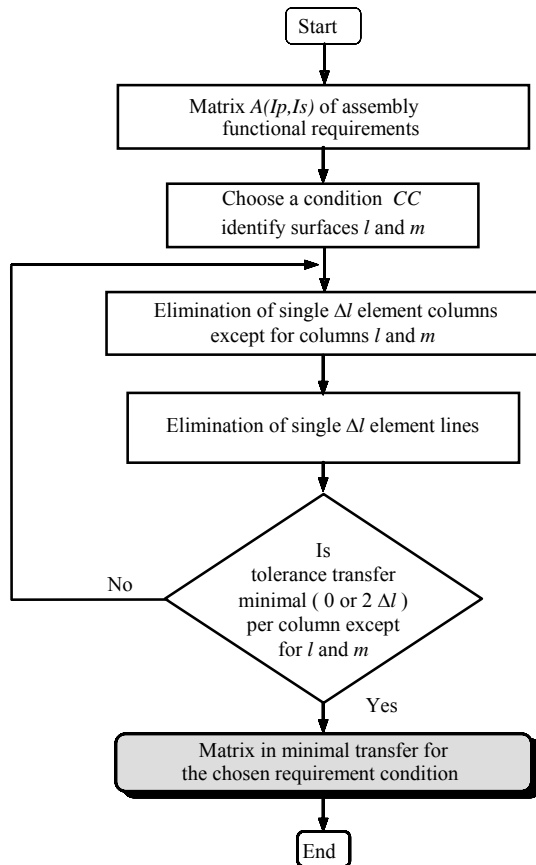


Fig. 3 Minimal transfer flowchart

Functional requirements verification based on standard minimal dispersions

The verification of the functional requirements is carried out by checking the feasibility of the assembly design in regards to the capabilities of the available manufacturing processes in the workshop in terms of minimal dispersions [11]–[12]. This condition is fulfilled when the manufacturing means can produce the design set dimensions imposed by the design office. In technical terms this condition is satisfied when the tolerance interval (IT) of the functional requirement CC is greater or equals the manufacturing stack up tolerance due to the summation of all dispersions Δl_i with reference minimal values [12] and given by (3).

$$IT_{CC} \geq \sum \Delta l_{ij} \quad (3)$$

C. Dispersions Optimization Procedure

Using the extracted tolerance chains, an optimization matrix of Ic lines and Id columns is built. Each line represents a functional requirement and each column represents a dispersion as shown in Fig. 4 for the sample example. It is well noticed that each line corresponds to a tolerance chain associated to a functional dimension chain. When a dispersion is present in a chain a variable x is affected to the corresponding position in the matrix. Otherwise the position takes the value zero [13].

Dispersions data matrix for design project :									
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT
1	$k_{2,3}$	x	x	x	x	0	0	0	1
2	$j_{4,5}$	x	x	0	x	x	x	x	2

Determine distribution order for rank 1										
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT	k'_1
1	$k_{2,3}$	x	x	x	x	0	0	0	1	0.25
2	$j_{4,5}$	x	x	0	x	x	x	x	2	0.333

Processing of line 1										
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT	k'_1
1	$k_{2,3}$	0.25	0.25	0.25	0.25	0	0	0	1	0.25

Save dispersions and determine distribution order for rank 2											
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT	k'_1	k'_2
1	$k_{2,3}$	0.25	0.25	0.25	0.25	0	0	0	1	0.25	-
2	$j_{4,5}$	0.25	0.25	0	0.25	x	x	x	2	0.3333	0.4166

Processing of line 2										
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT	k'_2
2	$j_{4,5}$	0.25	0.25	0	0.25	0.416	0.4166	0.4166	2	0.4166

Final matrix with optimized dispersions									
N°	CC	$\Delta l_1^{(A)}$	$\Delta l_1^{(F)}$	$\Delta l_2^{(A)}$	$\Delta l_3^{(F)}$	$\Delta l_3^{(G)}$	$\Delta l_4^{(G)}$	$\Delta l_5^{(A)}$	IT
1	$k_{2,3}$	0.25	0.25	0.25	0.25	0	0	0	1
2	$j_{4,5}$	0.25	0.25	0	0.25	0.4166	0.4166	0.4166	2

Fig. 4 Optimization for the sample example

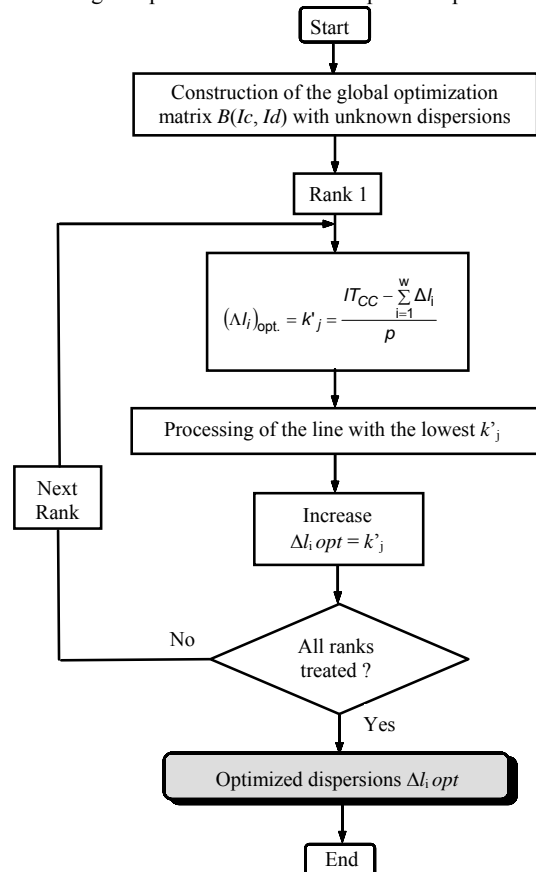


Fig. 5 Optimization flowchart

As the flowchart of Fig. 5 explains, the optimization process starts by computing a distribution coefficient k'_j for all the lines using (4); where w is the number of known dispersions, p the number of unknown dispersions and j the rank number.

$$(\Delta L_i)_{opt.} = k'_j = \frac{IT_{CC} - \sum_{i=1}^w \Delta L_i}{p} \quad (4)$$

Fig. 4 explains the optimization process for the sample example which is carried out by equal distribution for the line with the lowest k'_j . The computed dispersions of the line are then introduced in all the columns where they appear. The process is repeated for the remaining lines using the new ΔL_i values.

D. ISO Dimensions and Tolerances Synthesis Procedure

The designer generally determines, through classical design computations (material strength, weight,...), the limit values for design dimensions not to be exceeded. The dispersions method can be used to simulate and determine the optimal values of the functional dimensions which fulfil the functional requirements. Based on the fundamental model developed by Bourdet [7] and on the matrix format of the mechanical assembly, the average lengths L_i limited by two ΔL_i can then be calculated. For each part, the origin of basic average lengths are taken on the leftmost surface (surface 1, $L_1=0$). Using the functional requirements (CC) and the standard dimensions (CS) a system of equations is built to determine the basic average lengths L_i using (5) and (6) as follows :

$$(CC_{ij})_{moy} = L_j - L_i \quad (5)$$

$$(CS_{ij})_{moy} = L_j - L_i \quad (6)$$

The CS dimensions are selected to supplement the system among the functional dimensions of standard parts. In the case of unbounded dimensions, CC_{moy} and CS_{moy} are calculated by using the optimized dispersions by the following relations :

$$CC_{moy} = \frac{CC_{min} + (CC_{min} + \sum (\Delta L_i)_{opt.})}{2} \quad (7)$$

$$CS_{moy} = \frac{CS_{min} + (CS_{min} + \sum (\Delta L_i)_{opt.})}{2} \quad (8)$$

Equations (5) to (8) give a system of n equations and n unknown L_i . When the simulation lengths L_i are computed, the ISO average functional dimensions are calculated using (2). This procedure is automated as the flowchart of Fig. 6 shows. For the sample example there are five surfaces so four functional requirements are needed. The two functional conditions k and j are completed with two conditions as minimal dimensions in the dispersions matrix. These are given by the standard parts in the assembly. In the sample example, these are given by the nut G ($CS_{3,4}^{(G)} = 10$ mini) and the disc

F ($CS_{1,3}^{(F)} = 4$ mini).

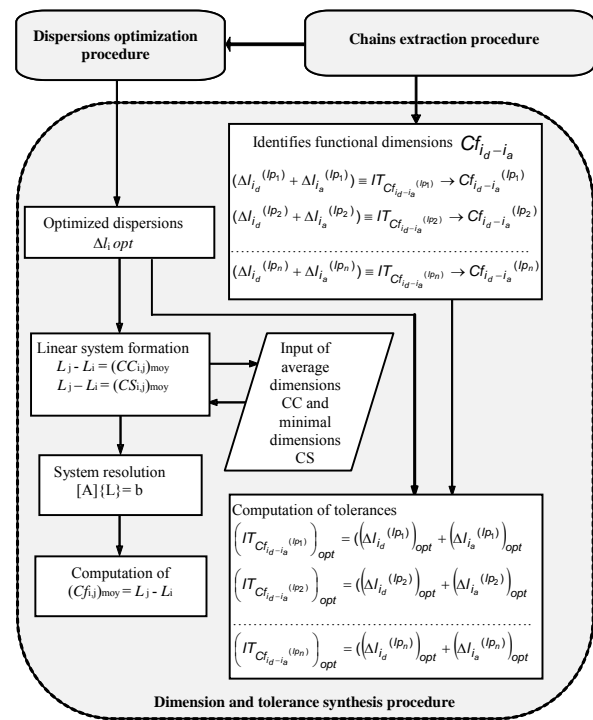


Fig. 6 Simulation flowchart

III. STATISTICAL APPROACH

A. Statistical Model

In the previous simulation, the tolerance synthesis procedure was based on an arithmetic equal increase of the dispersions values given by (3). However in reality, this distribution should take into account parameters such as the stochastic aspects of the machining dispersions as well as the complexity and the cost of the dimensions to be processed. To satisfy this objective, the arithmetic model (3) for the tolerance chain is reformulated into a statistical model which will assign the biggest tolerance to the most difficult dimension to machine. Assuming that all the components making up the dimension chain are independent and normally distributed, the statistical model (9) is obtained by applying the statistical parameter variance (σ^2) to the dimension chain (functional requirement CC_j and functional dimensions Cf_i).

$$\sigma_{CC_j}^2 = \sum_i \sigma_{Cf_i}^2 \quad (9)$$

Introducing K representing the probability of having a dimension within the tolerance interval T , the following relation where σ is the standard deviation can be written:

$$T_{CC_j} = K_{CC_j} \cdot \sigma_{CC_j} ; T_{Cf_i} = K_{Cf_i} \cdot \sigma_{Cf_i} \quad (10)$$

Replacing the standard deviations from (10) into (9) gives the following model for the design tolerance chain:

$$\left(\frac{T_{CC_j}}{K_{CC_j}}\right)^2 = \sum_{i=1}^{n_j} \left(\frac{T_{Cf_i}}{K_{Cf_i}}\right)^2 \quad (11)$$

In manufacturing practice K is equal to 6 [6]—[14] which will give from (11) the following root sum squares (RSS) statistical model:

$$T_{CC_j}^2 = \sum_{i=1}^{n_j} T_{Cf_i}^2 \quad (12)$$

B. Manufacturing Cost Based Tolerance Synthesis Model

Based on the cost-tolerance data in the literature [6]—[15], an approach which adequately describes the relationship between tolerance and associated cost is used [16]. This approach was previously developed for the optimization of manufacturing tolerances [17] and is now adapted for the synthesis of optimal design tolerances.

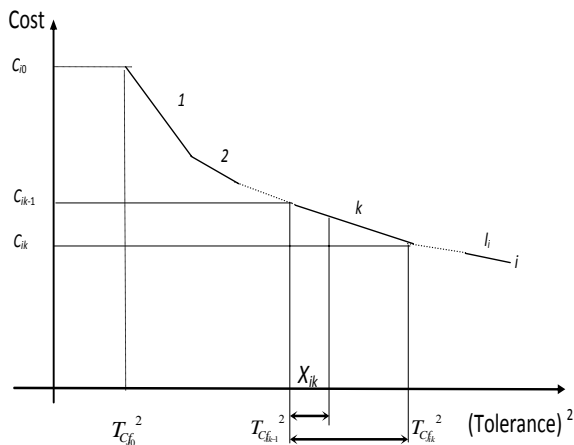


Fig. 7 Cost-tolerance model

As shown in Fig. 7, the cost curve for each dimension is approximated by a set of small linear segments. The objective function which is the sum of concave functions leads to the formulation of a linear programming problem. The tolerance and corresponding cost for design functional dimension Cf_i are given by (13) and (14).

$$T_{Cf_i}^2 = T_{Cf_{i0}}^2 + \sum_{k=1}^{l_i} X_{i,k} \quad ; \quad \forall i \quad (13)$$

$$C_i = C_{i0} + \sum_{k=1}^{l_i} V_{i,k} \cdot X_{i,k} \quad ; \quad \forall i \quad (14)$$

The slopes $V_{i,k}$ are computed as in

$$V_{i,k} = \frac{C_{i,k} - C_{i,k-1}}{T_{Cf_{i,k}}^2 - T_{Cf_{i,k-1}}^2} \quad ; \quad \forall i,k \quad (15)$$

The application of (13) and (14) to all the design functional dimensions affecting a functional requirement condition CC i.e. a design dimension chain, gives the following tolerance synthesis model for a dimension chain j :

minimize

$$C_{CC_j} = \sum_{i=1}^{n_j} (C_i - C_{i0}) = \sum_{i=1}^{n_j} \sum_{k=1}^{l_i} V_{i,k} \cdot X_{i,k}$$

subject to

$$\sum_{i=1}^{n_j} \sum_{k=1}^{l_i} X_{i,k} \leq T_{CC_j}^2 - \sum_{i=1}^{n_j} T_{Cf_{i0}}^2$$

$$0 \leq X_{i,k} \leq T_{Cf_{i,k}}^2 - T_{Cf_{i,k-1}}^2 \quad ; \quad \forall i,k$$

This optimization model represents a linear programming problem which is solved for the unit tolerance variables $X_{i,k}$ using the simplex technique [18]. The model can be separately applied to each tolerance chains j within the mechanical sub-assembly. However a number of these tolerances are common to several tolerance chains that the simulation module must take into account during the tolerance synthesis. If there are w dimension chains within the sub-assembly, each dimension chain j can have n_j dimensions where m_j dimensions are common to the other chains. This will result in the following:

$$w \text{ tolerance chains } \begin{cases} T_{Cf_{11}} & T_{Cf_{12}} & \dots & T_{Cf_{1m_1}} & \rightarrow m_1 \\ T_{Cf_{21}} & T_{Cf_{22}} & \dots & T_{Cf_{2m_2}} & \rightarrow m_2 \\ \dots & \dots & \dots & \dots & \dots \\ T_{Cf_{w1}} & T_{Cf_{w2}} & \dots & T_{Cf_{wm_w}} & \rightarrow m_w \end{cases}$$

In the process plan there will be m manufacturing dimensions common to two or more manufacturing chains; m is given then by:

$$m = m_1 \cap m_2 \cap \dots \cap m_w \quad (16)$$

Taking into account the fact of common functional dimensions to several dimension chains and in order to perform the optimization procedure to the whole mechanical sub-assembly, the final tolerance synthesis model to be integrated in the pre-project assembly simulation is reformulated as follows:

minimize

$$C_{Ass} = \sum_{j=1}^w \sum_{i=1}^{n_j-m_j} \sum_{k=1}^{l_i} V_{jik} \cdot X_{jik} + \sum_{t=1}^m \sum_{k=1}^{l_t} V_{tk} \cdot X_{tk}$$

subject to

$$\sum_{i=1}^{n_j-m_j} \sum_{k=1}^{l_i} X_{jik} + \sum_{t=1}^m \sum_{k=1}^{l_t} X_{tk} \leq T_{CC_j}^2 - \sum_{i=1}^{n_j-m_j} T_{Cf_{jio}}^2 - \sum_{t=1}^m T_{Cf_{t0}}^2 ;$$

$\forall j$

$$0 \leq X_{jik} \leq T_{Cf_{jik}}^2 - T_{Cf_{jik-1}}^2 ; \forall j, i, k$$

$$0 \leq X_{tk} \leq T_{Cf_{tk}}^2 - T_{Cf_{tk-1}}^2 ; \forall t, k$$

This tolerance synthesis model is built as a linear programming based optimization model [18]. The optimization process computes the unit tolerance variables X_{jik} and X_{tk} . The computed unit variables X_{jik} are then used to determine the individual tolerances for the functional dimensions i in each dimension chain j in the sub-assembly. On the other hand, the computed unit variables X_{tk} are used to determine the tolerances of the manufacturing dimensions common to two or more dimension chains in the sub-assembly.

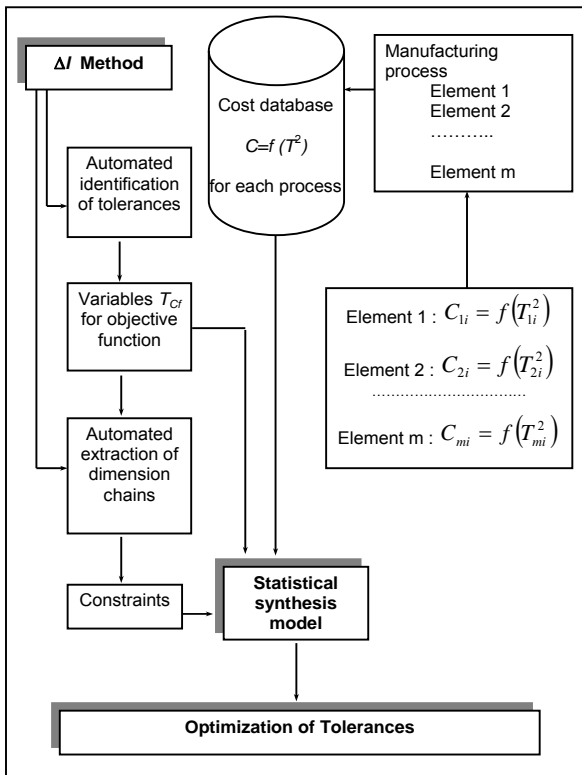


Fig. 8 Statistical optimization procedure

IV. MODEL APPLICATION AND TESTS

The developed statistical tolerance synthesis model has been programmed into the tolerance optimization procedure of the simulation module as illustrated in Fig. 8. This figure clearly shows that the model functional constraints are given

from the automatically extracted tolerance chains. The tasks of chain extraction and tolerance identification are performed using the ΔI method during the verification procedure of the simulation module. Fig. 9 and Fig. 10 show the screen results for the applied simulation to the sample example of Fig. 1.

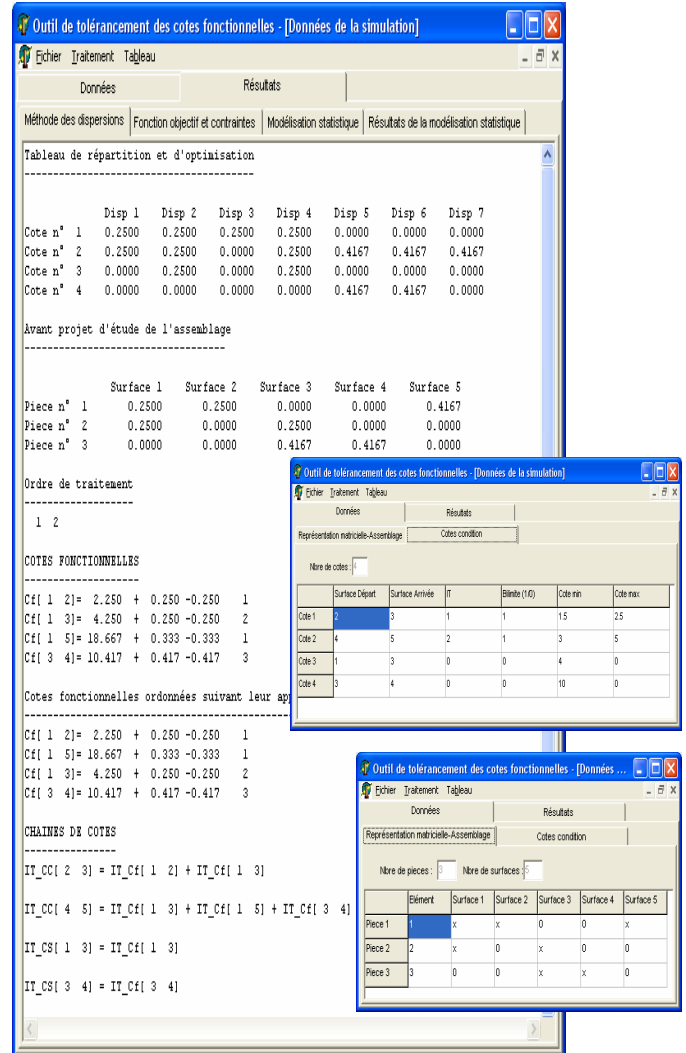


Fig. 9 Simulation data and ΔI optimization results

V. CONCLUSION

The objective of this work which is based on the approach of simultaneous engineering is to contribute to the development of a CAD/CAM tool for the simulation and synthesis of functional design dimensions expressed by their average values and their tolerance intervals. The dispersions CHAINES known as the ΔI method which proved to be reliable in the simulation of manufacturing dimensions was used to develop a methodology for the automation of the simulation. It also permits to express the design tolerances as function of the manufacturing tolerances represented by the dispersions. This methodology was constructed and tested manually step by step using a simplified example of a mechanical assembly. Afterwards, it was automated by the realization of a computer

program. The program was tested thereafter on the simpler example as well as on complex assemblies which are difficult or impossible to treat manually. The automatic treatment gave results agreeing with the manual processing for the illustrative example and showed the effectiveness of the automated simulation by solving the complicated examples. In addition, a statistical and cost based approach has been developed and integrated in the simulation module. This approach can be used for statistical tolerance synthesis.

make it possible to choose adequate and optimal average values and tolerance intervals for the functional dimensions among several possible solutions.

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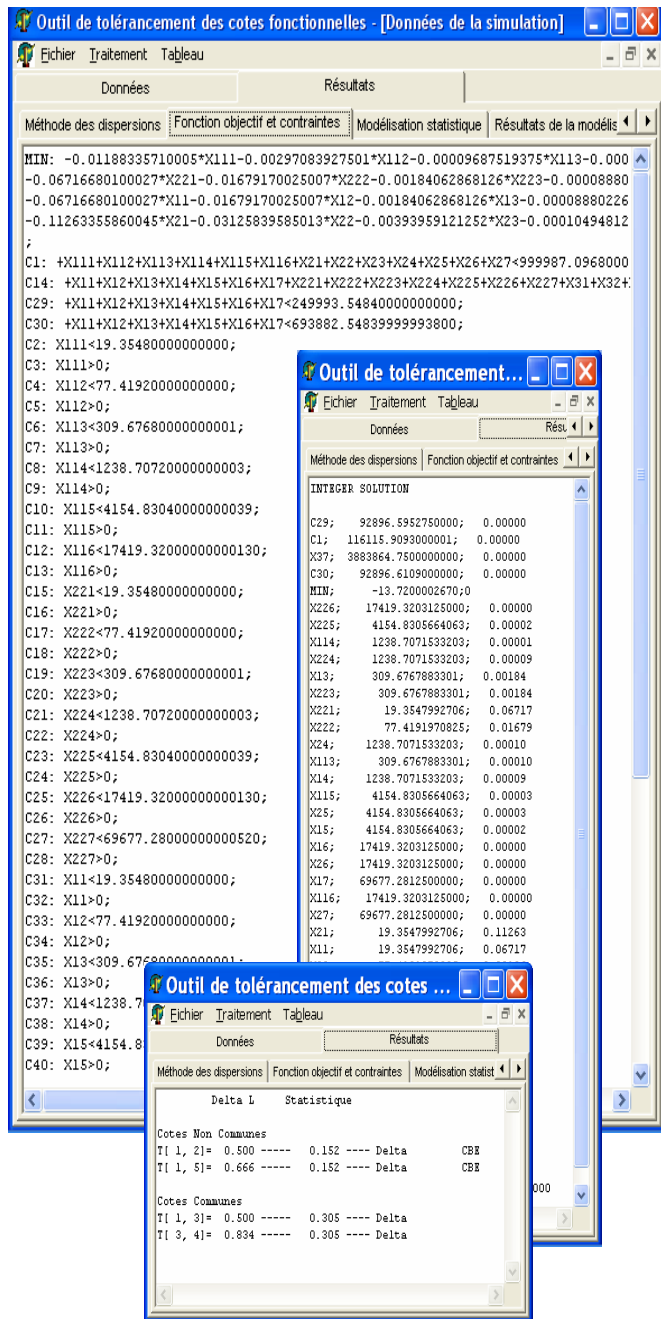


Fig. 10 Simplex data and statistical synthesis results

The final program can thus be used at will in order to simulate the functional requirements of design projects and to