Performance Improvement in Internally Finned Tube by Shape Optimization

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Abstract-Predictions of flow and heat transfer characteristics and shape optimization in internally finned circular tubes have been performed on three-dimensional periodically fully developed turbulent flow and thermal fields. For a trapezoidal fin profile, the effects of fin height h, upper fin widths d_1 , lower fin widths d_2 , and helix angle of fin γ on transport phenomena are investigated for the condition of fin number of N = 30. The CFD and mathematical optimization technique are coupled in order to optimize the shape of internally finned tube. The optimal solutions of the design variables (i.e., upper and lower fin widths, fin height and helix angle) are numerically obtained by minimizing the pressure loss and maximizing the heat transfer rate, simultaneously, for the limiting conditions of d_1 = 0.5~1.5 mm, d_2 = 0.5~1.5 mm, h= 0.5~1.5mm, γ = 10~30 degrees. The fully developed flow and thermal fields are predicted using the finite volume method and the optimization is carried out by means of the multi-objective genetic algorithm that is widely used in the constrained nonlinear optimization problem.

Keywords—Computational fluid dynamics, Genetic algorithm, Internally finned tube with helix angle, Optimization.

I. INTRODUCTION

THE high-integration trend in electronic devices inevitably has brought the increase of heat generation per unit area of the chips. The cooling technology is, therefore, becoming important along with the development of electronic devices for the steady operation of the core components. In order to obtain the thermal stability of the core electronic devices, internally finned tubes are widely used as one of the passive heat transfer enhancement techniques. Due to the trend of compactness in electronic devices, it is getting difficulties to have enough spaces available for the heat exchange from the components. Since the exchangers should be installed in the limited spaces, many factors such as the heat transfer, the size of the space available for the installment and the pressure drop are to be simultaneously considered and designed. The optimal design is, therefore, should be performed to obtain the optimal shape or structure of heat exchangers.

Performance analyses of internally finned tube for a thermal performance improvement have been performed numerically and experimentally for many years [1-3]. However, they have proposed the correlation equations for the design variables by considering only the flow and thermal characteristics of them.

In recent years, the use of commercial CFD codes for analyzing the flow and thermal fields in industrial applications has been dramatically increased due to their advanced computational capacity and analytic algorithm. In addition, many optimization techniques have been developed to obtain the optimal solutions. Therefore, much attention has been paid to the optimization of fluid/thermal systems by combining the CFD and optimization algorithm [4,5]. Lee et al.[4] obtained the optimal solutions by using a sequential quadratic programming (SQP) method, which is one of the local optimization methods, in an internally finned tube. However, they did not consider the effect of helix angle.

For the prediction of transport phenomena in fluid/thermal systems, high computational cost for function evaluation and the occurrence of numerical noise are commonly confronted. In addition, the optimal solutions by local optimization techniques may bring on a question whether the results are correct or not. Thus, a way to overcome the above mentioned problems is to introduce a global optimization technique.

In this study, the optimization is carried out to obtain the optimal fin shape which is attached at the tube inside wall with a rotational repeatedness. It can be completed when the heat transfer is maximized and the friction loss is minimized, simultaneously. Thermal and flow characteristics in the tube are predicted by using CFD in order to calculate two objective functions (i.e., heat transfer rate and friction coefficient). Genetic algorithm (GA) is adopted for optimization and an integration technology is developed to combine two different programs (that is, CFD and GA).

II. DEFINITION OF OPTIMAL DESIGN PROBLEM

A. Physical Model

The optimization problem considered in this study is to maximize the thermal performance of the internally finned tube which the fins are repeatedly attached at the tube wall with a helix angle (γ) as shown in Fig. 1. Fins, that have a trapezoidal cross-sectional area, are appeared repeatedly in the circumferential direction. Note that they are rotated around the center axis by an angle $\alpha = 2\pi/N$. Thus, only one module that contains one fin can be adopted as a computational domain and flow is the

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Fig. 1 Schematics of internally finned tube with helix angle

periodically fully developed in the main flow direction because the effect of entrance region can be ignored due to the small ratio of tube diameter to length (d/L = 1). It is assumed that the fluid (Pr = 7) is incompressible with constant properties and the flow is steady state and turbulent (Re=10,000). The temperature at tube wall included fins is kept at high temperature (T_w =350K) and at the inlet cold water (T_{in} =318K) is flow in.

B. Formulation for Optimization Problem

The optimization is to find the best design variables with the minimized/maximized objective function numerically. In this case, the design variables are commonly subjected to the constraint conditions. Thus, the nonlinear, constrained optimum design problem can be expressed mathematically as follows:

Find
$$\mathbf{X} = \{X_1, X_2, L X_N\}^T$$
 (1)

to minimize
$$F(\mathbf{X})$$

subject to $g_i(\mathbf{X}) \leq 0$

$$\mathbf{X}_{i}^{L} \le \mathbf{X}_{i} \le \mathbf{X}_{i}^{U} \text{ for } i = 1, N \tag{4}$$

where **X** represents the design variable vector and *N* is the number of design variables. $F(\mathbf{X})$ is the objective function which depends on the values of the design variables. $g_j(\mathbf{X})$ is the inequality constraints and \mathbf{X}_i^L and \mathbf{X}_i^U are the lower and upper limits of the design variables, respectively, and they simply limit the region of search for the optimization.

Design Variables: The geometric parameters which strongly influence the thermal performance of the tube are the fin height (*h*), upper and lower parts of fin (d_1 , d_2), and helix angle (γ), as shown in Fig. 1. Thus, four design variables are considered in this study (i.e., $\mathbf{X} = [h, d_1, d_2, \gamma]$).

Objective Functions: For a given operating condition, increasing the heat transfer rate, however, is accompanied with

increasing the pressure drop as a necessity. It is obvious from this phenomenon that a high thermal performance (or cooling efficiency) can be obtained both by maximizing the heat transfer rate and the pressure drop in the case of fixed volume of tube. Therefore, Nusselt number (Nu) and friction coefficient (f) have been generally adopted as the objective functions in this study that are defined as

$$Nu = \frac{hl_e}{k} , \quad f = \left(\frac{\partial P}{\partial z} l_c / \frac{1}{2} \rho V_c^2\right)$$
(5)

Constraints: The constrained conditions and the upper and lower limits are determined by considering the manufacturing conditions. Thus, in this study, the constraints are given as follows;

0.5 $mm \le h$, d, $d_2 \le 1.5 mm$, $0^\circ \le \gamma \le 20^\circ$ (6)

III. OPTIMIZATION

A. Genetic Algorithm

A genetic algorithm is a unique global optimum algorithm based on the survival of the fittest, the mechanism of natural selection and reproduction [6-8]. Each individual has the number of strings representing characteristics of an individual design instead of the gene in natural life. One generation consists of a group of individuals. The string belonging to each individual is decoded to design variables and its relative fitness in the generation is evaluated. The fitness shows the measure of how the individual adapts it to the given environments. The higher fitness individuals have the more chances to generate offspring, which have similar genes. According to evolution, the superior genes augment their numbers in the generation and contribute to improving average fitness consequently. In terms of using information coming from across design space, the GA is different from the typical optimization methods using a derivative of a design point. This unique feature can help to find global optima instead of local ones. Consequently, the global optimization is the GA's unique feature that can be applied to intricate and multi-objective design spaces, designs with noise, and robust designs.

B. Operators

(2)

(3)

In a generation, new individuals for the next generation can be obtained by using genetic operations such as selection, crossover and mutation. The selection introduces the direction of the evolution for the survival of the fittest and chooses parents from the genetic pool. The crossover is a process reproducing the superior individuals to evolve the generation globally. Since crossover and selection only rearranging the genes are deterministic processes, it is difficult to introduce completely new genes, which did not exist previously. The deficiency of searching design space leads a lot of danger to converge into the local optima after all. The mutation can prevent the dangers and keep its balance to find the global optima. Searching for the balance between cross-over and mutation can effectively lead the GA to the global optima. These genetic operations mimicking natural life can improve the average fitness of a generation constantly. A very new individual, which has an identical twin in the new generation, is ignored for keeping the diversity of the system.

Selection: In this study, owing to the multi-objectives, the tournament selection is used. Pairs of the individuals are picked at random from the population. The ones with higher fitness are copied into the mating pool. Numbers of tournaments, n competitions instead of one, are employed. As a result, this can manage the selection pressure. The high selection pressure leads to fast convergence, but finds the local optima instead of global ones because of insufficient searching. In contrast, the low selection pressure accounts for slow convergence. Steps constitute the selection of individuals for the mating pool below.

Is there a dominant individual in a pair of the individuals?
Is there a pareto individual in a pair of the individuals in the same generation?

3) Are both pareto individuals? The less crowded individual is selected.

4) The superior individual between two is selected.

These processes are close to the mathematical definitions of pareto optimum, and provide non-dominated solutions.

Crossover: The crossover that exchanges genes between two selected individuals can reproduce new offspring, which have similar genes but different characteristics from their parents. The crossover points are randomly chosen. The strings between the two crossover points will be exchanged simply for the two-point crossover. The frequency of crossover is governed by a given crossover rate or probability of a crossover. A larger crossover rate increases the recombination of building blocks, however, with an increasing probability of losing good strings. The number of crossover points has close relation to the survival of schema that is the meaningful pattern in the gene. Long and high fit schemas start from short and low fit ones. Taking into the consideration of schema's survival, the two-point crossover is more favorable [9,10]. In this study, the two-point crossover is selected and a single point is optional.

Mutation: the uniform mutation, which mutates all the genes at the same rate, was applied. The gene was represented by binary data and XOR binary operation was sufficient for the mutation. The mutation rate was set to be below 0.5% for the balance between exploration and exploitation that can warrant the global optimization.

Niche: The properties of adjacent individuals in the design space are similar to each other. For extending exploration, it is necessary to control the number of individuals inside the niche radius. Before sufficient exploration or at an early stage of evolution, the existence of local optima is very attractive. The offspring tends to gather around the local optimum points instead of global ones, which leads early mature convergence. The niche is able to give a change to explore the design space and prevent GA from early mature convergence. In this study, the niche distance employs the binary distance between two individuals instead of the n dimensional norm.

$$\frac{r_{ij}}{R} = \sum_{k=1}^{L} \left[\frac{\left| d_i - d_j \right|}{R} \right]_k = \sum_{k=1}^{L} \left[\frac{m \cdot \Delta x}{n \cdot \Delta x} \right]_k = \sum_{k=1}^{L} \left[\frac{m}{n} \right]_k \tag{7}$$

where $|d_i - d_j|_k$ is the distance between *i* and *j* individual in *k* variable and *m* is the binary distance. *n* is the niche binary distance. Using the niche radius in real design space could not represent all niches at once. The niche radiuses for each design variable are required. In the case of the binary distance, however, one niche radius is sufficient.

IV. FLOW AND THERMAL FIELDS

A. Governing Equations

In order to optimize the fin shape of internally finned tube for high thermal performance, heat transfer rate and pressure loss, which can be predicted by calculation of flow and thermal fields, are maximized and minimized, respectively. Therefore, it is important to understand the transport phenomena in the tube.

The physical problem considered in this study is the three-dimensional turbulent mixed convective flow of steady and incompressible fluid. The fluid properties are taken to be constant except for the density in the buoyancy term of the momentum equation. The effects of viscous dissipation and radiation heat transfer are assumed to be negligibly small.

Circumferential direction: All dependent variables including velocities and scalar quantities are periodically repeated along the circumferential direction for one period [11] and they are given by

$$\phi(\mathbf{s}) = \phi(\mathbf{s}, n\alpha) \tag{7}$$

where **s** is a position vector, α a rotational angle, and *n* the number of module. Equation (7) means that all flow conditions are moved by a period(α) in the circumferential direction.

Main flow direction: For a periodically fully developed flow, velocities and pressure are expressed the following equations;

$$u_i(\mathbf{s}) = u_i(\mathbf{s}, nL) \tag{8a}$$

$$p(\mathbf{s}) = -\beta_{1i}x_i + \beta(\mathbf{s}) \tag{8b}$$

where *L* means the length of repeated module, β is the linear component of a mean pressure gradient, and β /dis the periodic pressure. The term $\beta_{1i}x_i$ in (8b) represents the non-periodic pressure drop and becomes a driving force in the flow direction. In a fully developed flow field with constant wall temperature, the fluid temperature exponentially approaches to the wall temperature so that it is not consistent with any boundaries. Thus, it is necessary to define the appropriate temperature scale for a periodic flow [6].

B. Turbulent Modelling

Flow in a tube is turbulent so that the Reynolds averaged Navier-Stokes equation should be calculated for a steady and incompressible flow. In this study, the flow domain is divided into two regions such as near wall and fully turbulent regions and adopted a standard turbulent model [12] and low-Reynolds model, which is proposed by Norris and Reynold [13], respectively. According to this model, turbulent kinetic energy (k) and its dissipation rate (ε) are expressed in a tensor form as follows;

$$\frac{\partial}{\partial x_j} \left(\rho u_i k \right) = \frac{\partial}{\partial x_j} \left[\frac{\mu_i}{\sigma_k} \frac{\partial k}{\partial x_j} \right] + P_k - \rho \varepsilon \tag{9}$$

$$\varepsilon = \frac{k^{3/2}}{l_{\varepsilon}} \left(1 + \frac{C_{\varepsilon}}{\operatorname{Re}_{y}} \right)$$
(10)

where i = 1,2 and 3 denote *x*, *y*, and *z* directions, respectively. The term P_k in Eq. (9) stands for the production term. The model constants and various functions used in the turbulent model are detailed in References [12,13].

C. Boundary Conditions

A no-slip boundary condition for all solid walls is assigned for velocity. At the inlet of the tube, the coolant of a constant temperature ($T_{in} = 318$ K) flows downward with a constant velocity. The corresponding turbulent kinetic energy and its dissipation rate are $k_{in} = 1.5I_0^2 w^2$ and $\varepsilon_{in} = k_{in}^{3/2}/L_{\varepsilon}$, respectively. Here I_0 means the local turbulence intensity (= 0.1) and L_{ε} is a length scale for dissipation, taken here as D_h . The periodic boundary conditions are imposed for all dependent variables at inlet and exit as given in Eq.(7) and (8).

D. Numerical Procedure

The grid systems are altered automatically because fin shape is changed each iteration during optimization. For the length of the main flow direction, only three-layer grid system is adopted due to the negligible effect on flow characteristics and the length of each layer is 2mm. The prediction of flow and thermal fields is carried out by SARD-CD [14] which is one of the commercial flow/thermal prediction programs. The solutions are treated as converged ones when the sum of residual and the relative deviation of dependent variables between consecutive iterations are less than 10⁻⁵.

V. NUMERICAL METHODOLOGY FOR OPTIMIZATION

The following three programs are used to obtain the optimal design variables of the internally finned tube;

(1) The main program which defines various arrays and parameters,

(2) The analyzer that evaluates the objective functions (i.e., program for predicting the flow and thermal fields), and

(3) The optimizer which can solve a nonlinear optimization problem.

Once the objective functions (Nu and f) are obtained as the results of calculation of flow and thermal fields by the analyzer, the main program calls the optimizer to proceed with optimization. The optimizer may modify the design variables. When the optimizer requires new values of the objective functions, it returns to the main program and the analyzer is called to calculate them. In this step, the analyzer should generate a new grid system because new design variables are



Fig. 2 Numerical procedure for optimization of internally finned tube by the genetic algorithm

proposed by the optimizer. This process is repeated until the optimization is complete and is performed automatically. As a result of optimization, the optimal design variables and the corresponding pressure drop and heat transfer rate are obtained. Fig. 2 shows the numerical methodology for optimization using the genetic algorithm.

VI. RESULTS AND DISCUSSION

The optimal values of the design variables for trapezoidal fins in internally finned tube which fins are attached at the tube wall with helix angle are numerically acquired by using CFD and multi-objective GA method. Table I shows baseline geometry of fin and objective functions (f and Nu). The optimum factors used in a genetic algorithm are listed in Table II.

A. Validation for Computational Results

The flow and thermal characteristics in internally finned tube are complicated due to its geometry. Thus, the results of CFD are verified in a smooth tube and are shown in Table III. When the temperatures variations of wall and fluid are little, an experimental correlation for Nusselt number can be expressed as follows;

$$Nu = 0.023 \,\mathrm{Re}^{0.8} \,\mathrm{Pr}^{0.4} \tag{11}$$

Friction coefficient (f) for a smooth tube is given by [15],

$$f = (1.82 \log_{10} \text{Re} - 1.64)^{-2}$$
(12)

and the above equation (12) has an error of 6% in the rage of

0.5 < Pr < 200. As shown in Table III, the maximum error shows about 5.3% in a friction coefficient for Re = 20,000 while the error for other cases is less than 1%. Therefore, the results are good agreement with those of experiment.

TABLE I Initial Conditions and Their Objective Functions

Design variab	oles	Objective functions		
Fin height (<i>h</i>), Upper width (d_1), Lower width (d_2), Helix angle (γ),	1.0 mm 1.0 mm 0.5 mm 15 degree	Friction coeff. (f), 0.0499 Nusselt number (Nu), 116.03		

TABLE II Optimum Factors in Genetic Algorithm

Optimum factor	Value
Population	35
Generation	30
Cross over rate	0.8
Mutation rate	0.5%
Tournament level	2
Niche	3

TABLE III ALIDATION WITH A SMOOTH CIRCULAR TUBE

	Experiment[15]		This study	
Re	f	$Nu/Pr^{0.4}$	f	<i>Nu</i> / Pr ^{0.4}
10,000	0.03174	36.4525	0.0309	36.9911
15,000	0.02815	50.4198	0.0279	51.0155
20,000	0.02611	63.4676	0.0275	63.7267

B. Optimal Solutions

In general, the multi-objective optima cannot be defined uniquely except for the linearly independent system. Thus the multi-objective optimization technique can only optimize a vector-valued objective function. Unlike the single or weighted objective optimization, the solutions are not a single point, but a family of points known as the Pareto-optimal set. In case of multi-objective optimization, whole designs can be divided into dominated and non-dominated ones. Pareto optimal in the Pareto principle states that there are optimal solutions that cannot be improved upon without disadvantaging at least one objective. The non-dominated optima can be a Pareto set, which is placed on the front line of the design space. The gradient-based optimization technique makes objectives into a single objective with a weighting or a utility function. It, however, may distort the original design space and one may get the solution of one of the Pareto frontiers or the local optima. When the orders of magnitude of objectives are significantly different, the gradient-based optimizer aims for the biggest objective that contributes the most. The normalization of objectives can give some relief, but the basic distortion might still remain. In this study, owing to using multi-objective GA, simply MOGA, every objective is compared and evaluated independently. It is free from normalization and localization. For the optimization by genetic algorithm, 30 individuals for one population are used. The selection pressure for convergence acceleration is used as 2. In each tournament, two candidates are randomly selected, and through 2 competitions, the winner has a chance to become a parent for the reproduction. Two cutting lines are used to effectively maximize the life of schema, which means useful patterns in the gene. To keep the balance between exploitation and exploration, 0.5% of mutation rate is used. When a new offspring individual is found to be a genetic twin in the next generated.

Fig. 3 shows the typical Pareto sets among the total Pareto sets of 195. Filled circles represent the Pareto frontiers and hollow circles stand for the dominated solutions. Note that the Pareto #1 represents the individual which has the smallest value of friction coefficient, while Pareto #195 means the individual that the heat transfer rate is the highest. Both the Nusselt number and friction coefficient are linearly dependent. As shown in Fig. 3, since the objectives, the Nusselt number and the friction coefficient, are maximized and minimized, respectively, some Pareto individuals are placed from the bottom left corner to the top right corner, but others are located behind the Pareto frontier line.



Fig. 3 Pareto sets and dominated individuals with objectives

Fig. 4 depicts the variation of design variables $(h, d_1, d_2, and \gamma)$ for randomly selected Pareto frontiers. As shown in figure, the design variables according to the objective functions change with a tendency. The helix angle does not change but approaches to its upper limit value ($\gamma = 20^{\circ}$) over the Pareto #117. This means that the effect of helix angle on heat transfer rate in internally finned tubes is large compared with other variables. It can be also found that the height of fin is steadily lengthened as the number of Pareto frontier increases because it has a strong influence on the heat transfer rates. From the above two phenomena, it is concluded that for a fixed helix angle the longer fin height have an advantage of the improvement of heat transfer. Especially, the lower fin width (d_2) is for nothing in the optimal solutions for all Pareto frontiers. Fig. 4 also shows that the fin shape for Pareto #1 is similar with that of



Fig. 4 Variations of design variables for Pareto individuals

Pareto #36 except for the helix angle. In addition, for all Pareto frontiers, d_1 becomes to be larger than d_2 in spite of no-use of constraint.

To help the understanding of Fig. 4, the optimum values of the design variables and the corresponding objective functions for various Pareto frontiers are listed in Table IV. We are also able to compare each optimum solution from the table because it shows the results of the degree of importance of the two objective functions (i.e., Nu and f). The designer can be simply selected from Table IV for the most useful geometrical configurations of the internally finned tube. It seems to be that that the results of Pareto#195 can not be used as a optimal solution because the friction coefficient increases by 11% while Nusselt number merely augments by 0.9% from 165.8 to 164.33 compared with Pareto#191. Therefore, the optimal values for Pareto#195 are not appropriate for the optimal solutions within the range of this study.

VII. CONCLUSION

We numerically obtained the optimum design variables of internally finned tube with helix angle to minimize the friction coefficient and to maximize the heat transfer rate on the threedimensional periodically fully developed flow and thermal fields. The integration of multi-objective genetic algorithm for optimization technique and CFD was performed for completing

TABLE IV Optimal Solutions for Various Pareto Sets

			unit: m	m or degree		
	Design variables				Objectives	
	h	d_l	d_2	γ	$f(10^{-2})$	Nu
1	0.502	1.440	0.545	7.312	3.33	81.38
36	0.504	1.439	0.561	16.38	3.69	98.32
117	0.582	0.953	0.538	19.98	4.25	122.16
151	0.708	0.763	0.561	19.98	4.85	135.52
168	0.862	0.696	0.538	19.98	5.44	146.62
182	0.940	0.651	0.545	19.98	5.89	154.53
191	1.148	0.637	0.538	19.98	6.66	164.33
195	1.394	0.907	0.561	19.98	7.39	165.80

the optimization. As the results of optimization, the following conclusions were obtained: The Pareto frontier sets could be obtained for the multi-objective optimization problem. The most dominant design variables for the pressure drop and thermal resistance were the helix angle (γ) and the fin height (h) while the effect of lower fin width (d_2) on them was relatively small compared to the other two design variables. The results also showed that the optimal design variables for the Pareto #191, which is the case of heat transfer rate, were h =1.15 mm, $d_1 = 0.64$ mm, $d_2 = 0.54$ mm and $\gamma = 19.98^{\circ}$. In this case, Nusselt number for the optimum model was increased by 41.6 %, while the pressure drop was increased by 25.1 % compared to those of the baseline model. It was also found that the genetic algorithm for multi-objective problem can be a popular technique by comparing with a local optimization technique for its accuracy and efficiency. The results of this work can offer designers the information they need to select the optimal design variables corresponding to the preferred objective functions.

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