

# Elimination Noise by Adaptive Wavelet Threshold

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**Abstract**—Due to some reasons, observed images are degraded which are mainly caused by noise. Recently image denoising using the wavelet transform has been attracting much attention. Wavelet-based approach provides a particularly useful method for image denoising when the preservation of edges in the scene is of importance because the local adaptivity is based explicitly on the values of the wavelet detail coefficients. In this paper, we propose several methods of noise removal from degraded images with Gaussian noise by using adaptive wavelet threshold (Bayes Shrink, Modified Bayes Shrink and Normal Shrink). The proposed thresholds are simple and adaptive to each subband because the parameters required for estimating the threshold depend on subband data. Experimental results show that the proposed thresholds remove noise significantly and preserve the edges in the scene.

**Keywords**—Image denoising, Bayes Shrink, Modified Bayes Shrink, Normal Shrink.

## I. INTRODUCTION

NOISE may come in the form of thermal noise, measurement errors, or introduced by recording medium, transmission medium, and digitization. Over the last decades, there has been abundant interest for noise removal in signals and images. Image denoising is a field of engineering that studies methods used to recover an original scene from degraded observations. It is an area that has been explored extensively in the image processing, astronomical, and optics communities for some time. Recently, various wavelet based method have been proposed for the purpose of image denoising. The wavelet shrinkage method is a nonlinear image denoising procedure to remove noise by shrinking the empirical wavelet coefficients in the wavelet domain. The method is based on thresholding in the scene that each wavelet coefficient of the image is compared to a given threshold; if the coefficient is smaller than the threshold, then it is set to zero, otherwise it is kept or slightly reduced in magnitude. The intuition behind such an approach follows from the fact that the wavelet transform is efficient at energy compaction, thus small wavelet coefficients are more likely due to important image features, such as edges.

Originally, Donoho and Johnstone proposed the use of a universal threshold uniformly throughout the entire wavelet decomposition tree [6,7]. Then the use of the wavelet tree was

found to be more efficient [3,10,11]. Some methods of selecting thresholds that are adaptive to different spatial characteristics have recently been proposed and investigated [5,8,9]. In general, adaptive approaches have been found to be more effective than their global counterparts.

The paper is organized as follows. Section II introduces adaptive wavelet threshold (BS, MBS, NS). Detailed Experimental results reporting the performance of the proposed algorithm are given in section III. Finally, our conclusion is given in section IV.

## II. ADAPTIVE WAVELET THRESHOLD

### A. Bayes Shrink(BS)

Wavelet shrinkage is a method of removing noise from images in wavelet shrinkage, an image is subjected to the wavelet transform, the wavelet coefficients are found, the components with coefficients below a threshold are replaced with zeros, and the image is then reconstructed [5].

In particular, the BS method has been attracting attention recently as an algorithm for setting different thresholds for every subband. Here subbands are frequently bands that differ from each other in level and direction. The BS method is effective for images including Gaussian noise. The observation model is expressed as follows:  $Y = X + V$

Here  $Y$  is the wavelet transform of the degraded image,  $X$  is the wavelet transform of the original image, and  $V$  denotes the wavelet transform of the noise components following the Gaussian distribution  $N(0, \sigma_v^2)$ . Here, since  $X$  and  $V$  are mutually independent, the variances  $\sigma_y^2$ ,  $\sigma_x^2$  and  $\sigma_v^2$  of  $y$ ,  $x$ , and  $v$  are given by:

$$\sigma_y^2 = \sigma_x^2 + \sigma_v^2 \quad (1)$$

Let us present a method for deriving of the noise:

It has been shown that the noise standard derivation  $\sigma_v$  can be accurately estimated from the first decomposition level diagonal subband  $HH_1$  by the robust and accurate median estimator [7].

$$\hat{\sigma}_v = \frac{\text{median}(|HH_1|)}{0.6745} \quad (2)$$

2) Estimation of the variance of the degraded image  $y$ :  
The variance of the degraded image can be estimated as

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$$\hat{\sigma}_y^2 = \frac{1}{M} \sum_{m=1}^M A_m^2 \quad (3)$$

Where  $A_m$  are the coefficients of wavelet in every scale,  $M$  is the total number of coefficient of wavelet.

3) Calculation of the threshold value  $T$ :

$$T_{BS} = \frac{\hat{\sigma}_v^2}{\hat{\sigma}_x} \quad (4)$$

Where

$$\hat{\sigma}_x = \sqrt{\max(\hat{\sigma}_y^2 - \hat{\sigma}_v^2)} \quad (5)$$

Note that in the case where  $\hat{\sigma}_v^2 \geq \hat{\sigma}_y^2$ ,  $\hat{\sigma}_x^2$  is taken to be zero, i.e.  $T_{BS} \rightarrow \infty$ . Alternatively, in practice, one may choose  $T_{BS} = \max\{|A_m|\}$ , and all coefficients are set to zero.

In summary, the bayes shrink thresholding technique performs soft thresholding with adaptive, data driven, subband and level dependent near optimal threshold given by:

$$T_{BS} = \begin{cases} \frac{\hat{\sigma}_v^2}{\hat{\sigma}_x}, & \text{if } \hat{\sigma}_v^2 < \hat{\sigma}_y^2 \\ \max\{|A_m|\}, & \text{otherwise} \end{cases} \quad (6)$$

#### B. Modified Bayes Shrink (MBS)

There is the problem of noise not being sufficiently removed in an image processed using bayes shrink method. But modified bayes shrink remove noise better than bayes shrink. It performs its processing using threshold values that are different for each subband coefficient the threshold  $T$  can be determined as follows:

$$T_{MBS} = \frac{\beta \hat{\sigma}_v^2}{\hat{\sigma}_x} \quad (7)$$

$$\beta = \sqrt{\frac{\log M}{2 \times j}} \quad (8)$$

$M$  is the total of coefficients of wavelet.  $j$  is the wavelet decomposition level present in the subband coefficients under scrutiny.

#### C. Normal Shrink(NS)

In this section, we calculate the value of threshold by normal shrink [2].

$$T_{NS} = \frac{\lambda \hat{\sigma}_v^2}{\hat{\sigma}_y} \quad (9)$$

$$\lambda = \sqrt{\log\left(\frac{L_k}{J}\right)} \quad (10)$$

$L_k$  is length of the subband at  $k^{\text{th}}$  scale.  $J$  is the total number of decompositions.

We can summarize the process BS, MBS, NS as follows:

A) Perform Multiscale decomposition of the image corrupted by Gaussian noise using wavelet transform.

B) Estimate the noise variance ( $\hat{\sigma}_v^2$ ) and for each scale compute the scale parameter.

C) For details of total subbands at first compute the standard deviation  $\hat{\sigma}_y, \hat{\sigma}_x$  after compute threshold finally apply soft thresholding to the noisy coefficients.

D) Invert the multiscale decomposition to reconstruct the denoised image.

The procedure was explained in Fig.1

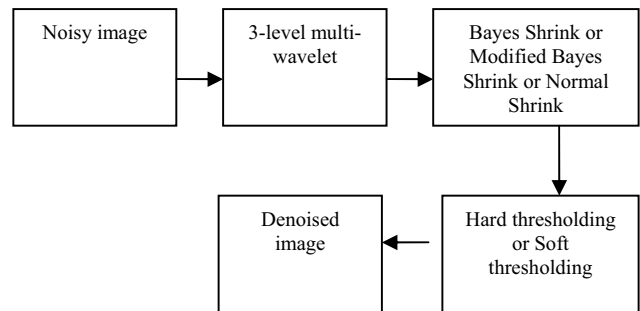


Fig. 1 Image denoising by adaptive wavelet threshold

### III. EXPERIMENTAL RESULTS

Our test consists of an image of peppers and house of size (256×256). The kind of noise is Gaussian with variance 0.06. In the test, Gaussian noise is added to original image. In this test, we used from several methods for image denoising.

Median filter and other nonlinear filters, as well as wiener filters and other optimization filters have been used as denoising methods for images with noise. A median filter is an effective filter for edge preservation which is not possible with conventional linear filter. As well as, the wiener filter is more effective than the conventional linear filter in preservation of the edges and high frequency components in images and it works efficiently when the noise is Gaussian noise.

The results show both of filter, denoise weakly as well as new blurriness occurs in the processed image. Therefore they remove a lot of details of original image during denoising.

In global threshold, we used from one threshold value. In the wavelet decomposition, the magnitude of the coefficients varies depending on the decomposition level. Hence, if all levels are processed with one threshold value the processed image may be overly smoothed so that sufficient information preservation is not possible and the image get blurry. Therefore the method is not suitable.

The result shows Bayes shrink performs denoising that is consistent with the human visual system that is less sensitive to the presence of noise in vicinity of edges. However, the presence of noise in flat regions of the image is perceptually more noticeable by the human visual system. Bayes shrink performs little denoising in high activity sub-regions to preserve the sharpness of edges but completely denoised the flat sub-parts of the image.

Performance of normal shrink is similar to bayes shrink. But normal shrink preserved edges better than noise removal

method using the bayes shrink method as well as removing noise better than bayes shrink.

The result shows The modified bayes shrink yields the best results for denoising and also adopts a thresholding strategy that not only performs well. But it is also intuitively appealing as well as the results of simulations performed showed that the modified bayes shrink preserved edges better than bayes shrink and normal shrink.

We can use from MSE and SNR as global measure of objective improvement. The value of MSE represents mean square error and SNR shows the value of removing noise. The equations are as follows:

$$MSE = 10 \log_{10} \left\{ \frac{1}{N^2} \sum_{x,y} [X(x,y) - \hat{X}(x,y)]^2 \right\} \quad (11)$$

$$SNR = 10 \log_{10} \left\{ \frac{\sum_{x,y} [X(x,y)]^2}{\sum_{x,y} [X(x,y) - \hat{X}(x,y)]^2} \right\} \quad (12)$$

For an  $N \times N$  image, where  $X(x,y)$  and  $\hat{X}(x,y)$  are the original image and denoised image.

The result shows all of the adaptive wavelet threshold (normal shrink, bayes shrink and modified bayes shrink) remove noise better than others. But, it depends on noise, one of the adaptive wavelet threshold is better. In general, in low noise, normal shrink is the best because it has minimum MSE and maximum SNR and in high noise, modified bayes shrink is the best because it has minimum MSE and maximum SNR (signal to noise ratio).

The results bring in the Tables (I, II, III, IV, V, VI, VII, and VIII).

TABLE I

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.001$  FOR REMOVING NOISE [NOISY PEPPERS] (LOW NOISE)

Method	MSE	SNR
Without filtering	42.22	24.40
Median filter	41.49	25.13
Wiener filter	38.92	27.71
Soft threshold	45.25	21.37
Hard threshold	40.91	25.72
Global threshold	43.17	23.45
Normal shrink	38.90	27.73
Bayes shrink	39	27.56
Modified Bayes shrink	39.03	27.59

TABLE II

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.01$  FOR REMOVING NOISE [NOISY PEPPERS] (LOW NOISE)

Method	MSE	SNR
Without filtering	52.07	14.55
Median filter	46.36	20.26
Wiener filter	45.33	21.29
Soft threshold	49.48	17.14
Hard threshold	46.82	19.80
Global threshold	47.85	18.77
Normal shrink	45.21	21.41
Bayes shrink	45.38	21.24
Modified Bayes shrink	45.41	21.21

TABLE III

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.03$  FOR REMOVING NOISE [NOISY PEPPERS] (HIGH NOISE)

Method	MSE	SNR
Without filtering	56.54	10.08
Median filter	50.16	16.46
Wiener filter	49.68	16.94
Soft threshold	50.75	15.87
Hard threshold	49.81	16.81
Global threshold	49.69	16.93
Normal shrink	48.20	18.42
Bayes shrink	48.05	18.58
Modified Bayes shrink	47.86	18.76

TABLE IV

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.06$  FOR REMOVING NOISE [NOISY PEPPERS] (HIGH NOISE)

Method	MSE	SNR
Without filtering	58.47	8.15
Median filter	52	14.62
Wiener filter	51.57	15.06
Soft threshold	51.23	15.39
Hard threshold	50.62	16
Global threshold	50.56	16.06
Normal shrink	51.57	15.06
Bayes shrink	49.18	17.44
Modified Bayes shrink	49.11	17.51

TABLE V

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.001$  FOR REMOVING NOISE [NOISY HOUSE] (LOW NOISE)

Method	MSE	SNR
Without filtering	42.24	25.09
Median filter	40.03	27.30
Wiener filter	39.50	28.82
Soft threshold	43.29	24.09
Hard threshold	39.79	27.53
Global threshold	41.05	26.27
Normal shrink	37.90	29.43
Bayes shrink	38.06	29.26
Modified Bayes shrink	38	29.33

TABLE VI

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.003$  FOR REMOVING NOISE [NOISY HOUSE] (LOW NOISE)

Method	MSE	SNR
Without filtering	46.95	20.37
Median filter	42.26	25.07
Wiener filter	40.99	26.34
Soft threshold	44.93	22.39
Hard threshold	41.93	25.39
Global threshold	43.25	24.08
Normal shrink	40.84	26.49
Bayes shrink	40.86	26.47
Modified Bayes shrink	40.88	26.46

TABLE VII

COMPARISON DIFFERENT METHODS WITH  $\sigma_v^2 = 0.03$  FOR REMOVING NOISE [NOISY HOUSE] (HIGH NOISE)

Method	MSE	SNR
Without filtering	56.66	10.67
Median filter	49.95	17.38
Wiener filter	49.44	17.88
Soft threshold	48.08	19.25
Hard threshold	46.95	20.38
Global threshold	47.26	20.07
Normal shrink	46.60	20.73
Bayes shrink	45.96	21.36
Modified Bayes shrink	45.93	21.39

TABLE VIII

COMPARISON DIFFERENT METHODS WITH  $\sigma_w^2 = 0.05$  FOR REMOVING NOISE [NOISY HOUSE] (HIGH NOISE)

Method	MSE	SNR
Without filtering	58.56	8.76
Median filter	51.94	15.39
Wiener filter	51.34	15.99
Soft threshold	48.65	18.67
Hard threshold	48.05	19.27
Global threshold	47.95	19.37
Normal shrink	47.90	19.43
Bayes shrink	47.17	20.15
Modified Bayes shrink	47.14	20.18

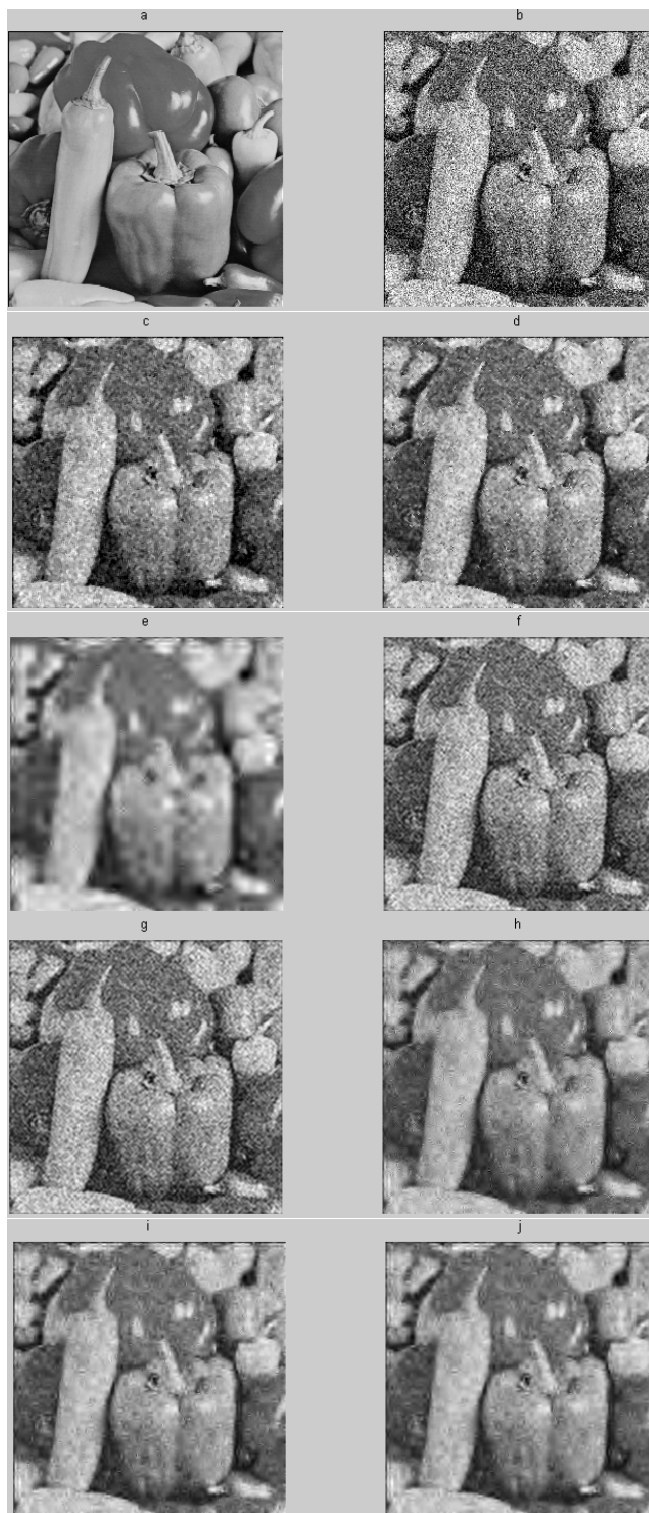




Fig 2,3 a) Original image. b) Gaussian noise with variance 0.06 is added to original image. c) Image denoising with median filter. d) Image denoising with wiener filter. e) Image denoising with global thresholding. f) Image denoising with stationary wavelet transform with hard threshold. g) Image denoising with stationary wavelet transform with soft threshold. h) Image denoising with Normal Shrink. i) Image denoising with Bayes Shrink. j) Image denoising with Modified Bayes Shrink (noisy pepper & noisy house)

#### IV. CONCLUSION

In our work, different methods for image denoising is proposed by adaptive threshold. In the proposed method, it depends on the noise, the adaptive wavelet threshold method (BS or MBS or NS) is applied in order to remove Gaussian noise. The results show in low noise, the normal shrink yields the best results for denoising because it has maximum SNR and minimum MSE. And in high noise, the modified bayes shrink yields the best results for denoising because it has maximum SNR and minimum MSE.

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#### REFERENCES

- [1] A.K.Katssagelous, *Digital Image Restoration*. New-York: springer-Verlag.1991.
- [2] Lakhwinder Kaur and Savita Gupta and R.C.Chauhan, "Image denoising using wavelet thresholding," punjab (148106), India.2003.
- [3] S.ZHONG, and V.Cherkassy, "image denoising using wavelet thresholding and model selection,"proc.IEEE.Int.Conf.on image processing.[ICIP].Vancouver.bc,Sept.2000.
- [4] Savita Gupta,"wavelet Based Image Compression Using Daubechies Filter," Bombay,NCC-2002.
- [5] S. Grace Chang, "adaptive Wavelet Thresholding for Image denoising and compression," IEEE transactions on Image processing, Vol.9, No.9 september.2000.
- [6] D.L.Donoho,"Denoising and soft thresholding," IEEE.transactions.information.Theory,VOL.41,PP.613-627,1995.
- [7] D.L.Donoho,"nonlinear wavelet methods for recovery of signals, densities, and spectra from indirect and noisy data," in proc. of symposia in applied mathematics, VOL, 00, PP.173-205, AMS, 1993.
- [8] S.G.Chang, B.YU, and martin Vetterli, "spatially adaptive wavelet thresholding with context modeling for image denoising," IEEE.trans.on image proc.,VOL.9,no.9,PP.1522-1531,2000.
- [9] S.G.Chang, B.YU, and martin Vetterli, "bridging compression to wavelet thresholding as a denoising method," in proc. conf. information sciences systems, Baltimore, MD, PP.568-573, 1997.
- [10] D.L.Donpho, and I.M.Johnstone, "adaptive to unknown smoothness via wavelet shrinkage," Journal of American statistical ASSOC.,VOL.90,NO.90,PP.1200-1224,1995.
- [11] D.L.Donoho, and I.M.Johnstone, "Ideal spatial adaptation via wavelet shrinkage,"Biometrika,VOL.81,PP.425-455,1994.
- [12] NM.Vatterli and J.Kovacevic, Wavelets and subband Coding. Englewood Cliffs, NJ, Prentice Hall, 1995.
- [13] A.K. Katssagelous and K.T.Lay, "Maximum Likelihood Blur Identification and Image restoration Using the EM algorithm," IEEE Transactions on Signal Processing, Vol. 39, No.3, March 1991.