

# Intuitionistic Fuzzy Points in Semigroups

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**Abstract**—The notion of intuitionistic fuzzy sets was introduced by Atanassov as a generalization of the notion of fuzzy sets. Y.B. Jun and S.Z. Song introduced the notion of intuitionistic fuzzy points. In this paper we find some relations between the intuitionistic fuzzy ideals of a semigroup  $S$  and the set of all intuitionistic fuzzy points of  $S$ .

**Keywords**—Semigroup, Regular(intra-regular) semigroup, Intuitionistic fuzzy point, Intuitionistic fuzzy subsemigroup, Intuitionistic fuzzy ideal, Intuitionistic fuzzy interior ideal, Intuitionistic fuzzy semiprime ideal, Intuitionistic fuzzy prime ideal.

## I. INTRODUCTION

A semigroup is an algebraic structure consisting of a non-empty set  $S$  together with an associative binary operation[4]. The formal study of semigroups began in the early 20th century. Semigroups are important in many areas of mathematics, for example, coding and language theory, automata theory, combinatorics and mathematical analysis. The concept of fuzzy sets was introduced by Lofti Zadeh[19] in his classic paper in 1965. Azirel Rosenfeld[13] used the idea of fuzzy set to introduce the notions of fuzzy subgroups. Nobuaki Kuroki[8], [9], [10] is the pioneer of fuzzy ideal theory of semigroups. The idea of fuzzy subsemigroup was also introduced by Kuroki[8], [10]. In [9], Kuroki characterized several classes of semigroups in terms of fuzzy left, fuzzy right and fuzzy bi-ideals. Others who worked on fuzzy semigroup theory, such as X.Y. Xie[16], [17], Y.B. Jun[5], [6], are mentioned in the bibliography. X.Y. Xie[16] introduced the idea of extensions of fuzzy ideals in semigroups. The notion of intuitionistic fuzzy sets was introduced by Atanassov[1], [2], [3] as a generalization of the notion of fuzzy sets. Pu and Liu[12] introduced the notion of fuzzy points. In [15], X.P. Wang, Z.W. Mo, W.J. Liu, in [18] Y.H. Yon and in [7] K.H. Kim characterized fuzzy ideals as fuzzy points of semigroups. Y.B. Jun and S.Z. Song introduced the notion of intuitionistic fuzzy points[6]. In this paper, we consider the semigroup  $\underline{S}$  of the intuitionistic fuzzy points of a semigroup  $S$ , and discuss some relations between the fuzzy subsemigroups(fuzzy bi-ideals, fuzzy interior ideals, fuzzy ideals, fuzzy prime ideals, fuzzy semiprime ideals) of  $S$  and the subsets of  $\underline{S}$ . Among other results we obtain some characterization theorems of regular and intra-regular semigroups in terms of intuitionistic fuzzy points.

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## II. PRELIMINARIES

In this section we discuss some elementary definitions that we use in the sequel.

**Definition II.1.** [11] If  $(X, *)$  is a mathematical system such that  $\forall a, b, c \in X, (a * b) * c = a * (b * c)$ , then  $*$  is called associative and  $(X, *)$  is called a *semigroup*.

**Definition II.2.** [11] A *subsemigroup* of a semigroup  $S$  is a non-empty subset  $I$  of  $S$  such that  $I^2 \subseteq I$ .

**Definition II.3.** [11] A subsemigroup  $I$  of a semigroup  $S$  is called an *interior ideal* of  $S$  if  $SIS \subseteq I$ .

**Definition II.4.** [11] A subsemigroup  $I$  of a semigroup  $S$  is called a *bi-ideal* of  $S$  if  $ISI \subseteq I$ .

**Definition II.5.** [11] A *left (right) ideal* of a semigroup  $S$  is a non-empty subset  $I$  of  $S$  such that  $SI \subseteq I$  ( $IS \subseteq I$ ). If  $I$  is both a left and a right ideal of a semigroup  $S$ , then we say that  $I$  is an *ideal* of  $S$ .

**Definition II.6.** [11] Let  $S$  be a semigroup. Then an ideal  $I$  of  $S$  is said to be (i) *prime* if for ideals  $A, B$  of  $S, AB \subseteq I$  implies that  $A \subseteq I$  or  $B \subseteq I$ , (ii) *semiprime* if for an ideal  $A$  of  $S, A^2 \subseteq I$  implies that  $A \subseteq I$ .

**Definition II.7.** [1], [2] The *intuitionistic fuzzy sets* defined on a non-empty set  $X$  as objects having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \},$$

where the functions  $\mu_A : X \rightarrow [0, 1]$  and  $\nu_A : X \rightarrow [0, 1]$  denote the degree of membership and the degree of non-membership of each element  $x \in X$  to the set  $A$  respectively, and  $0 \leq \mu_A(x) + \nu_A(x) \leq 1$  for all  $x \in X$ .

For the sake of simplicity, we shall use the symbol  $A = (\mu_A, \nu_A)$  for the intuitionistic fuzzy subset  $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle : x \in X \}$ .

**Definition II.8.** [6] Let  $\alpha, \beta \in [0, 1]$  with  $\alpha + \beta \leq 1$ . An *intuitionistic fuzzy point*, written as  $x_{(\alpha, \beta)}$ , is defined to be an intuitionistic fuzzy subset of  $S$ , given by

$$x_{(\alpha, \beta)}(y) = \begin{cases} (\alpha, \beta) & \text{if } x = y \\ (0, 1) & \text{otherwise} \end{cases}$$

**Definition II.9.** [14] A non-empty intuitionistic fuzzy subset  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy subsemigroup* of  $S$  if (i)  $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$ , (ii)  $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ .

**Definition II.10.** [14] An intuitionistic fuzzy subsemigroup  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy*

interior ideal of  $S$  if (i)  $\mu_A(xay) \geq \mu_A(a) \forall x, a, y \in S$ , (ii)  $\nu_A(xay) \leq \nu_A(a) \forall x, a, y \in S$ .

**Definition II.11.** [14] An intuitionistic fuzzy subsemigroup  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy bi-ideal* of  $S$  if (i)  $\mu_A(x\omega y) \geq \min\{\mu_A(x), \mu_A(y)\} \forall x, \omega, y \in S$ , (ii)  $\nu_A(x\omega y) \leq \max\{\nu_A(x), \nu_A(y)\} \forall x, \omega, y \in S$ .

**Definition II.12.** [14] A non-empty intuitionistic fuzzy subset  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy left(right) ideal* of  $S$  if (i)  $\mu_A(xy) \geq \mu_A(y)$  (resp.  $\mu_A(xy) \geq \mu_A(x)$ )  $\forall x, y \in S$ , (ii)  $\nu_A(xy) \leq \nu_A(y)$  (resp.  $\nu_A(xy) \leq \nu_A(x)$ )  $\forall x, y \in S$ .

**Definition II.13.** [14] A non-empty intuitionistic fuzzy subset  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy two-sided ideal* or an *intuitionistic fuzzy ideal* of  $S$  if it is both an intuitionistic fuzzy left and an intuitionistic fuzzy right ideal of  $S$ .

Alternative definition of Definition 2.13, is as follows.

**Definition II.14.** [14] A non-empty intuitionistic fuzzy subset  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy ideal* of  $S$  if (i)  $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$ , (ii)  $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ .

**Definition II.15.** [14] An intuitionistic fuzzy ideal  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy semiprime ideal* of  $S$  if (i)  $\mu_A(x) \geq \mu_A(x^2) \forall x \in S$ , (ii)  $\nu_A(x) \leq \nu_A(x^2) \forall x \in S$ .

**Definition II.16.** [14] An intuitionistic fuzzy ideal  $A = (\mu_A, \nu_A)$  of a semigroup  $S$  is called an *intuitionistic fuzzy prime ideal* of  $S$  if (i)  $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\} \forall x, y \in S$ , (ii)  $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ .

### III. MAIN RESULTS

Let  $\mathfrak{S}(S)$  be the set of all intuitionistic fuzzy subsets of a semigroup  $S$ . For each  $A = (\mu_A, \nu_A), B = (\mu_B, \nu_B) \in \mathfrak{S}(S)$ , the product of  $A$  and  $B$  is an intuitionistic fuzzy subset  $A \circ B$  defined as follows:

$$A \circ B = \{ \langle x, (\mu_A \circ \mu_B)(x), (\nu_A \circ \nu_B)(x) \rangle : x \in S \}$$

$$\text{where } (\mu_A \circ \mu_B)(x) = \begin{cases} \sup_{x=uv} [\min\{\mu_A(u), \mu_B(v)\} : u, v \in S] \\ 0, \text{ if for any } u, v \in S, x \neq uv \end{cases}$$

$$\text{and } (\nu_A \circ \nu_B)(x) = \begin{cases} \inf_{x=uv} [\max\{\nu_A(u), \nu_B(v)\} : u, v \in S] \\ 1, \text{ if for any } u, v \in S, x \neq uv \end{cases}$$

It is clear that  $(A \circ B) \circ C = A \circ (B \circ C)$ , and that if  $A \subseteq B$ , then  $A \circ C \subseteq B \circ C$  and  $C \circ A \subseteq C \circ B$  for any  $A, B, C \in \mathfrak{S}(S)$ . Thus  $\mathfrak{S}(S)$  is a semigroup with the product  $\circ$ .

Let  $\underline{S}$  be the set of all intuitionistic fuzzy points in a semigroup  $S$ . Then  $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in \underline{S}$ , where  $\alpha \wedge \gamma = \min(\alpha, \gamma)$  and  $\beta \vee \delta = \max(\beta, \delta)$  and  $x_{(\alpha, \beta)} \circ (y_{(\gamma, \delta)} \circ z_{(\eta, \theta)}) = (xyz)_{(\alpha \wedge \gamma \wedge \eta, \beta \vee \delta \vee \theta)} = (x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)}) \circ z_{(\eta, \theta)}$  for

any  $x_{(\alpha, \beta)}, y_{(\gamma, \delta)}, z_{(\eta, \theta)} \in \underline{S}$ . Thus  $\underline{S}$  is a subsemigroup of  $\mathfrak{S}(S)$ .

For any  $A = (\mu_A, \nu_A) \in \mathfrak{S}(S)$ ,  $\underline{A}$  denote the set of all intuitionistic fuzzy points contained in  $A = (\mu_A, \nu_A)$ , that is,  $\underline{A} = \{x_{(\alpha, \beta)} \in \underline{S} : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\}$ . If  $x_{(\alpha, \beta)} \in \underline{S}$ , then  $\alpha > 0$  and  $\beta < 1$ .

**Proposition III.1.** Let  $A = (\mu_A, \nu_A)$  and  $B = (\mu_B, \nu_B)$  be two intuitionistic fuzzy subsets of a semigroup  $S$ . Then

- (i)  $\underline{A \cup B} = \underline{A} \cup \underline{B}$ .
- (ii)  $\underline{A \cap B} = \underline{A} \cap \underline{B}$ .
- (iii)  $\underline{A \circ B} \supseteq \underline{A} \circ \underline{B}$ .

*Proof:* (i) Let  $x_{(\alpha, \beta)} \in \underline{A \cup B} \Leftrightarrow \{x \in \underline{S} : (\mu_A \cup \mu_B)(x) \geq \alpha \text{ and } (\nu_A \cap \nu_B)(x) \leq \beta\} \Leftrightarrow \{x \in \underline{S} : \max\{\mu_A(x), \mu_B(x)\} \geq \alpha \text{ and } \min\{\nu_A(x), \nu_B(x)\} \leq \beta\} \Leftrightarrow [x \in \underline{S} : \{\mu_A(x) \geq \alpha \text{ or } \mu_B(x) \geq \alpha\} \text{ and } \{\nu_A(x) \leq \beta \text{ or } \nu_B(x) \leq \beta\}] \Leftrightarrow \{x \in \underline{S} : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\} \text{ or } \{x \in \underline{S} : \mu_B(x) \geq \alpha \text{ and } \nu_B(x) \leq \beta\} \Leftrightarrow x_{(\alpha, \beta)} \in \underline{A} \text{ or } x_{(\alpha, \beta)} \in \underline{B} \Leftrightarrow x_{(\alpha, \beta)} \in \underline{A \cup B}$ . Hence  $\underline{A \cup B} = \underline{A} \cup \underline{B}$ .

(ii) Let  $x_{(\alpha, \beta)} \in \underline{A \cap B} \Leftrightarrow \{x \in \underline{S} : (\mu_A \cap \mu_B)(x) \geq \alpha \text{ and } (\nu_A \cup \nu_B)(x) \leq \beta\} \Leftrightarrow \{x \in \underline{S} : \min\{\mu_A(x), \mu_B(x)\} \geq \alpha \text{ and } \max\{\nu_A(x), \nu_B(x)\} \leq \beta\} \Leftrightarrow [x \in \underline{S} : \{\mu_A(x) \geq \alpha \text{ and } \mu_B(x) \geq \alpha\} \text{ and } \{\nu_A(x) \leq \beta \text{ and } \nu_B(x) \leq \beta\}] \Leftrightarrow \{x \in \underline{S} : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta\} \text{ and } \{x \in \underline{S} : \mu_B(x) \geq \alpha \text{ and } \nu_B(x) \leq \beta\} \Leftrightarrow x_{(\alpha, \beta)} \in \underline{A} \text{ and } x_{(\alpha, \beta)} \in \underline{B} \Leftrightarrow x_{(\alpha, \beta)} \in \underline{A \cap B}$ . Hence  $\underline{A \cap B} = \underline{A} \cap \underline{B}$ .

(iii)  $\underline{A \circ B} = \{x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} : x_{(\alpha, \beta)} \in \underline{A}, y_{(\gamma, \delta)} \in \underline{B}\} = \{(xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} : \mu_A(x) \geq \alpha \text{ and } \nu_A(x) \leq \beta, \mu_B(y) \geq \gamma \text{ and } \nu_B(y) \leq \delta\} = \{(xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} : \mu_A(x) \geq \alpha \text{ and } \mu_B(y) \geq \gamma, \nu_B(x) \leq \beta \text{ and } \nu_B(y) \leq \delta\} = \{(xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} : \min\{\mu_A(x), \mu_B(y)\} \geq \alpha \wedge \gamma, \max\{\nu_B(x), \nu_B(y)\} \leq \beta \vee \delta\} \leq \{(xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} : \sup_{x, y \in S} [\min\{\mu_A(x), \mu_B(y)\}] \geq \alpha \wedge \gamma, \inf_{x, y \in S} [\max\{\nu_A(x), \nu_B(y)\}] \leq \beta \vee \delta\} = \{(u)_{(\alpha \wedge \gamma, \beta \vee \delta)} : (\mu_A \circ \mu_B)(u) \geq \alpha \wedge \gamma, (\nu_B \circ \nu_B)(u) \leq \beta \vee \delta\} = \underline{A \circ B}$ . Hence  $\underline{A \circ B} \subseteq \underline{A} \circ \underline{B}$ . ■

**Theorem III.2.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy subsemigroup of  $S$ ,
- (ii)  $\underline{A}$  is a subsemigroup of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy subsemigroup of  $S$ . Let  $x_{(\alpha, \beta)}, y_{(\gamma, \delta)} \in \underline{A}$ . Then  $\mu_A(x) \geq \alpha > 0, \mu_A(y) \geq \gamma > 0$  and  $\nu_A(x) \leq \beta < 1, \nu_A(y) \leq \delta < 1$ . Since  $\mu_A(xy) \geq \min\{\mu_A(x), \mu_A(y)\} \geq \alpha \wedge \gamma$  and  $\nu_A(xy) \leq \max\{\nu_A(x), \nu_A(y)\} \leq \beta \vee \delta$ . Consequently,  $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in \underline{A}$ . This implies that  $\underline{A}^2 \subseteq \underline{A}$ . Hence  $\underline{A}$  is a subsemigroup of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is a subsemigroup of  $\underline{S}$ . Let  $x, y \in S$ . If  $\mu_A(x) = \mu_A(y) = 0$  and  $\nu_A(x) = \nu_A(y) = 1$ , then  $\min\{\mu_A(x), \mu_A(y)\} = 0 \leq \mu_A(xy)$  and  $\max\{\nu_A(x), \nu_A(y)\} = 1 \geq \nu_A(xy)$ . If  $\mu_A(x) = \mu_A(y) \neq 0$  and  $\nu_A(x) = \nu_A(y) < 1$ , then  $x_{(\mu_A(x), \nu_A(x))}, y_{(\mu_A(y), \nu_A(y))} \in \underline{A}$ . Since  $\underline{A}$  is a subsemigroup of  $\underline{S}$ , so we have  $(xy)_{(\mu_A(x) \wedge \mu_A(y), \nu_A(x) \vee \nu_A(y))} = x_{(\mu_A(x), \nu_A(x))} \circ y_{(\mu_A(y), \nu_A(y))} \in \underline{A}$  which implies that  $\mu_A(xy) \geq \mu_A(x) \wedge \mu_A(y) = \min\{\mu_A(x), \mu_A(y)\}$  and  $\nu_A(xy) \leq \nu_A(x) \vee \nu_A(y) = \max\{\nu_A(x), \nu_A(y)\}$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy subsemigroup of  $S$ . ■

**Theorem III.3.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy bi-ideal of  $S$ ,
- (ii)  $\underline{A}$  is a bi-ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy bi-ideal of  $S$ . Then  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy subsemigroup of  $S$ . Hence by Theorem 3.2,  $\underline{A}$  is a subsemigroup of  $\underline{S}$ . Let  $x_{(\alpha, \beta)}, z_{(\eta, \theta)} \in \underline{A}$  and  $y_{(\gamma, \delta)} \in \underline{S}$ . Then  $\mu_A(x) \geq \alpha > 0$ ,  $\mu_A(z) \geq \eta > 0$  and  $\nu_A(x) \leq \beta < 1$ ,  $\nu_A(z) \leq \theta < 1$ . Since  $\mu_A(xyz) \geq \min\{\mu_A(x), \mu_A(z)\} \geq \alpha \wedge \eta$  (since  $\gamma > 0$ ) and  $\nu_A(xyz) \leq \max\{\nu_A(x), \nu_A(z)\} \leq \beta \vee \theta$  (since  $\delta < 1$ ). Consequently,  $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} \circ z_{(\eta, \theta)} = (xyz)_{(\alpha \wedge \eta, \beta \vee \theta)} \in \underline{A}$ . This implies that  $\underline{A} \underline{S} \underline{A} \subseteq \underline{A}$ . Hence  $\underline{A}$  is a bi-ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is a bi-ideal of  $\underline{S}$ . Then  $\underline{A}$  is a subsemigroup of  $\underline{S}$ . Hence by Theorem 3.2,  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy subsemigroup of  $S$ . Let  $x, y, z \in S$ . If  $\mu_A(x) = \mu_A(z) = 0$  and  $\nu_A(x) = \nu_A(z) = 1$ , then  $\min\{\mu_A(x), \mu_A(z)\} = 0 \leq \mu_A(xyz)$  and  $\max\{\nu_A(x), \nu_A(z)\} = 1 \geq \nu_A(xyz)$ . If  $\mu_A(x) = \mu_A(z) \neq 0$  and  $\nu_A(x) = \nu_A(z) < 1$ , then  $x_{(\mu_A(x), \nu_A(x))}, z_{(\mu_A(z), \nu_A(z))} \in \underline{A}$ . Since  $\underline{A}$  is a bi-ideal of  $\underline{S}$ , so we have  $(xyz)_{(\mu_A(x) \wedge \mu_A(z), \nu_A(x) \vee \nu_A(z))} = (xyz)_{(\mu_A(x) \wedge \mu_A(z), \nu_A(x) \vee \nu_A(z))} = x_{(\mu_A(x), \nu_A(x))} \circ y_{(\mu_A(x), \nu_A(x))} \circ z_{(\mu_A(z), \nu_A(z))} \in \underline{A}$  which implies that  $\mu_A(xyz) \geq \mu_A(x) \wedge \mu_A(z) = \min\{\mu_A(x), \mu_A(z)\}$  and  $\nu_A(xyz) \leq \nu_A(x) \vee \nu_A(z) = \max\{\nu_A(x), \nu_A(z)\}$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy bi-ideal of  $S$ . ■

**Theorem III.4.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy interior ideal of  $S$ ,
- (ii)  $\underline{A}$  is an interior ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy interior ideal of  $S$ . Then  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy subsemigroup of  $S$ . Hence by Theorem 3.2,  $\underline{A}$  is a subsemigroup of  $\underline{S}$ . Let  $x_{(\alpha, \beta)}, z_{(\eta, \theta)} \in \underline{S}$  and  $y_{(\gamma, \delta)} \in \underline{A}$ . Then  $\mu_A(y) \geq \gamma > 0$  and  $\nu_A(y) \leq \delta < 1$ . Since  $\mu_A(xyz) \geq \mu_A(y) \geq \alpha \wedge \gamma$  (since  $\alpha, \eta > 0$ ) and  $\nu_A(xyz) \leq \nu_A(y) \leq \beta \vee \delta$  (since  $\beta, \theta < 1$ ). Consequently,  $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} \circ z_{(\eta, \theta)} = (xyz)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in \underline{A}$ . This implies that  $\underline{S} \underline{A} \underline{S} \subseteq \underline{A}$ . Hence  $\underline{A}$  is an interior ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is an interior ideal of  $\underline{S}$ . Then  $\underline{A}$  is a subsemigroup of  $\underline{S}$ . Hence by Theorem 3.2,  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy subsemigroup of  $S$ . Let  $x, y, z \in S$ . If  $\mu_A(y) = 0$  and  $\nu_A(y) = 1$ , then  $\mu_A(y) = 0 \leq \mu_A(xyz)$  and  $\nu_A(y) = 1 \geq \nu_A(xyz)$ . If  $\mu_A(y) \neq 0$  and  $\nu_A(y) < 1$ , then  $y_{(\mu_A(y), \nu_A(y))} \in \underline{A}$ . Since  $\underline{A}$  is an interior ideal of  $\underline{S}$ , so we have  $(xyz)_{(\mu_A(y), \nu_A(y))} = (xyz)_{(\mu_A(y) \wedge \mu_A(y), \nu_A(y) \vee \nu_A(y))} = x_{(\mu_A(y), \nu_A(y))} \circ y_{(\mu_A(y), \nu_A(y))} \circ z_{(\mu_A(y), \nu_A(y))} \in \underline{A}$  which implies that  $\mu_A(xyz) \geq \mu_A(y)$  and  $\nu_A(xyz) \leq \nu_A(y)$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy interior ideal of  $S$ . ■

**Theorem III.5.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy (right, left) ideal of  $S$ ,
- (ii)  $\underline{A}$  is a right(left) ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy right ideal of  $S$ . Let  $x_{(\alpha, \beta)} \in \underline{A}$  and  $y_{(\gamma, \delta)} \in \underline{S}$ . Then  $\mu_A(x) \geq \alpha > 0$  and  $\nu_A(x) \leq \beta < 1$ . Since  $\mu_A(xy) \geq \mu_A(x) \geq \alpha \wedge \gamma$  (since  $\gamma > 0$ ) and  $\nu_A(xy) \leq \nu_A(x) \leq \beta \vee \delta$  (since  $\delta < 1$ ). Consequently,  $x_{(\alpha, \beta)} \circ y_{(\gamma, \delta)} = (xy)_{(\alpha \wedge \gamma, \beta \vee \delta)} \in \underline{A}$ . This implies that  $\underline{A} \underline{S} \subseteq \underline{A}$ . Hence  $\underline{A}$  is a right ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is a right ideal of  $\underline{S}$ . Let  $x, y \in S$ . If  $\mu_A(x) = 0$  and  $\nu_A(x) = 1$ , then  $\mu_A(x) = 0 \leq \mu_A(xy)$  and  $\nu_A(x) = 1 \geq \nu_A(xy)$ . If  $\mu_A(x) \neq 0$  and  $\nu_A(x) < 1$ , then  $x_{(\mu_A(x), \nu_A(x))} \in \underline{A}$ . Since  $\underline{A}$  is a right ideal of  $\underline{S}$ , so we have  $(xy)_{(\mu_A(x) \wedge \mu_A(x), \nu_A(x) \vee \nu_A(x))} = x_{(\mu_A(x), \nu_A(x))} \circ y_{(\mu_A(x), \nu_A(x))} \in \underline{A}$  which implies that  $\mu_A(xy) \geq \mu_A(x)$  and  $\nu_A(xy) \leq \nu_A(x)$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy right ideal of  $S$ . Similarly we can prove the theorem for left ideal and ideal. ■

**REMARK 1.** It is clear that any ideal of a semigroup  $S$  is an interior ideal of  $S$ . It is also clear that any intuitionistic fuzzy ideal of a semigroup  $S$  is an intuitionistic fuzzy interior ideal of  $S$ .

**Definition III.6.** [9] A semigroup  $S$  is called regular if, for each element  $a$  of  $S$ , there exists an element  $x \in S$  such that  $a = axa$ . A semigroup  $S$  is called intra-regular if, for each element  $x \in S$ , there exist elements  $a, b \in S$  such that  $x = ax^2b$ .

**Theorem III.7.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a regular semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy (right, left) ideal of  $S$ ,
- (ii)  $\underline{A}$  is an interior ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Follows easily from Remark 1 and Theorem 3.4.

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is an interior ideal of  $\underline{S}$ . Let  $x \in S$ . Then there exists an element  $a \in S$  such that  $x = xax$  (since  $S$  is regular). If  $\mu_A(x) = 0$  and  $\nu_A(x) = 1$ ,

then  $\mu_A(x) = 0 \leq \mu_A(xy)$  and  $\nu_A(x) = 1 \geq \nu_A(xy)$ . If  $\mu_A(x) \neq 0$  and  $\nu_A(x) < 1$ , then  $x_{(\mu_A(x), \nu_A(x))} \in \underline{A}$  and  $y_{(\mu_A(x), \nu_A(x))} \in \underline{S}$ . Since  $\underline{A}$  is an interior ideal of  $\underline{S}$ , so we have  $(xy)_{(\mu_A(x), \nu_A(x))} = (xaxy)_{(\mu_A(x), \nu_A(x))} = ((xa)xy)_{(\mu_A(x) \wedge \mu_A(x) \wedge \mu_A(x), \nu_A(x) \vee \nu_A(x) \vee \nu_A(x))} = (xa)_{(\mu_A(x), \nu_A(x))} \circ x_{(\mu_A(x), \nu_A(x))} \circ y_{(\mu_A(x), \nu_A(x))} \in \underline{A}$  which implies that  $\mu_A(xy) \geq \mu_A(x)$  and  $\nu_A(xy) \leq \nu_A(x)$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy right ideal of  $S$ . The proof is similar in case of intuitionistic fuzzy left ideal and intuitionistic fuzzy ideal. ■

**Theorem III.8.** Let  $S$  be a semigroup. If for fixed  $\alpha, \beta \in [0, 1]$  with  $\alpha + \beta \leq 1$ ,  $f_{(\alpha, \beta)} : S \rightarrow \underline{S}$  is a function defined by  $f_{(\alpha, \beta)}(x) = x_{(\alpha, \beta)}$ , then  $f_{(\alpha, \beta)}$  is an injective homomorphism.

*Proof:* To show  $f_{(\alpha, \beta)}$  is a homomorphism: Let  $x, y \in S$ . Then  $f_{(\alpha, \beta)}(xy) = (xy)_{(\alpha, \beta)} = (xy)_{(\alpha \wedge \alpha, \beta \vee \beta)} = x_{(\alpha, \beta)} \circ y_{(\alpha, \beta)} = f_{(\alpha, \beta)}(x)f_{(\alpha, \beta)}(y)$ . Hence  $f_{(\alpha, \beta)}$  is an homomorphism.

To show  $f_{(\alpha, \beta)}$  is injective: Let  $x_1, x_2 \in S$ . Then  $f_{(\alpha, \beta)}(x_1) = f_{(\alpha, \beta)}(x_2) \Rightarrow (x_1)_{(\alpha, \beta)} = (x_2)_{(\alpha, \beta)} \Rightarrow x_1 = x_2$ .

Hence  $f_{(\alpha, \beta)}$  is an injective homomorphism. ■

**Theorem III.9.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of an intra-regular semigroup  $S$ . Then following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy (right, left) ideal of  $S$ ,
- (ii)  $\underline{A}$  is an interior ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Follows easily from Remark 1 and Theorem 3.4.

(ii)  $\Rightarrow$  (i) : Let us suppose that  $\underline{A}$  is an interior ideal of  $\underline{S}$ . Let  $x, y \in S$ . Then there exist elements  $a, b \in S$  such that  $x = ax^2b$  (since  $S$  is intra-regular). If  $\mu_A(x) = 0$  and  $\nu_A(x) = 1$ , then  $\mu_A(x) = 0 \leq \mu_A(xy)$  and  $\nu_A(x) = 1 \geq \nu_A(xy)$ . If  $\mu_A(x) \neq 0$  and  $\nu_A(x) < 1$ , then  $x_{(\mu_A(x), \nu_A(x))} \in \underline{A}$  and  $y_{(\mu_A(x), \nu_A(x))} \in \underline{S}$ . Since  $\underline{A}$  is an interior ideal of  $\underline{S}$ , so we have  $(xy)_{(\mu_A(x), \nu_A(x))} = (ax^2by)_{(\mu_A(x), \nu_A(x))} = ((ax)x(by))_{(\mu_A(x) \wedge \mu_A(x) \wedge \mu_A(x), \nu_A(x) \vee \nu_A(x) \vee \nu_A(x))} = (ax)_{(\mu_A(x), \nu_A(x))} \circ x_{(\mu_A(x), \nu_A(x))} \circ (by)_{(\mu_A(x), \nu_A(x))} \in \underline{A}$ . This implies that  $\mu_A(xy) \geq \mu_A(x)$  and  $\nu_A(xy) \leq \nu_A(x)$ . Hence  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy right ideal of  $S$ . The proof is similar in case of intuitionistic fuzzy left ideal and intuitionistic fuzzy ideal. ■

**Theorem III.10.** A semigroup  $S$  is intra-regular if and only if the semigroup  $\underline{S}$  is intra-regular.

*Proof:* Let  $a_{(\alpha, \beta)} \in \underline{S}$  and  $a \in S$ . Then there exist elements  $x, y \in S$  such that  $a = xa^2y$  (since  $S$  is intra-regular). So  $x_{(\alpha, \beta)}, y_{(\alpha, \beta)} \in \underline{S}$ . Then  $x_{(\alpha, \beta)} \circ a_{(\alpha, \beta)} \circ y_{(\alpha, \beta)} = x_{(\alpha, \beta)} \circ (a^2)_{(\alpha \wedge \alpha, \beta \vee \beta)} \circ y_{(\alpha, \beta)} = x_{(\alpha, \beta)} \circ (a^2)_{(\alpha, \beta)} \circ y_{(\alpha, \beta)} = (xa^2y)_{(\alpha \wedge \alpha, \beta \vee \beta)} = a_{(\alpha, \beta)}$ . Hence  $\underline{S}$  is intra-regular.

Conversely, let  $\underline{S}$  is intra-regular and  $a \in S$ . Then for any  $\alpha, \beta \in [0, 1]$ , there exist elements  $x_{(\gamma, \delta)}, y_{(\eta, \theta)} \in \underline{S}$  such that  $a_{(\alpha, \beta)} = x_{(\gamma, \delta)} \circ a_{(\alpha, \beta)} \circ y_{(\eta, \theta)} = x_{(\gamma, \delta)} \circ (a^2)_{(\alpha \wedge \alpha, \beta \vee \beta)} \circ y_{(\eta, \theta)} = x_{(\gamma, \delta)} \circ (a^2)_{(\alpha, \beta)} \circ y_{(\eta, \theta)} = (xa^2y)_{(\gamma \wedge \alpha \wedge \eta, \delta \vee \beta \vee \theta)}$ . This implies that  $a = xa^2y$  and  $x, y \in S$ . Hence  $S$  is intra-regular. ■

**Theorem III.11.** A semigroup  $S$  is regular if and only if the semigroup  $\underline{S}$  is regular.

*Proof:* Let  $a_{(\alpha, \beta)} \in \underline{S}$  and  $a \in S$ . Then there exists an element  $x \in S$  such that  $a = axa$  (since  $S$  is regular). So  $x_{(\alpha, \beta)} \in \underline{S}$ . Then  $a_{(\alpha, \beta)} \circ x_{(\alpha, \beta)} \circ a_{(\alpha, \beta)} = (axa)_{(\alpha \wedge \alpha, \beta \vee \beta)} = (axa)_{(\alpha, \beta)} = a_{(\alpha, \beta)}$ . Hence  $\underline{S}$  is regular.

Conversely, let  $\underline{S}$  is regular and  $a \in S$ . Then for any  $\alpha, \beta \in [0, 1]$ , there exists an element  $x_{(\gamma, \delta)} \in \underline{S}$  such that  $a_{(\alpha, \beta)} = a_{(\alpha, \beta)} \circ x_{(\gamma, \delta)} \circ a_{(\alpha, \beta)} = (axa)_{(\alpha \wedge \gamma \wedge \alpha, \beta \vee \delta \vee \beta)} = (axa)_{(\gamma \wedge \alpha, \beta \vee \delta)}$ . This implies that  $a = axa$  and  $x \in S$ . Hence  $S$  is regular. ■

**Lemma III.12.** [9] For a semigroup  $S$ , the following conditions are equivalent:

- (i)  $S$  is intra-regular (regular),
- (ii)  $L \cap R \subset LR$  (resp.  $R \cap L = RL$ ) holds for every left ideal  $L$  and right ideal  $R$  of  $S$ .

**Lemma III.13.** [14] For a semigroup  $S$ , the following conditions are equivalent:

- (i)  $S$  is intra-regular (regular),
- (ii)  $A \cap B \subset A \circ B$  (resp.  $B \cap A = B \circ A$ ) holds for every intuitionistic fuzzy left ideal  $A = (\mu_A, \nu_A)$  and intuitionistic fuzzy right ideal  $B = (\mu_B, \nu_B)$  of  $S$ .

**Theorem III.14.** For a semigroup  $S$ , the following conditions are equivalent:

- (i)  $S$  is intra-regular,
- (ii)  $\underline{A} \cap \underline{B} \subset \underline{A} \circ \underline{B}$  for every intuitionistic fuzzy left ideal  $A = (\mu_A, \nu_A)$  and intuitionistic fuzzy right ideal  $B = (\mu_B, \nu_B)$  of  $S$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $S$  be an intra-regular semigroup and  $A = (\mu_A, \nu_A)$ ,  $B = (\mu_B, \nu_B)$  are respectively intuitionistic fuzzy left and intuitionistic fuzzy right ideal of  $S$ . By Theorem 3.10,  $\underline{S}$  is an intra-regular semigroup. By Theorem 3.5,  $\underline{A}$  and  $\underline{B}$  are respectively left and right ideal of the semigroup  $\underline{S}$ . Hence by Lemma 3.12,  $\underline{A} \cap \underline{B} \subset \underline{A} \circ \underline{B}$ .

(ii)  $\Rightarrow$  (i) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy left and  $B = (\mu_B, \nu_B)$  be an intuitionistic fuzzy right ideal of  $S$  such that  $\underline{A} \cap \underline{B} \subset \underline{A} \circ \underline{B}$ . Let  $x \in S$ . If  $(\mu_A(x) = 0$  or  $\mu_B(x) = 0)$  and  $(\nu_A(x) = 1$  or  $\nu_B(x) = 1)$ , then  $0 = \mu_A(x) \wedge \mu_B(x) \leq (\mu_A \circ \mu_B)(x)$ , i.e.,  $(\mu_A \cap \mu_B)(x) \leq (\mu_A \circ \mu_B)(x)$  and  $1 = \nu_A(x) \vee \nu_B(x) \geq (\nu_A \circ \nu_B)(x)$ , i.e.,  $(\nu_A \cup \nu_B)(x) \geq (\nu_A \circ \nu_B)(x)$ . Hence  $A \circ B \supset A \cap B$ .

Again if  $(\mu_A(x) > 0$  and  $\nu_A(x) < 1)$  and  $(\mu_B(x) > 0$  and  $\nu_B(x) < 1)$ , then  $x_{(\mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x))} \in \underline{A}$  and  $x_{(\mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x))} \in \underline{B}$ .

Hence  $x_{(\mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x))} \in \underline{A} \cap \underline{B} \subset \underline{A} \circ \underline{B} \subset \underline{A} \circ \underline{B}$  (c.f. Proposition 3.1). Then  $(\mu_A \circ \mu_B)(x) \geq \mu_A(x) \wedge$

$\mu_B(x) = (\mu_A \cap \mu_B)(x)$  and  $(\nu_A \circ \nu_B)(x) \leq \nu_A(x) \vee \mu_B(x) = (\nu_A \cup \nu_B)(x)$  which implies that  $A \circ B \supseteq A \cap B$ . Hence by Lemma 3.13,  $S$  is intra-regular. ■

**Lemma III.15.** [14] Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy right ideal and  $B = (\mu_B, \nu_B)$  be an intuitionistic fuzzy left ideal of a semigroup  $S$ . Then  $A \circ B \subseteq A \cap B$ .

**Theorem III.16.** For a semigroup  $S$ , the following conditions are equivalent:

- (i)  $S$  is regular,
- (ii)  $\underline{B} \cap \underline{A} = \underline{B} \circ \underline{A}$  for every intuitionistic fuzzy left ideal  $A = (\mu_A, \nu_A)$  and intuitionistic fuzzy right ideal  $B = (\mu_B, \nu_B)$  of  $S$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $S$  be a regular semigroup and  $A = (\mu_A, \nu_A)$ ,  $B = (\mu_B, \nu_B)$  are respectively intuitionistic fuzzy left and intuitionistic fuzzy right ideal of  $S$ . By Theorem 3.11,  $\underline{S}$  is a regular semigroup. By Theorem 3.5,  $\underline{A}$  and  $\underline{B}$  are respectively left and right ideal of the semigroup  $\underline{S}$ . Hence by Lemma 3.12,  $\underline{A} \cap \underline{B} = \underline{A} \circ \underline{B}$ .

(ii)  $\Rightarrow$  (i) : Form (ii)  $\Rightarrow$  (i) we have  $A \circ B \supseteq A \cap B$ . Again by Lemma 3.15,  $A \circ B \subseteq A \cap B$ . Consequently,  $A \circ B = A \cap B$  and hence by Lemma 3.13,  $S$  is regular. ■

**Theorem III.17.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then the following are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy semiprime ideal of  $S$ ,
- (ii)  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy semiprime ideal of  $S$ . Then  $\mu_A(x) \geq \mu_A(x^2)$  and  $\nu_A(x) \leq \nu_A(x^2) \forall x \in S$ . Let  $x_{(\alpha, \beta)} \circ x_{(\alpha, \beta)} \in \underline{A}$ , i.e.,  $(x^2)_{(\alpha, \beta)} \in \underline{A}$ . Then  $\mu_A(x^2) \geq \alpha$  and  $\nu_A(x^2) \leq \beta$ . Since  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy semiprime ideal of  $S$ , so  $\mu_A(x) \geq \mu_A(x^2) \geq \alpha$  and  $\nu_A(x) \leq \nu_A(x^2) \leq \beta$ , which implies that  $x_{(\alpha, \beta)} \in \underline{A}$ . Hence  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let  $\underline{A}$  be a semiprime ideal of  $\underline{S}$ . Let  $\mu_A(x^2) = \alpha$  and  $\nu_A(x^2) = \beta$ . Then  $(x^2)_{(\alpha, \beta)} \in \underline{A}$ , i.e.,  $x_{(\alpha, \beta)} \circ x_{(\alpha, \beta)} \in \underline{A}$  implies that  $x_{(\alpha, \beta)} \in \underline{A}$  (since  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ ). Then  $\mu_A(x) \geq \alpha$  and  $\nu_A(x) \leq \beta$  which implies that  $\mu_A(x) \geq \mu_A(x^2)$  and  $\nu_A(x) \leq \nu_A(x^2)$ . Hence  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy semiprime ideal of  $S$ . ■

**Lemma III.18.** [14] For a semigroup  $S$  following conditions are equivalent:

- (i)  $S$  is an intra-regular semigroup,
- (ii) every intuitionistic fuzzy ideal  $A = (\mu_A, \nu_A)$  of  $S$  is an intuitionistic fuzzy semiprime ideal of  $S$ .

**Theorem III.19.** For any non-empty intuitionistic fuzzy subset  $A = (\mu_A, \nu_A)$  of an intra-regular semigroup  $S$  the following conditions are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy ideal of  $S$ ,
- (ii)  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic

fuzzy ideal of an intra-regular semigroup  $S$ . Then by Lemma 3.18,  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy semiprime ideal of  $S$  and hence by Theorem 3.17,  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let  $\underline{A}$  is a semiprime ideal of  $\underline{S}$ . Then by Theorem 3.17,  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy semiprime ideal of  $S$  and hence  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy ideal of  $S$ . ■

**Theorem III.20.** Let  $A = (\mu_A, \nu_A)$  be a non-empty intuitionistic fuzzy subset of a semigroup  $S$ . Then the following are equivalent:

- (i)  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy prime ideal of  $S$ ,
- (ii)  $\underline{A}$  is a prime ideal of  $\underline{S}$ .

*Proof:* (i)  $\Rightarrow$  (ii) : Let  $A = (\mu_A, \nu_A)$  be an intuitionistic fuzzy prime ideal of  $S$ . Then  $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$  and  $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ . Let  $x_{(\alpha, \beta)} \circ y_{(\alpha, \beta)} \in \underline{A}$ , i.e.,  $(xy)_{(\alpha \wedge \alpha, \beta \vee \beta)} = (xy)_{(\alpha, \beta)} \in \underline{A}$ . Then  $\mu_A(xy) \geq \alpha$  and  $\nu_A(xy) \leq \beta$  which implies that  $\max\{\mu_A(x), \mu_A(y)\} \geq \alpha$  and  $\min\{\nu_A(x), \nu_A(y)\} \leq \beta$ . Then  $(\mu_A(x) \geq \alpha$  or  $\mu_A(y) \geq \alpha)$  and  $(\nu_A(x) \leq \beta$  or  $\nu_A(y) \leq \beta)$ , i.e.,  $(\mu_A(x) \geq \alpha$  and  $\nu_A(x) \leq \beta)$  or  $(\mu_A(y) \geq \alpha$  and  $\nu_A(y) \leq \beta)$ , i.e.,  $x_{(\alpha, \beta)} \in \underline{A}$  or  $y_{(\alpha, \beta)} \in \underline{A}$ . Hence  $\underline{A}$  is a prime ideal of  $\underline{S}$ .

(ii)  $\Rightarrow$  (i) : Let  $\underline{A}$  be a prime ideal of  $\underline{S}$ . Let  $\mu_A(xy) = \alpha$  and  $\nu_A(xy) = \beta$ . Then  $x_{(\alpha, \beta)} \circ y_{(\alpha, \beta)} = (xy)_{(\alpha, \beta)} \in \underline{A}$  implies that  $x_{(\alpha, \beta)} \in \underline{A}$  or  $y_{(\alpha, \beta)} \in \underline{A}$  (since  $\underline{A}$  is a prime ideal of  $\underline{S}$ ). Then  $(\mu_A(x) \geq \alpha$  and  $\nu_A(x) \leq \beta)$  or  $(\mu_A(y) \geq \alpha$  and  $\nu_A(y) \leq \beta)$ , i.e.,  $(\mu_A(x) \geq \alpha$  or  $\mu_A(y) \geq \alpha)$  and  $(\nu_A(x) \leq \beta$  or  $\nu_A(y) \leq \beta)$ , i.e.,  $\max\{\mu_A(x), \mu_A(y)\} \geq \alpha$  and  $\min\{\nu_A(x), \nu_A(y)\} \leq \beta$ , i.e.,  $\max\{\mu_A(x), \mu_A(y)\} \geq \mu_A(xy)$  and  $\min\{\nu_A(x), \nu_A(y)\} \leq \nu_A(xy)$ .....(A). Since  $\underline{A}$  is a prime ideal of  $\underline{S}$ , so  $\underline{A}$  is an ideal of  $\underline{S}$ . So, by Theorem 3.5,  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy ideal of  $S$ . Then  $\mu_A(xy) \geq \max\{\mu_A(x), \mu_A(y)\}$  and  $\nu_A(xy) \leq \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ .....(B). Combining (A) and (B) we have,  $\mu_A(xy) = \max\{\mu_A(x), \mu_A(y)\}$  and  $\nu_A(xy) = \min\{\nu_A(x), \nu_A(y)\} \forall x, y \in S$ . Hence  $A = (\mu_A, \nu_A)$  is an intuitionistic fuzzy prime ideal of  $S$ . ■

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