Abstract—This paper presents an efficient emission constrained economic dispatch algorithm that deals with nonlinear cost function and constraints. It is then incorporated into the dynamic programming based hydrothermal coordination program. The program has been tested on a practical utility system having 32 thermal and 12 hydro generating units. Test results show that a slight increase in production cost causes a substantial reduction in emission.

Keywords—Emission constraint, Hydrothermal coordination, and Economic dispatch algorithm.

I. INTRODUCTION

The economic dispatch of an electrical power system is the determination of the generation levels that minimizes the system total generation cost such that the system constraints are satisfied.

In the classical economic dispatch, a set of coordination equations is solved using the Lagrange multiplier. Equal incremental cost method is normally used to solve the coordination equations and the Lagrange multiplier is updated from an initial guess in order to make the total generation equal to the system demand plus losses. The constrained problem is normally solved by fixing any unit violating the maximum or minimum limit during iteration to that limit without dispatching it further. Fahmideh-Vojdani and Galiana [1] showed that this limit fixing technique may not yield an optimal solution and proposed a practical algorithm for quadratic cost function. Their algorithm decides which generation level to be fixed during iterations.

The solution method to dynamic programming based hydrothermal coordination problem includes the economic dispatch problem as a subproblem. Many economic dispatches, normally more than a thousand, need to be performed in the course of a 24 hours study. Therefore, the design of an efficient economic dispatch subroutine is crucial to the performance of a dynamic programming based hydrothermal coordination program.

In recent years, several economic emission dispatch strategies have been proposed [2]-[6]. The paper [7] suggests an efficient economic dispatch algorithm that handles nonlinear cost function and constraints, but the algorithm was developed for a system which did not consider emission constraint. In this paper, the formulation [7] has been modified to cope with emission constraint. It is then incorporated into the dynamic programming based hydrothermal coordination program [8]. The program has been tested on a practical utility system. Test results show that a slight increase in production cost reduces the emission substantially.

II. PROBLEM FORMULATION

A. Notations

The list of symbols used in this paper is as follows:

- $M$ - number of thermal units
- $T$ - number of periods for dividing the scheduling time horizon
- $i$ - Index of the thermal unit
- $t$ - time index
- $P_i$ - MW power output of a generating unit
- $P_{max}$ - maximum MW power of a generating unit
- $P_{min}$ - minimum MW power of a generating unit
- $D_t$ - demand in period $t$
- $P_{loss_t}$ - transmission loss in period $t$
- $C_i(.)$ - production cost function of the $i$-th thermal unit
- $E_i(.)$ - emission function of the $i$-th thermal unit
- $b_0, b_1, b_2$ - coefficients for production cost function
- $e_0, e_1, e_2$ - coefficients for emission function
- $B_0, B_1, B$ - transmission loss coefficients

B. Objective function and constraints

Mathematically, the economic dispatch problem can be expressed as follows:

$$\sum_{i=1}^{M} C_i(P_{i,t})$$

subject to the energy balance equation

$$\sum_{i=1}^{M} P_{i,t} = P_{loss_t} + D_t$$

and to the generation limits

$$P_{i,t} \leq P_{i,t} \leq P_{max}$$
The new constraint function is the emission function. The total emission from the system is

\[ \sum_{i=1}^{M} E_i(P_{i,t}) \quad (4) \]

III. PROPOSED APPROACH

The cost objective function in (1) is then augmented by constraint (4) using Lagrange multiplier \( \lambda \), called the emission weighting factor, as follows

\[ \min F = \sum_{i=1}^{M} [C_i(P_{i,t}) + \lambda E(P_{i,t})] \]

(5)

For the optimization problem (5), (2) the necessary conditions for optimality are [9]

\[ \frac{\partial F}{\partial P_{i,t}} = \lambda [1 - \frac{\partial P_{loss,i}}{\partial P_{i,t}}] \]

(6)

\[ \sum_{i=1}^{M} P_{i,t} = P_{loss,i} + D_i \]

(7)

where \( \lambda \), the Lagrange multiplier is an unknown value to be determined and \( \frac{\partial F}{\partial P_{i,t}} \) is known as the incremental cost function.

Thermal unit cost is represented as quadratic function of its generation. Sulfur oxide emissions are assumed proportional to each unit’s cost curve. This is a reasonable assumption since such emissions are proportional to fuel consumption [10]. This leads to the following cost and emission functions

\[ C_i(P_{i,t}) = b_{0,i} + b_{1,i} P_{i,t} + b_{2,i} P_{i,t}^2 \]

\[ E_i(P_{i,t}) = e_{0,i} + e_{1,i} P_{i,t} + e_{2,i} P_{i,t}^2 \]

If we neglect the transmission loss (\( P_{loss,i} = 0 \)), (6) and (7) can be written as

\[ b_{1,i} + 2b_{2,i} P_{i,t} + \omega_i e_1 + 2\omega_i e_2 P_{i,t} = \lambda \]

(8)

\[ \sum_{i=1}^{M} P_{i,t} = D_i \]

(9)

From (8),

\[ P_{i,t} = \frac{\lambda - b_{1,i} - \omega_i e_1}{2(b_{2,i} + \omega_i e_2)} \]

(10)

Putting the value of \( P_{i,t} \) from (10) in (9):

\[ \frac{\lambda - b_{1,i} - \omega_i e_1}{2(b_{2,i} + \omega_i e_2)} = D_i \]

or

\[ \lambda = D_i + \frac{\sum_{i=1}^{M} b_{1,i} + \omega_i e_1}{2(b_{2,i} + \omega_i e_2)} \]

(11)

Omitting generation limit constraints, (10) and (11) determine the analytical solution for the lossless case. The availability of an analytical solution to (8) and (9) substantially increases the efficiency of the classical algorithm [11].

If we consider transmission loss, the equations to be solved are as follows,

\[ b_{1,i} + 2b_{2,i} P_{i,t} + \omega_i e_1 + 2\omega_i e_2 P_{i,t} = \lambda [1 - \frac{\partial P_{loss,i}}{\partial P_{i,t}}] \]

(12)

\[ \sum_{i=1}^{M} P_{i,t} = P_{loss,i} + D_i \]

(13)

These are nonlinear equations of unknown variables \( P_{i,t} \) and \( \lambda \) and they can only be solved iteratively. Let \( \lambda^{\text{old}} \) and \( P_{i,t}^{\text{old}} \) be approximate solutions to (12) and (13). It is necessary to find new approximations

\[ \lambda^{\text{new}} = \lambda^{\text{old}} + \delta \lambda \]

\[ P_{i,t}^{\text{new}} = P_{i,t}^{\text{old}} + \delta P_{i,t} \]

Now, the transmission loss is represented by the following expression

\[ P_{loss,i} = \sum B_{i,j} P_{j,t} + \sum B_{i,j} P_{j,t} + B_0 \]

(14)

Using (14), \( \lambda^{\text{new}} \) and \( P_{i,t}^{\text{new}} \) are, to the first order, given by:

\[ 2(b_{2,i} + \omega_i e_2) P_{i,t}^{\text{new}} + b_{1,i} + \omega_i e_1 = \lambda^{\text{new}} [1 - 2 \sum B_{i,j} P_{j,t}^{\text{old}} - B_1] - 2 \lambda^{\text{old}} \sum B_{i,j} P_{j,t}^{\text{old}} \]

(15)

\[ \sum (1 - 2 \sum B_{i,j} P_{j,t}^{\text{old}} - B_1) P_{j,t}^{\text{new}} = P_{loss,i} + D_i - (2 \sum B_{i,j} P_{j,t}^{\text{old}} + B_1) P_{j,t}^{\text{old}} \]

(16)

In the classical solution algorithm, the last term in (15) as well the term \( 2 \sum B_{i,j} P_{j,t}^{\text{old}} + B_1 \) are neglected. To retain the classical form and to improve convergence, it is possible to include only the \( i \)-th term of the summation, i.e.

\[ 2 \lambda^{\text{old}} B_{i,j} \delta P_{j,t}^{\text{old}}, \text{ or rather } 2 \lambda^{\text{old}} B_{i,j} (P_{j,t}^{\text{new}} - P_{j,t}^{\text{old}}). \]

We now have

\[ 2(b_{2,i} + \omega_i e_2) B_{i,j} P_{j,t}^{\text{old}} + b_{1,i} + \omega_i e_1 - 2 \lambda^{\text{old}} B_{i,j} P_{j,t}^{\text{old}} = \lambda^{\text{new}} [1 - 2 \sum B_{i,j} P_{j,t}^{\text{old}} - B_1] - 2 \lambda^{\text{old}} \sum B_{i,j} P_{j,t}^{\text{old}}. \]

\[ \lambda^{\text{old}} B_{i,j} \delta P_{j,t}^{\text{old}}, \text{ or rather } 2 \lambda^{\text{old}} B_{i,j} (P_{j,t}^{\text{new}} - P_{j,t}^{\text{old}}). \]
If we denote, 
\[ P_{i,j}^* = P_{i,j}^{\text{new}} - \sum (2 \sum B_{i,j} P_{i,j}^{\text{old}} + B_i P_{i}^{\text{old}}) \]

(17) If we denote, 
\[ P_{i,j}^* = P_{i,j}^{\text{new}} - \sum (2 \sum B_{i,j} P_{i,j}^{\text{old}} + B_i P_{i}^{\text{old}}) \]

(17) can be written in the equivalent lossless form [1] as 
\[ 2b^2_i P_{i,j}^* + b_i^* = \lambda_{i,j}^{\text{new}} \]
\[ \sum P_{i,j}^* = b^* \]

We can now apply the technique [1] to fix the generation constraints (3).

IV. TEST RESULTS

This emission constrained economic dispatch algorithm has been coded in ‘C’ language for use in the dynamic programming based hydrothermal coordination program [8] developed in ‘C’ language too. The program has been tested on a practical utility system using the data of the generating units and system demands. The test system consists of 32 thermal and 12 hydro generating units, of which seven are gas turbine units. The total thermal capacity of the system is 3,640 Megawatt (MW) and the total hydro capacity is 848 MW. Numerical results presented here are based on three data sets: Case 1, Wednesday; Case 2, Saturday; Case 3, Sunday. The scheduling horizon is 24 hours in all cases. A summary of the system characteristics and parameters for these data sets is shown in Table 1.

<table>
<thead>
<tr>
<th>System</th>
<th>Number of units</th>
<th>Total capacity or requirements (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steam units</td>
<td>25</td>
<td>3360</td>
</tr>
<tr>
<td>Gas turbine units</td>
<td>7</td>
<td>280</td>
</tr>
<tr>
<td>Hydro units</td>
<td>12</td>
<td>848</td>
</tr>
<tr>
<td>All units</td>
<td>44</td>
<td>4488</td>
</tr>
<tr>
<td>Peak demand</td>
<td></td>
<td>2198</td>
</tr>
<tr>
<td>Minimum demand</td>
<td></td>
<td>1563</td>
</tr>
<tr>
<td>Maximum reserve</td>
<td></td>
<td>166</td>
</tr>
</tbody>
</table>

To avoid the voluminous amount of results, the complete solution process will be presented for Case 2 only. However summary of results of all cases will be presented at the end. For Case 2, using the hydrothermal coordination program without the emission constraint (ω=0 for all units), a schedule shown in Table 2 was suggested whose cost of operation is $2,024,321. The total emission from this schedule was 766.54 tons. The emissions from each unit are shown in Table 3. Emissions from units 1, 2, 9 and 10 were found to be quite high.

<table>
<thead>
<tr>
<th>Hour</th>
<th>Thermal Units</th>
<th>Hydro Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>111111111</td>
<td>1001100111</td>
</tr>
<tr>
<td>2</td>
<td>111111111</td>
<td>1001100111</td>
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<tr>
<td>3</td>
<td>111111111</td>
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<tr>
<td>4</td>
<td>111111111</td>
<td>1001100111</td>
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<tr>
<td>5</td>
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<tr>
<td>6</td>
<td>111111111</td>
<td>1001100111</td>
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<tr>
<td>7</td>
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<td>1001100111</td>
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<tr>
<td>8</td>
<td>111111111</td>
<td>1001100111</td>
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<tr>
<td>9</td>
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<tr>
<td>10</td>
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<tr>
<td>11</td>
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<tr>
<td>12</td>
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<td>1001100111</td>
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<tr>
<td>13</td>
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<td>15</td>
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<tr>
<td>21</td>
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<td>1001100111</td>
</tr>
<tr>
<td>24</td>
<td>111111111</td>
<td>1001100111</td>
</tr>
</tbody>
</table>

In order to reduce emissions from the above mentioned four units, hydrothermal coordination program was rerun using ω=200 for these particular units. The schedule obtained corresponds to an operating cost of $2,034,143. The deviation of this schedule from the earlier one was as follows: -Thermal unit 10 was not committed during hours 1 to 4 -Thermal units 17 and 18 were committed during hours 1 to 8 -All hydro units had nonzero generation during hours 2 to 4

The total emission from this schedule was 744.35 tons. The increase of $9822 i.e. 0.49% in total cost in this run over the
previous one was due to inclusion of emission constraint. But
this caused a reduction of 22.19 tons i.e. 2.89% in total
emission. Cost and emission summary of all three cases are
shown in Table 4.

<table>
<thead>
<tr>
<th>Data Set</th>
<th>( \omega = 0 ) for all units</th>
<th>( \omega \neq 0 ) for all units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>Emission</td>
<td>Cost</td>
</tr>
<tr>
<td>Case 1</td>
<td>2,150,712</td>
<td>802.22</td>
</tr>
<tr>
<td>Case 2</td>
<td>2,024,321</td>
<td>766.54</td>
</tr>
<tr>
<td>Case 3</td>
<td>1,641,014</td>
<td>644.48</td>
</tr>
</tbody>
</table>

Emission constraint was enforced through a set of
weighting factors. Higher emitters were given higher
weighting factors to limit the emission to a greater extent. In
the study system, only four units were found to be larger
emitters of sulphur oxide. The emissions from them contribute
to more than 60% of the total emissions. Hence, it is quite
logical to use higher weighting factors for those units only
while keeping the weighting factor equal to zero for other
units as the emissions from them are quite low. The reduction
of emission is done by shifting some loadings of these units to
more expensive units thus resulting in higher operating cost.
However, the percentage reduction in emission obtained is
much higher than the percentage increase in operating cost.

V. CONCLUSIONS

An efficient economic dispatch algorithm for dealing with
nonlinear functions such as the thermal cost, transmission loss
and emission constraint is developed. It is then incorporated
into the dynamic programming based hydrothermal
coordination program. The program has been tested on a
practical utility system using the data of the generating units
and system demands. Emission weighting factor is varied to
mitigate the impact of the emission of the corresponding unit.
Higher emitters of sulphur oxide are given a higher weighting
factor in order to limit the emissions to a greater extent.
The reduction of emissions is accomplished by shifting some
loading of these higher emitter units to more expensive units.
This results in higher operating cost. However, it has been
observed that a slight increase in production cost causes a
substantial reduction in emission.

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