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Improving Co-integration Trading Rule Profitability with Forecasts from an Artificial Neural Network

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***Abstract*—**Co-integration models the long-term, equilibrium relationship of two or more related financial variables. Even if co-integration is found, in the short run, there may be deviations from the long run equilibrium relationship. The aim of this work is to forecast these deviations using neural networks and create a trading strategy based on them. A case study is used: co-integration residuals from Australian Bank Bill futures are forecast and traded using various exogenous input variables combined with neural networks. The choice of the optimal exogenous input variables chosen for each neural network, undertaken in previous work [1], is validated by comparing the forecasts and corresponding profitability of each, using a trading strategy.

***Keywords*—**Artificial neural networks, co-integration, forecasting, trading rule.

I. Introduction

ACCORDING to co-integration theory, if bonds or bank bills with different maturities are co-integrated, there exists a long term equilibrium relationship between them, so that, even if short term deviations from equilibrium may occur, market forces are expected to act to restore the equilibrium in the long run.

Many authors have investigated the co-integration relationship using many different financial assets. However, there has been little investigation regarding the factors that influence the long term co-integration relationship.

The identification of exogenous variables that have significant effects on the temporary deviation from the co-integration equilibrium has important economic implications because, if they are predictable, they can be used for forecasting future directions of the yield rate of bank bills.

In this work neural networks are used to forecast the deviation from long-run co-integration equilibrium (characterized by the residuals from the co-integration model). Neural networks are powerful nonlinear regression models that have had considerable success in modeling nonlinear financial variables.

In previous work [1], ten candidate variables were considered to see if they statistically significantly minimize mean squared error of the co-integration residual forecasts (using the neural network). In this work, we validate the choice of exogenous variables in [1] by converting the forecasts of the various neural networks with different exogenous input variables into trading rules. The key question is: do the most accurate neural networks with the best subset of exogenous input variables provide the most profitable trading rules? If so, we have validated the methodology which was used to choose them.

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II. Background

In this Background section, we briefly consider the underlying concepts of co-integration, bank-bill futures and neural networks.

A. An Introduction to Australian Bank Bill Futures

In this work we create a co-integration trading rule on Australian 90 day bank-bill futures.

The Australian 90 day bank bill futures contract traded on the Sydney Futures Exchange (SFE) is a leading interest rate futures contract based on the Australian 90 day bank accepted bills. The first such bill contract launched outside the US, it was initially listed on October 17, 1979. Currently, with an estimated daily turnover of approximately $4 billion, it has become one of the most actively traded short-term interest rate products trading in Australia and is considered a benchmark indicator for Australian short term interest rates[1].

The term to maturity is the amount of time until a contract matures. Futures contracts mature at a predetermined time in the future. The month in which a contract matures is known as the delivery month. The delivery months for the 90-Day Bank Bill futures contract are up to 20 quarter months un the future. A trader, in January, trading the nearest future would trade the March delivery month future, and the term to maturity would be 3 months. The second nearest or next expiry/delivery month would be the next quarter (June) and the days to maturity would be 6 months. Once the nearest Bank Bill future (the March expiry contract) has expired, it no longer is the nearest contract. There is a rollover effect, which causes a readjustment of the terms to maturity. At the expiry date of March, the second nearest expiry contract, the June expiry contract, becomes the nearest contract and all the other following delivery months contracts such as September and December expiry contracts roll over in a similar fashion.

[1] Sources: Sydney Futures Exchange Ltd. "90 day bank bill futures and options, November 2000", p, 5

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B. An Introduction to the Co-integration of Bank Bill Futures

The objective of this section is to investigate the term structure of Australian 90 day bank bill futures using co-integration analysis. The focus is on the relationship between the closest expiry contract and the following four nearby serial contracts as a portfolio (as these are the most heavily transacted contracts).

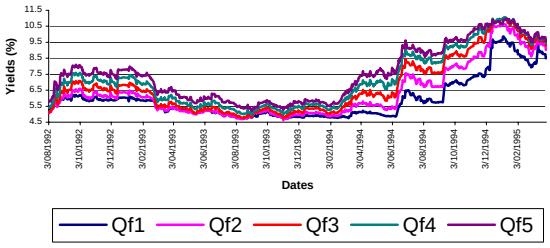

Fig. 1 The five nearest Australian 90 day bank bill futures

Evidence of co-integration exists in Figure 1. Qf1 to Qf5 indicate the five contracts from 3 months to 15 months. The figure shows that these five series display similar price behavior over time. They tend to move up and down together (but not exactly). They are highly correlated and it is this behavior that indicates we may find co-integration.

To create a co-integrating portfolio we can construct a portfolio which is a combination of the last four contracts designed to mimic the first contract (Equation 1). We find that the residuals from this portfolio:- u, are stationary and mean-reverting to some equilibrium value. This implies that we have found a co-integration relationship between the contracts.

Statistically, we can test for a co-integration relationship between the nearest contract (Qf1) and the next 4 nearest bank bill futures (Qf2 to Qf5) contracts by estimating the following relationship:-

$$Qf1 = a*Qf2 + b*Qf3 + c*Qf4 + d*Qf5 + u \quad (1)$$

Using the appropriate co-integration test on the residuals (Engel-Granger single equation test) we find statistically significant evidence of co-integration.

Essentially this tests if u, the residuals, are stationary (a consequence is that they mean-revert around an equilibrium value).

C. *An Introduction to Neural Networks*

Once the co-integration relationship is estimated (in Equation 1) artificial neural networks (ANNs) are used to estimate the residuals u. These residuals correspond to short term deviations from the long run co-integration equilibrium relationships between the first five contracts.

An artificial neural network is a computational system with structure and function inspired by the biological structure of the human brain. When a neuron receives the input signals from neighboring neurons, its task is to organize those signals and produce outputs to neighboring neurons. In an ANN, each neuron is interconnected with neurons in the previous layer from which it receives stimuli and with neurons in the subsequent layer to which it sends information.

Recently, neural networks have emerged as a powerful tool to be used where statistical methods are traditionally employed. From the statistician's point of view, neural networks are analogous to non parametric, non linear regression models [2]. Unlike conventional parametric models such as linear regression, neural networks have two unique characteristics:

- They do not require *a priori* specification of the functional relationship between variables.
- Neural networks are known to be a universal function approximator.

So, in our context, the aim of using ANNs is to estimate the residuals from Equation 1, u=f(.) where we choose f(.) to be an ANN.

D. The Exogenous Variables

There are many inputs that may be important in forecasting the co-integrating residuals, u.

As domestic markets are exposed to global market interactions, a large part of the domestic futures market can also be influenced by offshore futures market behavior. This is particularly true for a small economy like Australia, that is more heavily dependent upon the global economy. In addition, previous research indicates that the US financial markets are the most influential in the world and price movements have important information content that influences other domestic markets. For this reason, US financial market data are mainly used here.

Firstly, the Standard and Poor's 500 ($\triangle S\&P500_t$ #10) stock index futures, which is considered to be the most widely used benchmark for U.S. equity performance as well as for the performance of U.S economy is selected.

Also used are the Australian currency futures against U.S dollar ($\triangle au_t$ #7) because trading of Australian dollars in the foreign exchange market is highly related to the US dollar and international competitiveness is dependent on the price of the Australian currency against the U.S currency. Thus, the weaknesses or strengths of the Australian currency against the U.S currency may impact on the yield rate.

The oil ($\triangle o_t$ #9) and gold futures ($\triangle g_t$ #8), which are the most frequently traded commodity futures in the world, may contain different information. These are raw materials and any price changes may create a trading signal that may impact on the short term deviation.

On the domestic side, 3 year ($\triangle 3y_t$ #5) and 10 year interest rates futures ($\triangle 10y_t$ #6) contracts[2] traded on the SFE were chosen as well as the spread between 90 day bank bills ($\triangle ss_t$, #3). It is expected that the information contained in the

[2] SFE's 3 year and 10 year treasury bond futures contracts are Commonwealth Treasury Bonds that are fixed interest securities issued by the Federal Government to satisfy its borrowing requirement. Due to their dominance in the Australian fixed interest market they are regarded as being the benchmark indicators of long and medium term interest rates in Australia (SFE Ltd, 2000).

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yield movements of these contracts may influence the yield movement of 90 day BABs ($\triangle 90y_t$, #4).

As well as the market information data, the auto-correlation of the residuals from the co-integration relationship (u_{t-1} #2 and u_{t-3} #1) are considered because the residuals suggest that some serial correlations, which imply the relationship between the sequential residuals series, are not independent of each other.

In previous work [1], we modeled the nonlinear relationship between the co-integrating residuals and the other 10 individual inputs chosen as follows:

$$u_t = f(u_{t-3}, u_{t-1}, ss_t, \Delta 90y_t, \Delta 3y_t, \Delta 10y_t, \Delta au_{t-1}, \Delta g_{t-1}, \Delta o_{t-1}, \Delta s \,\&\, p_{t-1}) + e_t \quad (2)$$

where: u_t is the co-integrating residual at t day; e_t is a noise parameter; and f(.) is a neural net.

We found that none of the exogenous variables we added as inputs to the artificial neural network produced statistically significantly, more accurate forecasts.

However, we did find that an ANN including variables (#2, #4, #9) did produce more accurate forecasts of the co-integration residuals, u. Do these forecasts, although not statistically better, produce more profitable trading strategies?

III. Trading Strategy

The aim of this section is to evaluate the performance of the input variables chosen by forecasting the co-integration residuals, u, using various subsets of the exogenous input variables using equation (2). Using a simple trading rule, the forecasting model generates trading signals for determining buying and selling futures contracts as a result of the changes in yield spread. The performances of these inputs are then analyzed by considering the trading rule profits and losses and Sharpe ratio.

A. Out-of Sample Period

The out of sample data is from the 31/3/1995 to the 28/3/1997. The five contracts in out-of-sample periods also confirm a co-movement trend (Figure 2). As compared to the in-sample, the yield rates in out-of-sample period shows steadily decrease. The main cause was the weakness in the Australian economy. Also, the yield rates of the nearest expiry contract exceed the rates of the nearby contracts during the sample period from May 3, 1995 to February 15, 1996 and July 31, 1995 to November 30, 1996. In these two periods, inverted yield rates were observed on three occasions during January 12, 1995 to June 12, 1995 and January 29, 1996 to February 20, 1996 and November 25, 1996 to November 28, 1996.

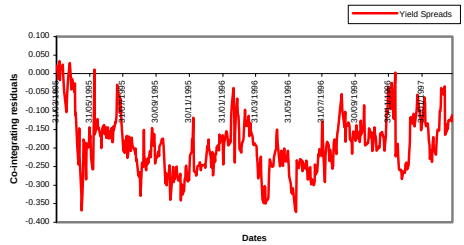

Fig. 2 The out-of-sample deviations from equilibrium, u_t.

Figure 2 shows that the short and long run yield rates tend to move together. However, the short term yield rates persistently deviate under the long term equilibrium rates in the sample period. Consequently, the yield spreads graphed in Figure 2 show negative deviations from the long term equilibrium.

B. The Technical Trading Strategy

Since there is a co-integration relationship between the five bank bill futures contracts out-of-sample, a portfolio can be constructed based on the co-integrating equilibrium relationship:

The yield rate of the first contract = the yield rates of nearby contracts in a portfolio + mis-pricing of the yield spread.

This can be written as follows:-

$$S_t = L_t + U_t \quad (3)$$

where S_t stands for the short term contract,

L_t is the long term contract,

U_t is the yield spread between S_t and L_t

The general principle of the portfolio trading is to take a portfolio position with the assumption that any mis-pricing between short and long term contracts at each time interval can be forecast. Profit or loss is calculated by the difference of the value of the position of the portfolio. Thus, the profitability of this trading strategy depends on the accuracy of forecasting of the changes in yield spreads.

Mathematically, the neural network yield spread forecasting model[3] can be expressed as follows:

$$u_{t+1} = f(x_i) + e_t \quad (4)$$

[3] Five fold cross validation [3] uses 80 % of data for training and the remaining 20% for testing. There are 675 days of historical data for input selection. The data sets are divided into a 80 % data set consisting of 540 patterns of training set and a 20% testing set consisting of 135 patterns. When the first sampling is completed, the testing set used in first sample is subsumed into the training set and the last 135 patterns is used in the first training set reserve for the second testing set.

In the context of cross-validation input selection, a forward selection process is used:- the single best-input variable is chosen first, and then inputs are added, one at a time so that, at each stage the variable chosen is the one which produces the least errors. This procedure begins by evaluating each of the input variables individually and selecting the one which gives the lowest cross validation errors. An additional input is added to the previous input set and the best combination is again chosen on the basis of which input variables gives rise to the lowest cross validation errors

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u_{t+1} = the prediction of yield spread (co-integrating residual) at time t+1; and, x_i = the input variables where i= the number of inputs.

Here, the interest is in the neural network yield spread prediction (instead of obtaining the difference between the prediction of the yield spreads and their actual value as was done in the input selection process, [1]). The model is used to discover the future direction of spread movements.

To validate the yield spread forecasting model, we use a turning point trading rule. The turning point trading rule is a trading strategy that depends on the changes in the futures yield spread, as indicated below:

$$\Delta u_{t+1} = \hat{u}_{t+1} - u_t \tag{5}$$

where $\hat{u}_{t+1}$ = neural network yield spread prediction; u_t = the yield spread from the co-integrating relation; Δu_{t+1} = the changes in yield spread at time t+1.

This turning point trading strategy attempts to monitor the changes in direction of future yield movements. If the predicted future yield spread rates are higher than the previous rates, then the forecasted yield change will be positive and imply spread rates will goes up. On the other hand, if the changes are negative, it will go down. Thus, the change indicates the future yield spread movements in relation to the previous position.

The trading rule can explained as follows: buy the short term contract and sell the long term if the changes are positive

$$Trading \quad Position_{t+1} = \begin{cases} If \quad U_{t+1} - U_t > 0 \quad then, \\ 1 \quad Buy \quad L_{t+1} \\ -1 \quad Sell \quad S_{t+1} \end{cases}$$

As the yield spread refers to the mis-pricing between a short and long term contract as a portfolio, the positive movements of the yield spread change indicate the relatively higher short term yield rate in respect to the previous rate. Thus, if tomorrow's prediction of the yield spread rate is greater than today's the yield rate of the short term contract will be relatively higher than long term yield in respect to today's rate. This is a signal to take a long position for short term contract and short position for the nearby contracts.

On the other hand, if tomorrow's prediction is less than today's, the long term rate is relatively stronger. Thus, the trading strategy is to do the opposite of the above rule and buy short on the long contract and long on the short contract.

IV. Results

In previous work, [1], it was found that 10 subsets of input combinations and each individual combination of groups were indistinguishable statistically. However, the input combination #2,#4,#9 was identified as the best, based on the mean squared error alone.

Figure 3 and 4 show that the highest total profit was also obtained by combining the input #2,#4,#9 (lag 1 residuals, 90 day bank bill changes and oil prices changes). This result suggests the best combination of inputs based on mean squared co-integration residual forecasting error also translates into optimal trading profitability. Although the total profits between input groups do not appear to be significantly different, the result shows that the profitability of the trading rule is consistent with the optimal choice of inputs made in [1]. Therefore, this return performance evaluation provides evidence about the usefulness of input selection of the previous work [1].

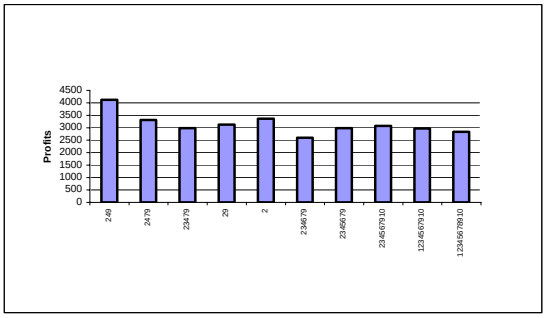

Fig. 3 Total profits

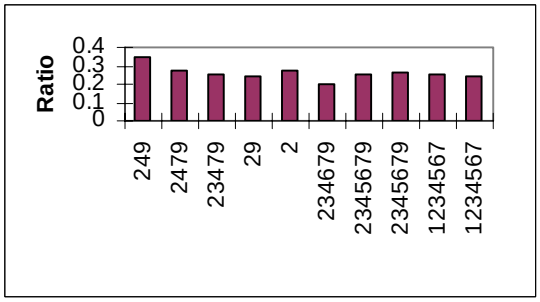

Fig. 4 Sharpe ratio

V. Conclusion

In this work we have validated the input selection procedure of earlier work. A trading rule constructed with the most accurate co-integration residual forecasts using optimal (although not statistically significant) exogenous input variables proves to be the most profitable also. Thus validating the previous work [1].

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