A quantitative Tool for Analyze Process Design

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Abstract—Some quality control tools use non metric subjective information coming from experts, who qualify the intensity of relations existing inside processes, but without quantifying them.

In this paper we have developed a quality control analytic tool, measuring the impact or strength of the relationship between process operations and product characteristics. The tool includes two models: a qualitative model, allowing relationships description and analysis; and a formal quantitative model, by means of which relationship quantification is achieved.

In the first one, concepts from the Graphs Theory were applied to identify those process elements which can be sources of variation, that is, those quality characteristics or operations that have some sort of prelacy over the others and that should become control items. Also the most dependent elements can be identified, that is those elements receiving the effects of elements identified as variation sources. If controls are focused in those dependent elements, efficiency of control is compromised by the fact that we are controlling effects, not causes.

The second model applied adapts the multivariate statistical technique of Covariance Structural Analysis. This approach allowed us to quantify the relationships. The computer package LISREL was used to obtain statistics and to validate the model.

Keywords—characteristics matrix; covariance structure analysis; LISREL

I. INTRODUCTION

IN different industries, as in the automotive, complex products are manufactured in complex processes. This complexity implies, for products, that lot of characteristics (dimensional, physical…) are required to fully characterize the product. In processes complexity means the presence of multiple steps and operations, in which the product is conformed, modified or assembled.

To understand the influences between characteristics and operations has a great interest for planning control in an optimal way, that is, with the best results and the minimum effort.

Technical knowledge and practical experience show that some characteristics and operations can be identified as the key for a good process performance, while other can be recognized as specially conflictive or difficult to control.

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Identify both can be the base for an effective control strategy, allowing quality and productivity improvements.

In the automotive industry, a simple tool was used for analyzing relationships between product characteristics and process operations. This tool, called Characteristics Matrix, was used in the process for defining Production Control Plans [1]. With a purely qualitative approach this tool produce a description of these relationships and this information was used, basically in an instinctive and logical way, for the Control Plan definition.

TABLE I
EXAMPLE OF CHARACTERISTICS MATRIX CHARACTERISTIC

Operation	Characteristic					
	Dim 1	Dim 2	Dim 3	Dim 4	Dim 5	Dim 6
Incoming material	*					
Cut ID	C	*				
Cut face	C		*			
Cut Dim C4		C	L	*•	•	•
Cut Dim C5		C	L	•	*•	•
Cut Dim C6		C	L	•	•	*•
Finish OD	*	C	L			

C= characteristic used for part clamping; L= characteristic used for part locating or positioning; *= characteristic created or modified in this operation; •= related characteristics (i.e.: produced in the same operation). Dim1 = Outer Diameter; Dim2 = Internal Diam.; Dim3 = Length; Dim4 = External diam.; Dim5 = External diam.; Dim6 = External diameter.

Carrión, Jabaloyes and López [2] worked with this matrix and advance beyond the original approach, using also a qualitative method. This method allows the recognition of higher order relationships (that is, indirect relations) between characteristics and operations. The method also informs about the potential critical nature and dependency of the items analyzed. Table 1 shows a simple example of Characteristics Matrix. In this paper, an approach based in the Mic-Mac method allows the analysis in deep of the relationships and to obtain the motricity and dependency of each item analyzed. Motricity represents the impact that an item has over the rest of characteristics and operations, while dependency measures to what point an item is depending of other characteristics and operations.

II. A NEW PROPOSAL FOR EVALUATING PROCESS RELATIONSHIP

With the basis of previous papers, our work follows a double direction. First we continue in a qualitative analysis of the relationships, using new graphic tools. Second we try to quantify relationships using a multivariate method. Each of these approaches or models is useful for a different phase in the Control Plan definition process. This double approach is presented in figure 1.

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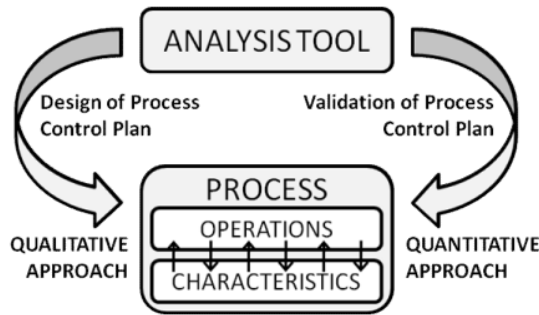International Journal of Mathematical and Computational Sciences
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Fig. 1 Double structure of the proposed tool

III. QUALITATIVE MODEL

The Quantitative Model can be applied both in design phase and in production phase, and in consequence is useful also for validating control plans. Objectives of the model are: i) Describe relations existing among process operations and quality characteristics generated by these operations. ii) Identify process elements of special relevance, basing in its motricity (that is, in the influence in other process steps and characteristics, see [2]). iii) Detect underlying dependency relations of some process elements. iv) Provide information useful to prepare control plans for manufacturing processes, or to redesign existing control plans.

For its application, the starting point is a relational diagram. The nodes represent the considered elements (operations and characteristics) and connectors, arrow lines, the sense of their relations. Figure 2 shows the diagram for the considered example.
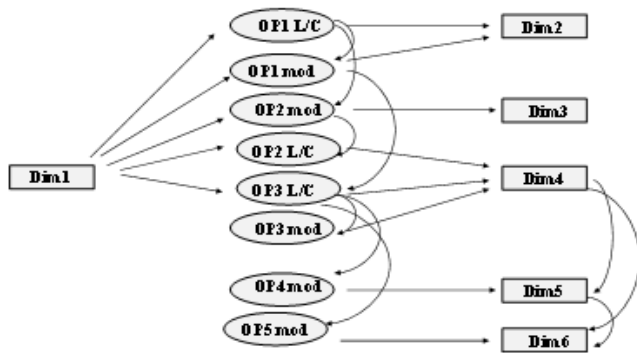

Fig. 2 Relational Diagram

Direct relations shown in the diagram, are condensed in the Adjacency Matrix A, whose elements are defined as follows [3]:

$$a_{ij} = \begin{cases} 1, \text{ if } v_i \text{ is a node adjacent to node } v_j \\ 0, \text{ if it isn't} \end{cases}$$

Table II shows the Adjacency Matrix for the case presented. Adjacency Matrix A is a square matrix whose rows and columns are in the same order: MOD operations, LC operations and the dimensions or quality characteristics.

		OP 1Mod	/ 2Mod	M 3Mod	o 4Mod	d 5Mod	OP 1L/C	L/C 2L/C	3L/C	C DIM1	A DIM2	R DIM3	A DIM4	C DIM5	T DIM6	Σ	Motric
OP	1Mod	0	0	0	0	0	0	0	1	0	1	0	0	0	0	2	9%
/	2Mod	0	0	0	0	0	0	1	0	0	0	1	0	0	0	2	9%
M	3Mod	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	4%
o	4Mod	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	4%
d	5Mod	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	4%
OP	1L/C	1	1	0	0	0	0	0	0	0	1	0	0	0	0	3	13%
	2L/C	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	4%
L/C	3L/C	0	0	1	1	1	0	0	0	0	0	0	1	0	0	4	17%
C	DIM1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	5	22%
A	DIM2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
R	DIM3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
A	DIM4	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2	9%
C	DIM5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	4%
T	DIM6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0%
	Σ	2	2	1	1	1	1	2	2	0	2	1	3	2	3	23	
	Depend	9%	9%	4%	4%	4%	4%	9%	9%	0%	9%	4%	13%	9%	13%		100%

Adjacency Matrix A is a square matrix whose rows and columns are in the same order: MOD operations, LC operations and the dimensions or quality characteristics.

The sum by rows gives as result the intensity of the role of the nodes as source of relations, corresponding to the dominion degree or motricity of the elements of the process. Summing by columns produces the dependency of the elements, that is, the degree of the intensity of their role as relations receptors. With those values figure 3 can be drawn, showing motricity and dependency for the different elements.
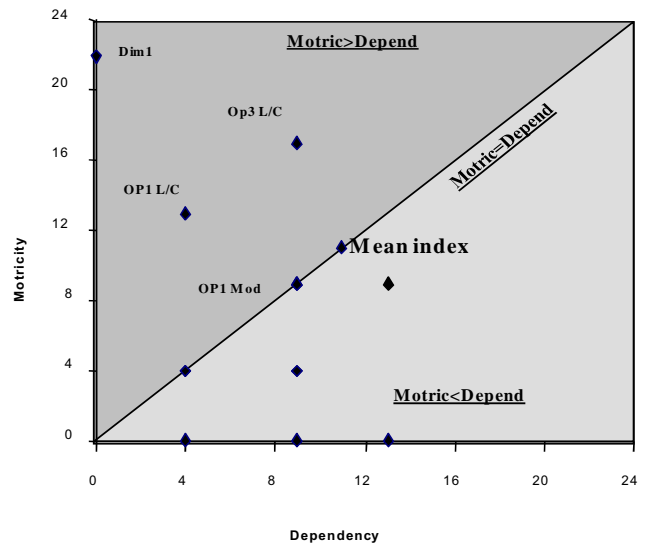

Fig. 3 Motricity vs. Dependency

The great importance of the part initial diameter is confirmed, because the corresponding characteristic, DIM1, has the higher motricity value and a null dependency. DIM1 is the independent variable of the model. The rest of the characteristics, remain in the lower area (dependency greater than motricity). Operations 1 and 2 (OP1 and OP2) present similar values for motricity and dependency, with values near the mean index. In consequence they must be controlled by the effects they can transmit to the rest of elements.

By successive powers of the Adjacency Matrix, upper order influences can be identified, because those new matrices informs about longer paths, not shown in matrix A. It can be defined as [3]:

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$$\begin{cases} \text{If } \langle v_i, v_j \rangle \text{ is an edge, the assigned value is } 1 \\ \text{If not, the value is } 0 \\ \text{If } \langle v_i, v_j \rangle \times \langle v_j, v_k \rangle = 1 \Rightarrow \langle v_i v_j v_k \rangle \text{ is a path} \end{cases}$$

In general, elements

$$a_{ij}^{(k)}$$

from matrix Ak indicate the number of length k paths existing between nodes i and j [4]:

$$a_{ij}^{(k)} = \sum_{m=1}^{n} a_{im} a_{mj}^{(k-1)}$$

Any path's length is k≤n-1, where n is the number of nodes. Relationship has k-1 intermediate nodes. Using a digraph for representation, successive powers of adjacency matrix converge to a null matrix.

On the other hand, matrix shows

$$B_k = \sum_{k=1}^{k} A^k = A + A^2 + A^3 + \cdots + A^k$$

all paths of length lower or equal to n that exist between two specific nodes, [5]. In the present case k=7, because powers of A begin to be null after A8.

TABLE III
POWERS SUMS MATRIX OF ADJACENCY MATRIX

		OP 1Mod	/ 2Mod	M 3Mod	o 4Mod	d 5Mod	OP 1L/C	2L/C	L/C 3L/C	C Dim1	A Dim2	R Dim3	A Dim4	C Dim5	T Dim6	Σ
OP	1Mod	0	0	1	1	1	0	0	1	0	1	0	2	3	6	16
/	2Mod	0	0	0	0	0	0	1	0	0	0	1	1	1	2	6
M	3Mod	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4
o	4Mod	0	0	0	0	0	0	0	0	0	0	0	0	1	1	2
d	5Mod	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
OP	1L/C	1	1	1	1	1	0	1	1	0	2	1	3	4	8	25
	2L/C	0	0	0	0	0	0	0	0	0	0	0	1	1	2	4
L/C	3L/C	0	0	1	1	1	0	0	0	0	0	0	2	3	6	14
C	Dim1	2	2	3	3	3	1	3	3	0	3	2	9	12	24	70
A	Dim2	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
R	Dim3	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A	Dim4	0	0	0	0	0	0	0	0	0	0	0	0	1	2	3
C	Dim5	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1
T	Dim6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	Σ	3	3	6	6	6	1	5	5	0	6	4	19	27	55	146

$B_7 =$

With basis in this matrix we can conclude that in the analyzed process, there are 146 way of relations between operations and characteristics (see Table 3). It is confirmed that Dim1, external diameter, is the more determinant characteristic, as is the origin of 70 of those relationships (48% of total). Adequate control of Dim1 (incoming material) is highly important. In decreasing importance order we have OP1 L/C (17% relationships), OP1 Mod (11%) and OP3 L/C (9%). Highly dependent characteristics are Dim4, Dim5 and Dim6, with dependency values of 13, 18 and 37% respectively.

IV. QUANTITATIVE MODEL

Objectives of this model are: 1) To confirm and quantify relations identified and described by the qualitative model. 2) To give statistical support in decision making about production processes validation and control. 3) To estimate parameter and variance-covariance matrices. 4) To evaluate model using statistical tools. 5) To quantify relations existing among operations and among operations and quality characteristics. 6) To serve as quantitative (statistical) basis for the definition of process management strategies.

For the development of this model we used a multivariate technique: the Covariance Structure Analysis. This includes a series of models known as Latent Variable Analysis [6], Confirmatory Factorial Analysis [7], Covariance Structure Models [8], Structural Equations Models SEM [9] and Structural Linear Relations LISREL [10].

The Covariance Structure Model includes three sub-models: First and second are models for measurements, relating observed variables with latent variables or factors; the third sub-model is a structural model relating endogenous factors among them and with exogenous factors.

Their expressions are:

Measurements sub-model for observed exogenous variables, x: $\qquad x = \Lambda_x \xi + \delta$

Measurements sub-model for observed endogenous variables, y: $\qquad y = \Lambda_y \eta + \varepsilon$

Structural equation sub-model:

$$\eta = B\eta + \Gamma\xi + \zeta$$

Where η is the endogenous factors vector; ξ is the exogenous factors vector; ε and δ are the measurement error vectors and ξ is equations error vector.

Eight matrices are involved in the model: Four of them are parameter matrices and the other four are variance-covariance matrices. Parameter matrices correspond to the coefficient matrices for the measurement models, or factorial loading matrices Λx y Λy, and to the structural coefficient matrices B yΓ, involved in the third equation.

Variance-covariance matrices are: Φ is the variance-covariance matrix for exogenous factors; Ψ in the variance-covariance matrix for equations errors; Θδ and Θε are, respectively, the variance-covariance matrices for the errors in measurements of exogenous and endogenous variables.

V. APPLICATION CASE

The case used for illustration is the same presented in the first part of this paper. Two alternative models were prepared, adjusting previously the number of factors, given that this number should be lower than the number of observed variables.

We present one of these models. Its relational diagram is shown in figure 5. In this figure, ovals represent factors, in the present case corresponding to operations, and rectangles correspond to the observed variables.

The applied model, based in the relational diagram (fig. 4) is:

- Measurement equations for endogenous quality characteristics, y:

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$$y = \Lambda_y \eta + \varepsilon$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \end{pmatrix} = \begin{bmatrix} \lambda_{11} & 0 & 0 & 0 & 0 \\ 0 & \lambda_{22} & \lambda_{23} & 0 & 0 \\ 0 & 0 & \lambda_{33} & 0 & 0 \\ 0 & 0 & 0 & \lambda_{44} & \lambda_{45} \\ 0 & 0 & 0 & \lambda_{54} & \lambda_{55} \\ 0 & 0 & 0 & \lambda_{64} & \lambda_{65} \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{pmatrix}$$

Where:

η_1 is the external diameter measurement operation for the bar coming from the supplier.

η_2, OP1 L/C, represents the clamping operation by the bar external diameter.

η_3, OP1 Mod, represents mechanizing operation for internal diameter and cutting the length of the bar.

η_4, OP2 L/C, represents location operations using final cutted face and clamping by the external diameter.

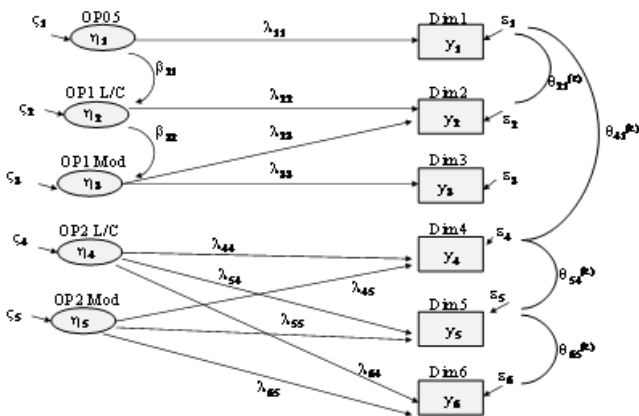η_5, OP2 Mod, represents mechanizing operations for dimensions 4, 5 and 6, all performed in the same equipment.

Fig. 4 Relational diagram for the case studied.

• Structural equation:

$$\eta = B\eta + \zeta$$

$$\begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \beta_{21} & 0 & 0 & 0 & 0 \\ 0 & \beta_{32} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{pmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \\ \eta_5 \end{pmatrix} + \begin{pmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \\ \zeta_4 \\ \zeta_5 \end{pmatrix}$$

In the following paragraphs we resume some results obtained using the computing package LISREL. Results presented correspond to the formulated model, to the relations between process operations and to relations between operations and generated quality characteristics.

A. Results referred to the model.

Matrix in table 4 shows three important values: -0.49, -0.58 and –0.63. This is a consequence of multicolineality problems present in the model, given that the input data matrix S is close to a non positive defined matrix.

TABLE IV
ESTIMATIONS CORRELATION MATRIX

	λ_{23}	λ_{45}	λ_{54}	λ_{64}	β_{21}	β_{32}	θ^ε_{21}	θ^ε_{41}	θ^ε_{54}	θ^ε_{65}
λ_{23}	1.00									
λ_{45}	0.01	1.00								
λ_{54}	0.00	0.11	1.00							
λ_{64}	0.00	-0.20	-0.11	1.00						
β_{21}	-0.49	0.17	-0.08	-0.18	1.00					
β_{32}	-0.25	-0.17	0.08	0.15	-0.31	1.00				
θ^ε_{21}	0.27	-0.18	0.09	0.17	-0.61	0.16	1.00			
θ^ε_{41}	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00		
θ^ε_{54}	-0.01	-0.58	-0.63	0.17	-0.04	0.04	0.04	0.00	1.00	
θ^ε_{65}	0.00	-0.07	0.00	0.08	0.06	-0.05	-0.06	0.00	0.04	1.00

Matrix in table 5 shows good fitting to the model. This result is confirmed with the Chi Square value ($X^2 = 11.81$ with 11 degrees of freedom and a significance level of 0.38).

TABLE V
STANDARDIZED RESIDUALS

	y_1	y_2	y_3	y_4	y_5	y_6
y_1	-0.63					
y_2	0.00	-1.81				
y_3	0.39	-2.34	-1.84			
y_4	0	-0.80	0.10	-0.63		
y_5	-0.40	-0.60	-1.39	0.00	-0.63	
y_6	0	0	-0.90	0.00	0.00	-0.10

B. Results referred to relations between operations.

Matrix in table 6 presents the structural coefficients between operations. There, η_1 is the external diameter measurement operation for the bar coming from the supplier, η_2 is the clamping operation by the bar external diameter and η_3 is the mechanizing operation for internal diameter and cutting the length of the bar.

Indirect effects correspond to the matrix in table 7. No direct relation was previously identified between operations η_1 and η_3, but there is an indirect effect of 23%, that can be interpreted as the magnitude of the impact transmitted to operation η_3 when a change is produced in operation η_1. There is a direct effect of operation η_1 over operation η_2. We can understand that operation η_2 is transmitting the effect form operation η_1 to operation η_3.

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Vol:6, No:1, 2012

TABLE VI
STANDARDIZED DIRECT EFFECTS MATRIX, FOR RELATIONS BETWEEN OPERATIONS

$$
\hat{B} = \begin{bmatrix}
 & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\
\eta_1 & 0 & 0 & 0 & 0 & 0 \\
\eta_2 & -0.31 & 0 & 0 & 0 & 0 \\
\eta_3 & 0 & -0.74 & 0 & 0 & 0 \\
\eta_4 & 0 & 0 & 0 & 0 & 0 \\
\eta_5 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

TABLE VII
STANDARDIZED INDIRECT EFFECTS MATRIX

$$
\begin{bmatrix}
 & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\
\eta_1 & 0 & 0 & 0 & 0 & 0 \\
\eta_2 & 0 & 0 & 0 & 0 & 0 \\
\eta_3 & 0.23 & 0 & 0 & 0 & 0 \\
\eta_4 & 0 & 0 & 0 & 0 & 0 \\
\eta_5 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

C. Results referred to relations between operations and quality characteristics

Table VIII presents the factorial loading matrix of operations over quality characteristics, where is the bar external diameter (from the supplier), is the part inner diameter and is the part length.

TABLE VIII
MATRIX FOR THE DIRECT EFFECTS OF OPERATIONS OVER QUALITY CHARACTERISTICS

$$
\hat{\Lambda}_y = \begin{bmatrix}
 & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\
y_1 & 0.49 & 0 & 0 & 0 & 0 \\
y_2 & 0 & 0.15 & -0.07 & 0 & 0 \\
y_3 & 0 & 0 & 0.22 & 0 & 0 \\
y_4 & 0 & 0 & 0 & 0.50 & 0.00 \\
y_5 & 0 & 0 & 0 & 0.00 & 0.49 \\
y_6 & 0 & 0 & 0 & 0.00 & 0.37
\end{bmatrix}
$$

From matrix in table IX, we can conclude that operation η_1 has no direct effect over quality characteristic y_2, but there is an indirect negative impact of 6% of any change produced in this operation, affecting characteristic y_2. In the same way, although there is no direct effect of the operation η_2 over characteristic y_3, there is and indirect negative effect of 16%, implying that a change in clamping operation η_2 will affect negatively part length (dimension y_3).

TABLE IX
MATRIX FOR THE STANDARDIZED INDIRECT EFFECTS OF OPERATIONS OVER QUALITY CHARACTERISTICS

$$
\begin{bmatrix}
 & \eta_1 & \eta_2 & \eta_3 & \eta_4 & \eta_5 \\
y_1 & 0 & 0 & 0 & 0 & 0 \\
y_2 & -0.06 & 0.05 & 0 & 0 & 0 \\
y_3 & 0.05 & -0.16 & 0 & 0 & 0 \\
y_4 & 0 & 0 & 0 & 0 & 0 \\
y_5 & 0 & 0 & 0 & 0 & 0 \\
y_6 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

VI. CONCLUSIONS

As the main results of this research, we can mention:

- We have contributed with statistical arguments to the analysis of the relations between operations in a manufacturing process and quality characteristics created in this process.

- It is important to remark that with proposed model it is possible to quantify relations between operations, although those are non measurable variables.

- The situations in which this method can be applied, meaning generally processes under control, are favourable for the presence of multicolinearity problems, and even considering this fact the method proposed allows the prosecution of the analysis.

- A quantitative approach has been introduced in a merely qualitative problem.

- The application of the multivariate technique of Covariance Structure Analysis in manufacturing processes is a field of high potential possibilities.

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