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Efficient Spectral Analysis of Quasi Stationary Time Series

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***Abstract*—** Power Spectral Density (PSD) of quasi-stationary processes can be efficiently estimated using the short time Fourier series (STFT). In this paper, an algorithm has been proposed that computes the PSD of quasi-stationary process efficiently using off-line autoregressive model order estimation algorithm, recursive parameter estimation technique and modified sliding window discrete Fourier Transform algorithm. The main difference in this algorithm and STFT is that the sliding window (SW) and window for spectral estimation (WSA) are separately defined. WSA is updated and its PSD is computed only when change in statistics is detected in the SW. The computational complexity of the proposed algorithm is found to be lesser than that for standard STFT technique.

***Keywords*—** Power Spectral Density (PSD), quasi-stationary time series, short time Fourier Transform, Sliding window DFT.

I. INTRODUCTION

Time-frequency analysis plays a central role in signal analysis. A global Fourier transform of a time signal is of little practical value as it looses the time aspect, if any, of the signal. Quasi-stationary signals which are evolving with time in an unpredictable way (like a speech signal or a biomedical signal) necessitate the notion of frequency analysis that keeps track of the time aspect of the signal as well [1, 5].

Short Time Fourier transform (STFT) proposed by Gabor in 1946 provide a means to capture the time information while computing the PSD [2, 3]. It is based upon a sliding window of a pre-selected fixed data length that slides with the occurrence of each new sample point of the time series [1]. The discrete Fourier transform (DFT) of this window is computed after each sample time shift. STFT is, thus, classified as a fixed resolution method for time-frequency analysis.

Maximum Entropy Method (MEM) [5, 7, 9] for estimating the PSD provides an alternate method for computing the PSD of a set of time series. Frequency resolution of MEM PSD is better than the periodogram approach using DFT. However, PSD through MEM of a sliding window, to keep track of time, for non-stationary signals has not attracted much attention in literature [5].

MEM requires the identification of the underlying autoregressive (AR) process governing the time series. Identification of the AR process can be done using offline techniques or recursive techniques. Offline techniques are found to be more suitable for estimating the AR model order. The AR parameters can be estimated offline or recursively, as the situation demands.

In this paper, an algorithm for efficient computation of quasi-stationary time series is presented. This involves computation of the DFT of a window, as in STFT, which slides with time after occurrence of each new sample point. This algorithm utilizes offline forward backward linear predictor algorithm of Marple [6] for estimating the model order, Recursive Least Squares (RLS) [8] for estimating the AR parameters and the modified recursive sliding window DFT algorithm for computing the DFT. The modified sliding window DFT is based on a *Window for Spectral Analysis* (WSA) concept. The WSA is the window that records only those samples that affect the PSD due to AR model variability in a quasi-stationary process. Thus, the PSD is computed only for the data points that affect the PSD, skipping the unnecessary samples (implying samples that do not affect the PSD). This results in reduced computational complexity of the algorithm.

Recently an algorithm that computes the DFT of the sliding window recursively, for the purpose of computing the STFT has been published [4]. This algorithm has been modified by inclusion of sample skipping concept. The samples that do not affect the PSD are skipped and not included in the WSA.

Section II of this paper introduces the modified sliding window DFT scheme. Section III describes the new algorithm that proposes the efficient procedure for determining the PSD of a quasi-stationary process. Section IV presents the evaluation of the computational complexity of the proposed algorithm as compared to the existing techniques. Simulation results have been presented in section V.

II. MODIFIED SLIDING WINDOW DFT

Consider a quasi-stationary time series of length N. A stationary segment of length M is identified as the time window where $M < N$. In STFT, the DFT of the window length M is taken and this window then slides on to the next sample as it occurs. This process of sliding the fixed length window causes inclusion of the new sample and exclusion of the first sample, after sliding, from the window.

Assume that the window at some fixed instant of time has data samples that may be represented by the vector $\bar{y}_1$. Assume that the DFT of this vector is given by $\bar{Y}_1$. On arrival of the next sample in the data stream, the window is moved causing inclusion of the new sample but exclusion of the first element from the data vector. In this case the updated sample vector would be represented by $\bar{y}_2$ and its corresponding DFT by $\bar{Y}_2$.

E. Jacobsen and R. Lyons [4] derived a recursive formula for computing the DFT of the sliding window in an efficient

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manner. This recursive formulation is given by,

$$Y_2[k] = e^{j2\pi k/M}\left(Y_1[k] + x_M - x_0\right) \quad (1)$$

where x_0 is the element excluded by the sliding window and x_M is the element introduced in the sliding window.

We propose the following postulate for improving the computational efficiency further for calculating the STFT of a time series representing a quasi-stationary process.

Postulate: The DFT $Y_2[k]$ in equation (1) would remain unchanged from $Y_1[k]$ if the included sample x_M forms part of the same stationary process that constituted the data samples vector $\bar{y}_1$.

Proof: Since by definition of the wide sense stationary process (WSS) the autocorrelation function is independent of time [10]. Thus it implies the proof of the above postulate.□

Thus, the sliding window recursive formula of (1) may be applied on a quasi-stationary time series by skipping the computation of the DFT until a change in the statistics is detected for the time series under study. The recursive modified sliding window DFT may be represented by (2),

$$Y_2[k] = e^{j2\pi k/M}\left(Y_1[k] + x_{M+q} - x_0\right) \quad (2)$$

Where x_{M+q} represents the sample, which heralds the change in the statistics of the stationary time series and consequently resulting the computational saving and q is a positive integer.

III. Algorithm

The proposed algorithm employs two windows. One is the sliding window (SW) which slides sample by sample (as is usual in STFT) and other is the window for spectral analysis (WSA). DFT of the SW is calculated at its initial location along with the order of the AR model using Marple's algorithm [6]. RLS is then used to find out the AR parameters. Now as SW slides a sample, the next sample is predicted using the AR parameters obtained from RLS. The estimated sample is compared with the actual sample value and the error is compared with a preset threshold. Whenever the error exceeds the specified threshold, Marple's algorithm is invoked for computation of the revised AR model order. The new model order may or may not be different from the previous. However, the underlying AR model is updated. This implies change in the statistics of the time series and requirement for updating the spectrum. The WSA is updated by the introduction of the sample that heralds the change in the underlying process. The updated spectrum of the WSA is computed employing the sliding window based DFT algorithm and this constitutes the updated PSD. Thus the spectrum is updated only when the underlying process indicates a change resulting in skipping of samples in the formulation of WSA that do not constitute any change in the underlying process.

The whole algorithm can be briefly described by the pseudo code given below.

1. *Formulate first SW with the first M samples.*
2. *Set WSA to first SW.*
3. *Estimate order using Marple's algorithm for SW.*
4. *Compute DFT using FFT for WSA.*
5. *Estimate AR parameters from the M samples of SW using RLS algorithm.*
6. *Forecast the M+1st sample from the AR model developed for SW.*
7. *Update the SW by inclusion of the M+1st and removal of 1st sample.*
8. *Compare the M+1st estimated sample with the true sample.*
   - *If error is greater than a threshold*
     - *Then update WSA by inclusion of the new true sample. Set Flag.*
     - *Else WSA is not updated. Reset Flag.*
9. *If Flag is set*
   - *Then recalculate order using the Marple's algorithm on SW*
   - *If order has changed from the previous order*
     - *Then re-initialize the RLS algorithm for new order and apply to complete current SW for estimation of the AR parameters.*
     - *Else estimate the AR parameters using the RLS with previous parameters and covariance matrix estimates.*
   - *Compute the DFT of WSA using the sliding DFT using* (2).
   - *Reset Flag.*
10. *Else continue from step* 6.

Determination of Threshold: The value of threshold can be adjusted from the apriori knowledge of the system and how much the error in prediction can be tolerated. So this is a flexible parameter in the algorithm depending on the apriori knowledge.

IV. Complexity Comparison

Let N be the size of the quasi-stationary signal and let M be the size of the sliding window within the signal. Assume the order of the AR model representing the stationary sliding window signal to be P. Also assume K as representing the number of times the error between the original signal sample and the forecast sample crosses the threshold level. The complexity of algorithms being used in the proposed algorithm is shown in Table 1. The asymptotic complexity of Marple's algorithm is $O(NMP + NP^2)$, and of RLS is $O(NP^2)$, and of FFT is $O(NM\log_2 M)$. The same for sliding DFT is $O(NM)$.

The complexity of the proposed algorithm is computed as under.

- In the initialization step, Marple's Algorithm, RLS and FFT are applied once.
- After initialization, Marple's Algorithm, RLS and sliding DFT are executed K times.
- Each forecast requires P multiplications and P-1 additions. This step is carried out N-M times.

Since K << N, the asymptotic complexity of the proposed algorithm becomes $O(\mathrm{K}P^2+\mathrm{K}M)$ which is lower than the complexity of algorithms described in Table 1. This observation is especially important as the sliding DFT [4] proposes an efficient recursive method for STFT computation. The algorithm proposed in this paper improves upon the efficiency for computing the PSD of quasi-stationary processes.

Table 1 Complexity of Algorithms for one Iteration

Algorithm	# of Real Mult.	# of Real Div.	# of Additions	Remarks
Marple's [7]	$MP + 8P^2 + M + 7P - 8$	$5P+3$	$MP + 9P^2 + 2M + 25P - 3$	Estimates P and AR Parameters.
RLS	$4P^2 + 4P$	$2P$	$2P+2$	Estimates AR parameters for known P.
FFT	$4M\log_2 M$		$4M\log_2 M$	
Sliding DFT	$4M$		$4M$	

V. Simulation Results

The threshold was set to 10^{-4} for all the simulations. Suppose a signal

$$y(t) = \sin(2\times 500\pi t) + \sin(2\times 1000\pi t) + 10\delta(t - t_0)$$

is sampled at 8 kHz. It is a quasi-stationary signal in the sense that a delta function occurs at t_0 = 0.192 m sec implying 1536th sample. Assume the SW to consist of 128 samples.

The proposed algorithm is applied to the samples of $y(t)$. Figure 1(a) depicts the PSD when WSA contains samples 17 to 143 (a total of 127 samples) and sample 1536 (to complete 128 samples). This PSD is identical to the one that is obtained for samples 1408 to 1536. Figure 1(b) shows the same PSD with added white Gaussian noise (SNR = 0.1648 dB). Again the PSD is identical to un-skipped SW PSD.

Consider another signal

$$f(t) = \sin(2\times 500\pi t) + \sin(2\times 1000\pi t)$$

sampled at 8 *k*Hz. SW size is taken as 64 samples. Figure 2 shows the power spectral density estimate of the last SW and WSA, which in this case is the first SW because the signal is stationary. It is observed that WSA presents better results.

Now consider the third signal [9]

$$f(t) = \sin(2\times 7.25\pi t + \pi/4)$$

sampled at 100 Hz. Window size was 128 samples. Figure 3 shows the PSD of the last window SW along with the PSD of WSA (the first SW). From this figure, again it is observed that WSA presents better results.

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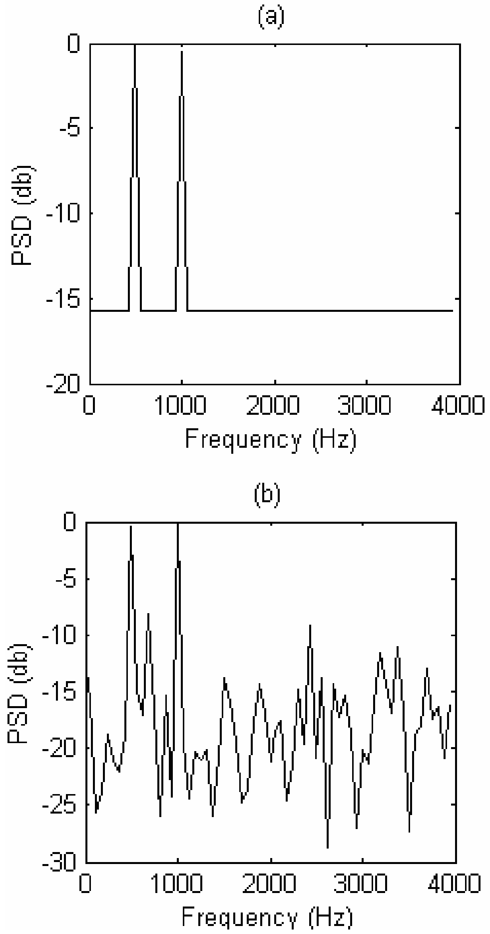

Figure 1. Plots for Normalized PSD. (a) No noise case. (b) SNR 0.1648 dBs

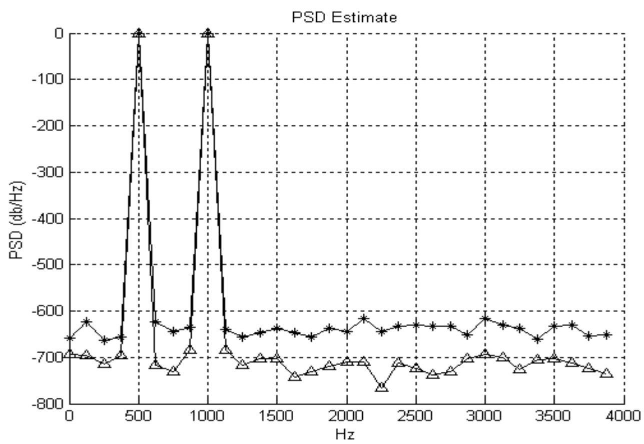

Figure 2. Δ shows the PDS plot for WSA and * shows the PDS plot for last SW for example 1.

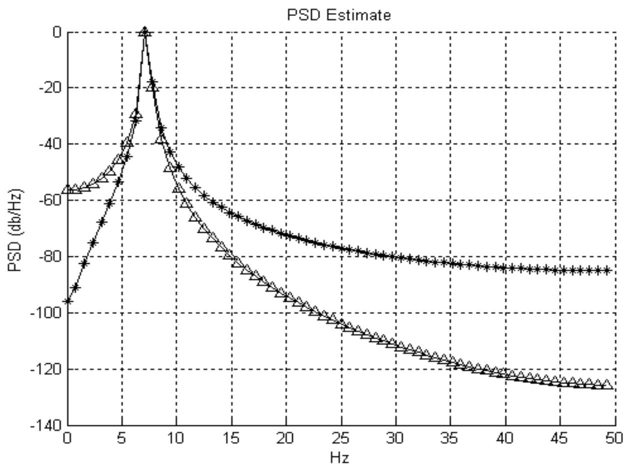

Figure 3. Δ shows the PDS plot for WSA and * shows the PDS plot for last SW for example 2.

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