World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:5, No:12, 2011

Ruin probability for a Markovian risk model with two-type claims

Dongdong Zhang, and Deran Zhang*

Abstract—In this paper, a Markovian risk model with two-type claims is considered. In such a risk model, the occurrences of the two type claims are described by two point processes $\{N_i(t), t \geq 0\}, i = 1, 2$, where $\{N_i(t), t \geq 0\}$ is the number of jumps during the interval $(0, t]$ for the Markov jump process $\{X_i(t), t \geq 0\}$. The ruin probability $\Psi(u)$ of a company facing such a risk model is mainly discussed. An integral equation satisfied by the ruin probability $\Psi(u)$ is obtained and the bounds for the convergence rate of the ruin probability $\Psi(u)$ are given by using key-renewal theorem.

Keywords—Risk model; Ruin probability; Markov jump process; Integral equation

I. Introduction

The classical risk model has been extensively studied since the work of Cramer[3], and has been generalized to various Markovian risk model which have been studied extensively $[1, 2, 4, 7]$. Recently, many authors have studied continuous-time risk models involving two classes of claims. Yuen et al. [9] consider the non-ruin probability for a correlated risk process involving two dependent classes of insurance risks, with exponential claims, which can be transformed into a surplus process with two independent classes of insurance risks, for which one claim number process is Poisson and the other is a renewal process with Erlang(2) claim inter-arrival times. Li and Garrido [5] consider a risk process with two classes of independent risks, namely the compound Poisson process and the renewal process with generalized Erlang(2) inter-arrivals times. A further extension was given by Li and Lu [6]. They derive a system of integro-differential equations for the Gerber-Shiu expected discounted penalty functions, when the ruin is caused by a claim belonging either to the first or to the second class and obtained explicit results when the claim sizes are exponentially distributed. Zhang et al. [8] extended the model of Li and Lu [6], by considering the claim number process of the second class to be a renewal process with generalized Erlang(n) inter-arrival times.

In this paper, we mainly consider a Markovian risk model with two-type claims. Integral equation for the ruin probability is found and the bounds for the convergence rate of the ruin probability are given.

Let $(\Omega, \mathcal{F}, \mathrm{P})$ be a complete probability space containing all objects defined in the following, $(\mathcal{S}_i, \mathcal{B}_i)(i = 1, 2)$ be two measurable spaces where $\mathcal{S}_i$ is a subset of real line R and $\mathcal{B}_i$ is a Borel $\sigma-$algebra on $\mathcal{S}_i$. Consider the risk model

$$U(t) = u + ct - \sum_{k=1}^{N_1(t)} Y_k - \sum_{k=1}^{N_2(t)} Z_k, \tag{1}$$

where $u = U(0) \geq 0$ is the initial surplus, $c > 0$ is the premium income rate, $\{Y_k, k \geq 1\}$ are i.i.d nonnegative random sequence with common distribution function $F_1$and mean value μ_1; $\{Z_k, k \geq 1\}$ are also i.i.d nonnegative random sequence but with common distribution function F_2 and mean value μ_2, $Y = \{Y_k, k \geq 1\}$ and $Z = \{Z_k, k \geq 1\}$ denote the two-type claim processes; $N_i(t)$ is the number of jumps during the interval $(0, t]$ for the Markov jump process $X_i = \{X_i(t), t \geq 0\}$ on space $\mathcal{S}_i$ with bounded intensity function $\lambda_i(x)$ and jumping measure $Q_i(x, B)$. Throughout this paper, we always assume that X_i is stationary ergodic with initial stationary distribution $q_i(\cdot)$, i.e., $\int_B \lambda_i(x) q_i(\mathrm{d}x) = \int_{\mathcal{S}_i} \lambda_i(x) Q_i(x, B) q_i(\mathrm{d}x)$ and X_1, X_2, Y, Z are mutually independent.

Let

$$\begin{aligned}
&T = \inf\{t \geq 0 : U(t) < 0\}, (\inf \Phi = \infty)\\
&\Psi(u) = \mathrm{P}\left(T < \infty | U(0) = u\right),\\
&R(u) = 1 - \Psi(u),\\
&\Psi_x(u) = \mathrm{P}\left(T < \infty | U(0) = u, X_1(0) = x\right),\\
&R_x(u) = 1 - \Psi_x(u), x \in \mathcal{S}_1,\\
&\widetilde{\Psi}_y(u) = \mathrm{P}\left(T < \infty | U(0) = u, X_2(0) = y\right),\\
&\widetilde{R}_y(u) = 1 - \widetilde{\Psi}_y(u), y \in \mathcal{S}_2,\\
&\Psi_{xy}(u) = \mathrm{P}\left(T < \infty | U(0) = u, X_1(0) = x, X_2(0) = y\right),\\
&R_{xy} = 1 - \Psi_{xy}(u), x \in \mathcal{S}_1, y \in \mathcal{S}_2.
\end{aligned}$$

We call T the time of ruin, $\Psi(u)$ the ruin probability, $R(u)$ the survival probability. Obviously, we have

$$\begin{aligned}
\Psi(u) &= \int_{\mathcal{S}_1} \int_{\mathcal{S}_2} \Psi_{xy}(u) q_2(\mathrm{d}y) q_1(\mathrm{d}x)\\
&= \int_{\mathcal{S}_1} \Psi_x(u) q_1(\mathrm{d}x)\\
&= \int_{\mathcal{S}_2} \widetilde{\Psi}_y(u) q_2(\mathrm{d}y).
\end{aligned}$$

Let

$$\rho = \frac{c - \mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) - \mu_2 \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x)}{\mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) + \mu_2 \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x)}$$

be the relative security loading. Throughout the paper, we always assume that $\rho > 0$.

D.D. Zhang is with the School of Educational Science, Fuyang Normal College, Fuyang 236032, China
D.R. Zhang is with the School of Mathematics and Computational Science, Fuyang Normal College, Fuyang 236032, China.
Corresponding author's e-mail: (zhderan@163.com)

II. Integral equation of ruin probability

Lemma 2.1 Under the assumption that $\rho > 0$, we have $\lim\limits_{u\to\infty} \Psi(u) = 0$.

Proof Put $Y(t) = U(t) - u$, since $\lim\limits_{t\to\infty} \frac{N_i(t)}{t} = \int_{\mathcal{S}_i} \lambda_i(x) q_i(\mathrm{d}x), i = 1, 2$, then

$$\begin{aligned}\lim_{t\to\infty} \frac{Y_t}{t} &= \lim_{t\to\infty} \left(c - \frac{1}{t}\sum_{k=1}^{N_1(t)} Y_k - \frac{1}{t}\sum_{k=1}^{N_2(t)} Z_k \right) \\ &= c - \lim_{t\to\infty} \frac{1}{N_1(t)} \sum_{k=1}^{N_1(t)} Y_k \cdot \frac{N_1(t)}{t} \\ &\quad - \lim_{t\to\infty} \frac{1}{N_2(t)} \sum_{k=1}^{N_2(t)} Z_k \cdot \frac{N_2(t)}{t} \\ &= c - \mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) - \mu_2 \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x).\end{aligned}$$

By the theory of Markov process and the assumption that $\lambda_i(x)$ is bounded, it is clear that X_i has only finite jumps during the interval $(0, \tau]$, thus $\inf\limits_{t\geq 0} Y_t$ is finite with probability one and thus

$$\lim_{u\to\infty} \Psi(u) = \lim_{u\to\infty} \mathrm{P}(\inf_{t\geq 0}(u + Y(t) < 0) = 0,$$

then Lemma 2.1 is proved.

Corollary 2.1 For $\Psi_x(u), x \in \mathcal{S}_1$, $\widetilde{\Psi}_y(u), y \in \mathcal{S}_2$ we have

$$\begin{aligned}&\lim_{u\to\infty} \Psi_x(u) = 0, \quad q_1(\cdot) - a.e. \quad x \in \mathcal{S}_1; \\ &\lim_{u\to\infty} \widetilde{\Psi}_y(u) = 0, \quad q_2(\cdot) - a.e. \quad y \in \mathcal{S}_2.\end{aligned}$$

Proof Since $\Psi(u) = \int_{\mathcal{S}_1} \Psi_x(u) q_1(\mathrm{d}x) = \int_{\mathcal{S}_2} \widetilde{\Psi}_y(u) q_2(\mathrm{d}y)$, by the dominated convergence theorem, we can get

$$\begin{aligned}0 = \lim_{u\to\infty} \Psi(u) &= \int_{\mathcal{S}_1} \lim_{u\to\infty} \Psi_x(u) q_1(\mathrm{d}x) \\ &= \int_{\mathcal{S}_2} \lim_{u\to\infty} \widetilde{\Psi}_y(u) q_2(\mathrm{d}y).\end{aligned}$$

Obviously, it is that $\lim\limits_{u\to\infty} \tilde{\Psi}_x(u) > 0, q_1(\cdot) - a.e. \ \ x \in \mathcal{S}_1$ and $\lim\limits_{u\to\infty} \widetilde{\Psi}_y(u) > 0, q_2(\cdot) - a.e. \ \ y \in \mathcal{S}_2$, thus

$$\begin{aligned}&\lim_{u\to\infty} \Psi_x(u) = 0, \quad q_1(\cdot) - a.e. \quad x \in \mathcal{S}_1; \\ &\lim_{u\to\infty} \widetilde{\Psi}_y(u) = 0, \quad q_2(\cdot) - a.e. \quad y \in \mathcal{S}_2,\end{aligned}$$

the proof of Corollary 2.1 is completed.

In the following, by using the backward differential technique, we give an integral equation satisfied by the ruin probability $\Psi(u)$.

Theorem 2.1 If the relative security loading $\rho > 0$, then

$$\begin{aligned}\Psi(0) &= \frac{1}{c}\left(\mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) + \mu_2 \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x) \right), \\ \Psi(u) &= \frac{1}{c} \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) \int_u^\infty \overline{F}_1(z)\mathrm{d}z \\ &\quad + \frac{1}{c}\int_0^u \left[\int_{\mathcal{S}_1} \lambda_1(x) \Psi_x(u - z) q_1(\mathrm{d}x) \right] \overline{F}_1(z)\mathrm{d}z \\ &\quad + \frac{1}{c} \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x) \int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\quad + \frac{1}{c}\int_0^u \left[\int_{\mathcal{S}_2} \lambda_2(y) \widetilde{\Psi}_y(u - z) q_2(\mathrm{d}y) \right] \overline{F}_2(z)\mathrm{d}z,\end{aligned}$$

where $\overline{F}_i(z) = 1 - F_i(z), i = 1, 2$.

Proof Using the backward differential technique, we have

$$\begin{aligned}R_{xy}(u) &= (1 - \lambda_1(x)\triangle)(1 - \lambda_2(y)\triangle) R_{xy}(u + c\triangle) \\ &\quad + \lambda_1(x) \triangle (1 - \lambda_2(y)\triangle) \int_{\mathcal{S}_1} Q_1(x, \mathrm{d}x_1) \times \\ &\quad \int_0^{u+c\triangle} R_{x_1 y}(u + c\triangle - z)\mathrm{d}F_1(z) \\ &\quad + \lambda_2(y) \triangle (1 - \lambda_1(x)\triangle) \int_{\mathcal{S}_2} Q_2(y, \mathrm{d}y_1) \times \\ &\quad \int_0^{u+c\triangle} R_{x y_1}(u + c\triangle - z)\mathrm{d}F_2(z) + \circ(\triangle). \qquad (2)\end{aligned}$$

Thus

$$\begin{aligned}cR'_{xy}(u) =& (\lambda_1(x) + \lambda_2(y)) R_{xy}(u) \\ &- \lambda_1(x) \int_{\mathcal{S}_1} Q_1(x, \mathrm{d}x_1) \int_0^u R_{x_1 y}(u - z)\mathrm{d}F_1(z) \\ &- \lambda_2(y) \int_{\mathcal{S}_2} Q_2(y, \mathrm{d}y_1) \int_0^u R_{x y_1}(u - z)\mathrm{d}F_2(z).\end{aligned}$$

Replacing u by t and integrating from $t = 0$ to $t = u$, we obtain

$$\begin{aligned}&c(R_{xy}(u) - R_{xy}(0)) \\ &= \lambda_1(x) \int_0^u R_{xy}(t)\mathrm{d}t \\ &- \lambda_1(x) \int_0^u \int_{\mathcal{S}_1} Q_1(x, \mathrm{d}x_1) \int_0^t R_{x_1 y}(t - z)\mathrm{d}F_1(z)\mathrm{d}t \\ &+ \lambda_2(y) \int_0^u R_{xy}(t)\mathrm{d}t \\ &- \lambda_2(y) \int_0^u \int_{\mathcal{S}_2} Q_2(y, \mathrm{d}y_1) \int_0^t R_{x y_1}(t - z)\mathrm{d}F_2(z)\mathrm{d}t \\ &= \lambda_1(x) \int_0^u R_{xy}(t)\mathrm{d}t - \lambda_1(x) \int_{\mathcal{S}_1} Q_1(x, \mathrm{d}x_1) \int_0^u R_{x_1 y}(t)\mathrm{d}t \\ &+ \lambda_1(x) \int_{\mathcal{S}_1} Q_1(x, \mathrm{d}x_1) \int_0^u \overline{F}_1(z) R_{x_1 y}(u - z)\mathrm{d}z \\ &+ \lambda_2(y) \int_0^u R_{xy}(t)\mathrm{d}t - \lambda_2(y) \int_{\mathcal{S}_2} Q_2(y, \mathrm{d}y_1) \int_0^u R_{x y_1}(t)\mathrm{d}t \\ &+ \lambda_2(y) \int_{\mathcal{S}_2} Q_2(y, \mathrm{d}y_1) \int_0^u \overline{F}_2(z) R_{x y_1}(u - z)\mathrm{d}z. \qquad (3)\end{aligned}$$

Integrating both sides of Eq.(3) about $q_1(\cdot)$ and $q_2(\cdot)$, we get

$$c[R(u)-R(0)] = \int_0^u \left[\int_{\mathcal{S}_1} \lambda_1(x) R_x(u-z) q_1(\mathrm{d}x)\right] \overline{F}_1(z)\mathrm{d}z + \int_0^u \left[\int_{\mathcal{S}_2} \lambda_2(y) \widetilde{R}_y(u-z) q_2(\mathrm{d}y)\right] \overline{F}_2(z)\mathrm{d}z.$$

Let $t \to \infty$ in the above equation, by the dominated convergence theorem, then

$$c[R(\infty)-R(0)] = \int_0^\infty \left[\int_{\mathcal{S}_1} \lambda_1(x) R_x(\infty) q_1(\mathrm{d}x)\right] \overline{F}_1(z)\mathrm{d}z + \int_0^\infty \left[\int_{\mathcal{S}_2} \lambda_2(y) \widetilde{R}_y(\infty) q_2(\mathrm{d}y)\right] \overline{F}_2(z)\mathrm{d}z.$$

It follows from Corollary2.1 that

$$c\Psi(0) = \mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}y) + \mu_2 \int_{\mathcal{S}_2} \lambda_2(y) q_2(\mathrm{d}y),$$

then

$$\Psi(0) = \frac{1}{c}\left(\mu_1 \int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) + \mu_2 \int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x)\right),$$

and

$$\begin{aligned}\Psi(u) &= \Psi(0) - \frac{1}{c}\times \\ &\quad \int_0^u \left[\int_{\mathcal{S}_1} \lambda_1(x)(1-\Psi_x(u-z)) q_1(\mathrm{d}x)\right] \overline{F}_1(z)\mathrm{d}z \\ &\quad - \frac{1}{c}\int_0^u \left[\int_{\mathcal{S}_2} \lambda_2(y)(1-\widetilde{\Psi}_y(u-z) q_2(\mathrm{d}y)\right] \overline{F}_2(z)\mathrm{d}z \\ &= \frac{1}{c}\int_{\mathcal{S}_1} \lambda_1(x) q_1(\mathrm{d}x) \int_u^\infty \overline{F}_1(z)\mathrm{d}z \\ &\quad + \frac{1}{c}\int_0^u \left[\int_{\mathcal{S}_1} \lambda_1(x)\Psi_x(u-z) q_1(\mathrm{d}x)\right] \overline{F}_1(z)\mathrm{d}z \\ &\quad + \frac{1}{c}\int_{\mathcal{S}_2} \lambda_2(x) q_2(\mathrm{d}x) \int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\quad + \frac{1}{c}\int_0^u \left[\int_{\mathcal{S}_2} \lambda_2(y)\widetilde{\Psi}_y(u-z) q_2(\mathrm{d}y)\right] \overline{F}_2(z)\mathrm{d}z.\end{aligned}$$

Thus the theorem is completed.

III. Bounds for convergence rate of ruin probability

Let $h_i(r) = \int_0^{+\infty} e^{rx}\mathrm{d}F_i(x) - 1, \widetilde{\lambda}_i = \sup_{x\in\mathcal{S}_i}\{\lambda_i(x)\}, \widehat{\lambda} = \inf_{x\in\mathcal{S}_i}\{\lambda_i(x)\}, i=1,2$. In the following, we assume that

$$\widetilde{\rho} = \frac{c}{\widetilde{\lambda}_1\mu_1 + \widetilde{\lambda}_2\mu_2} - 1 > 0, \widehat{\rho} = \frac{c}{\widehat{\lambda}_1\mu_1 + \widehat{\lambda}_2\mu_2} - 1 > 0,$$

and assume that there exists a real number $r_\infty > 0$ such that $h_i(r) \to \infty$ when $r \to \infty$ (we allow for the possibility $r_\infty = \infty$).

Lemma 3.1 Under the above assumptions, there exist $\widetilde{R}, \widehat{R}$ such that

$$\frac{\widetilde{\lambda}_1}{c}h_1(\widetilde{R}) + \frac{\widetilde{\lambda}_2}{c}h_2(\widetilde{R}) = \widetilde{R}, \quad \frac{\widehat{\lambda}_1}{c}h_1(\widehat{R}) + \frac{\widehat{\lambda}_2}{c}h_2(\widehat{R}) = \widehat{R}.$$

The proof of Lemma 3.1 is omitted.

Theorem 3.1 For the probability $\Psi(u)$, we have

$$\limsup_{u\to\infty} e^{\widetilde{R}u}\Psi(u) \le \frac{1+\widetilde{\rho}}{(1+\widetilde{\rho})\left(\frac{\widetilde{\lambda}_1}{c}h_1'(\widetilde{R}) + + \frac{\widetilde{\lambda}_2}{c}h_2'(\widetilde{R}) - 1\right)}, \tag{4}$$

$$\liminf_{u\to\infty} e^{\widehat{R}u}\Psi(u) \ge \frac{1+\widehat{\rho}}{(1+\widehat{\rho})\left(\frac{\widehat{\lambda}_1}{c}h_1'(\widehat{R}) + + \frac{\widehat{\lambda}_2}{c}h_2'(\widehat{R}) - 1\right)}. \tag{5}$$

Proof By theorem 2.1, we have

$$\begin{aligned}\Psi(u) &\le \frac{\widetilde{\lambda}_1}{c}\int_u^\infty \overline{F}_1(z)\mathrm{d}z + \frac{\widetilde{\lambda}_2}{c}\int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\quad + \frac{\widetilde{\lambda}_1}{c}\int_0^u \Psi(u-z)\overline{F}_1(z)\mathrm{d}z + \frac{\widetilde{\lambda}_2}{c}\int_0^u \Psi(u-z)\overline{F}_2(z)\mathrm{d}z \\ &= \frac{\widetilde{\lambda}_1}{c}\int_u^\infty \overline{F}_1(z)\mathrm{d}z + \frac{\widetilde{\lambda}_2}{c}\int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\quad + \int_0^u \Psi(u-z)\left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\overline{F}_2(z)\right)\mathrm{d}z.\end{aligned}$$

Multiplying the above inequality by $e^{\widetilde{R}u}$, we have

$$\begin{aligned}& e^{\widetilde{R}u}\Psi(u) \\ &\le \frac{\widetilde{\lambda}_1}{c}e^{\widetilde{R}u}\int_u^\infty \overline{F}_1(z)\mathrm{d}z + \frac{\widetilde{\lambda}_2}{c}e^{\widetilde{R}u}\int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\quad + \int_0^u e^{\widetilde{R}(u-z)}\Psi(u-z)e^{\widetilde{R}z}\left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\overline{F}_2(z)\right)\mathrm{d}z.\end{aligned}$$

Thus, by lemma 3.1, we have that

$$\int_0^\infty e^{\widetilde{R}z}\left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\overline{F}_2(z)\right)\mathrm{d}z = 1,$$

and then

$$\begin{aligned} 0 &\le \lim_{u\to\infty} \frac{\widetilde{\lambda}_1}{c}e^{\widetilde{R}u}\int_u^\infty \overline{F}_1(z)\mathrm{d}z + \frac{\widetilde{\lambda}_2}{c}e^{\widetilde{R}u}\int_u^\infty \overline{F}_2(z)\mathrm{d}z \\ &\le \lim_{u\to\infty}\int_u^\infty e^{\widetilde{R}z}\left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\overline{F}_2(z)\right)\mathrm{d}z = 0,\end{aligned}$$

so by the key-renewal theorem, we obtain

$$\limsup_{u\to\infty} e^{\widetilde{R}u}\Psi(u) \le \frac{c_1}{c_2},$$

where

$$c_1 = \int_0^\infty e^{\widetilde{R}u}\int_u^\infty \left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\int_u^\infty \overline{F}_2(z)\right)\mathrm{d}z\mathrm{d}u,$$

$$c_2 = \int_0^\infty z e^{\widetilde{R}z}\left(\frac{\widetilde{\lambda}_1}{c}\overline{F}_1(z) + \frac{\widetilde{\lambda}_2}{c}\int_u^\infty \overline{F}_2(z)\right)\mathrm{d}z.$$

So from the two above equations, we can get

$$c_1 = \frac{\widetilde{\rho}}{\widetilde{R}(1+\widetilde{\rho})}, c_2 = \frac{1}{\widetilde{R}}\left(\frac{\widetilde{\lambda}_1}{c}h_1'(\widetilde{R}) + + \frac{\widetilde{\lambda}_2}{c}h_2'(\widetilde{R}) - 1\right).$$

Then the proof of (4) is completed.

We can get the proof of (5) by imitating the above proof of (4).□

REFERENCES

[1] Asmussen S. Risk theory in a Markovian environment. Scand Actuarial J, 1989, 66-100.
[2] Blaszczyszyn B, Rolski T. Expansions for Markov-modulated systems and approximations of ruin probability. J appl Prob, 1996, 33:57-70.
[3] Cramer H. On the mathematical Theory of risk. Skandia Jubilee Volume, Stockholm, 1930.
[4] Helena Jasiulewicz, Probability of ruin with variable premium rate in a Markovina environment. Insurance: Mathematics and Economics 2001; 29: 291-296.
[5] Li S, Garrido J. Ruin probabilities for two classes of risk processes. ASTIN Bulletin 2005; 35: 61-77.
[6] Li S, Lu Y. On the expected discounted penalty functions for two classes of risk processes. Insurance: Mathematics and Economics 2005; 36: 179-193.
[7] Wang H.X. Yan Y.Z. et al. Markovian risk process. Applied Mathematics and mechanics(English edition), 2007,28(7):955-962.
[8] Yang H, Zhang Z. On a class of renewal risk model with random income. Applied Stochastic Models in Business and Industry 2009; 25(6): 678¨C695.
[9] Yuen K.C., Guo J., Wu X. On a correlated aggragate claims model with Poisson and Erlang risk processes, Insurance: Mathematics and Economics 2002; 31: 205-214.