

Optimal Manufacturing Scheduling for Dependent Details Processing

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Abstract—The increasing competitiveness in manufacturing industry is forcing manufacturers to seek effective processing schedules. The paper presents an optimization manufacture scheduling approach for dependent details processing with given processing sequences and times on multiple machines. By defining decision variables as start and end moments of details processing it is possible to use straightforward variables restrictions to satisfy different technological requirements and to formulate easy to understand and solve optimization tasks for multiple numbers of details and machines. A case study example is solved for seven base moldings for CNC metalworking machines processed on five different machines with given processing order among details and machines and known processing time's duration. As a result of linear optimization task solution the optimal manufacturing schedule minimizing the overall processing time is obtained. The manufacturing schedule defines the moments of moldings delivery thus minimizing storage costs and provides mounting due-time satisfaction. The proposed optimization approach is based on real manufacturing plant problem. Different processing schedules variants for different technological restrictions were defined and implemented in the practice of Bulgarian company RAIS Ltd. The proposed approach could be generalized for other job shop scheduling problems for different applications.

Keywords—Optimal manufacturing scheduling, linear programming, metalworking machines production, dependant details processing.

I. INTRODUCTION

THE manufacturing industries have to cope with increasing competitiveness in today's world market, so they must rely on innovative approaches in all aspects of manufacturing technology. As a result of the progress in computer technology the using of operations research methods is constantly emerging. Scheduling is concerned with allocation of resources over time so as to execute the processing tasks required to manufacture a given set of products [1]. The scheduling is a key factor for manufacturing productivity. Effective manufacture scheduling can improve on-time

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delivery, reduce inventory, cut lead times, and improve the utilization of bottleneck resources [2]. The simplest scheduling problem is the single machine sequencing problem [3]. Minimizing the total completion time is one of the basic objectives studied in the scheduling literature. The shortest processing time dispatching rule will give an optimal schedule in the single machine case if the tool life is considered infinitely long [4]. The scheduling with sequence-dependent setups is recognized as being difficult and most existing results in the literature focus on either a single machine or several identical machines [5]-[7]. The real-life scheduling problems usually have to consider multiple no identical machines. Most of the processing machines needed to process the jobs are available in the manufacturer's own factory and are of fixed (finite) number. Sometimes, certain details must be ordered to a third party companies to complete very specific processing as molding for example. In cases like that, the processing schedules are to be agreed for delivery times from the third-party processing. That means generating a schedule to process all jobs, so as to minimize the total cost, including the satisfaction of the due dates of the jobs [8]. Different manufacturing environments induce different scheduling constraints, some of which may be very specific to the problem under consideration [9].

Because of the combinatorial nature of scheduling problems, it is often difficult to obtain optimal schedules, especially within in real life conditions. Many heuristic methods are developed and implemented that consider the due dates, the criticality of operations, the operation processing times, and the machines utilization [10]-[13]. Due to the complexity of the real time scheduling problems most of methods are targeted to one or two machines scheduling.

The proposed here scheduling approach concerns a practical problem of scheduling of multiple job shops with fixed processing time data and predetermined details order processing among more then two different machines. The relationship between the processing details sequence and machines occupation might have a significant impact on overall manufacturing process performance.

II. PROBLEM FORMULATION

The current paper is inspired by a real life problem from a company producing CNC metalworking machines practice. It uses some base machines details molded in an outside company. They are expensive and heavyweight details which

are not easy to transport and need considerable space to store if delivered early then needed. Those moldings have to be additionally processed on specific machines to be ready for preliminary fixed mounting moment of time.

The problem studied in the paper could be described as: to get optimal manufacturing schedule if given (i) the number N of details to process, (ii) the number M of processing machines, (iii) the each detail technological processing sequence on different machines (usually some subset of the set of all available machines), (iv) the processing duration time for every detail on a particular processing machine, (v) the restriction of only one detail processing on a given machine at each moment of time, (vi) the predetermined order of processing of the details. That means optimal distribution over the time of each detail processing among machines complying with the given technological processing sequences and times, satisfying the restriction (v) and optimizing the overall processing time. The resulted manufacturing schedule should define the start and end processing moments of time for each particular detail. The start processing moments are used to specify the delivery times for outside ordered moldings. The end processing moments should ensure the best utilization of the processing machines.

III. THE MODEL AND OPTIMIZATION TASK

Let us introduce the model variables as time moments for starting of the detail i processing on the machine j , i.e. X_{ij} , where $i=1,2,\dots,N$ are the indexes of the details and $j=1,2,\dots,M$ are indexes of the machines. The jobs processing times T_{ij} of each detail i on machine j are constant and known. There exist two types of technological restrictions, expressing

- the order of each detail processing and
- the sequence of different details processing.

The first set of restrictions represents the technological detail processing sequence on particular machines considering the duration of processing on each machine. They can be expressed as constraints using the mere fact that operation on machine $(j+1)$ of detail i can start after the time duration T_{ij} for completing of operation on machine j , i.e. $X_{i(j+1)} - X_{ij} > T_{ij}$.

The second sets of constraints consider the preliminary given processing order among different details. These restrictions take into account the fact that next detail can start processing on the same machine after completing of the previous detail processing for time duration T_{ij} , i.e. $X_{(i+1)j} - X_{ij} > T_{ij}$. They also secure processing of only one detail at a time on a particular machine.

The generalized goal of optimal scheduling is to minimize the overall costs. Although many costs could be considered for optimization, the minimizing of details processing time duration is one of most frequently used. It provides the effective machines utilization and serves the optimization of details delivering and storage. The overall details processing time duration can be defined as difference between end processing moment of the last detail and start processing moment of the first detail and if the processing starts at

moment zero moment then the objective can be minimization of the end processing moment of the last detail.

Using those considerations an optimization task can be formulated:

$$\min X_{NEnd} \quad (1)$$

subject to

$$X_{i,j+1} - X_{ij} \geq T_{ij} \quad (2)$$

$$X_{i+1,j} - X_{ij} \geq T_{ij} \quad (3)$$

$$X_{ij} \geq 0 \quad (4)$$

where $i=1, 2, \dots, N$ and $j=1, 2, \dots, M$.

This generalized single objective linear optimization problem transforms to a complex combinatorial optimization problem when applied to a real life task formulation.

IV. CASE STUDY EXAMPLE

A producer of metalworking CNC machines needs seven base details (D1, D2, ..., D7) for mounting of each machine. Those base details come from third-party as heavyweight moldings that should be additionally processed by means of five CNC machines (M1, M2, ..., M5). The processing should comply with given technological instructions as operations sequence and duration for each particular detail on the corresponding machines shown in Table I.

TABLE I
 TECHNOLOGICAL INSTRUCTIONS OF DETAILS PROCESSING

Details processing order	Operation	Machine	Job duration, hours
D1	O ₁₁	M1	8
	O ₁₂	M2	6
	O ₁₄	M4	6
D2	O ₂₁	M1	8
	O ₂₂	M2	10
	O ₂₄	M4	6
D3	O ₃₁	M1	8
	O ₃₃	M3	8
	O ₃₂	M2	8
	O ₃₄	M4	4
D4	O ₄₁	M1	4
	O ₄₂	M2	1
	O ₄₃	M3	2
D5	O ₅₁	M1	4
	O ₅₂	M2	12
	O ₅₃	M3	4
D6	O ₅₅	M5	8
	O ₆₁	M1	6
D6	O ₆₃	M3	8
	D7	O ₇₃	M3
O ₇₄		M4	8

The optimal processing schedule should define:

- start moments of each detail processing, used to determine the moldings delivery time from the third-party company,

- end moments of each detail processing on a particular machine used to define the availability of the machine for the next operation,
- effective utilization of the machines,
- minimal overall processing time.

Using Table I data the decision variables are introduced as starting moments of time for processing of details:

- X_{11}, X_{12}, X_{14} – for D1 on machines M1, M2, M4;
- X_{21}, X_{22}, X_{24} – for D2 on machines M1, M2, M4;
- $X_{31}, X_{33}, X_{32}, X_{34}$ – for D3 on machines M1, M3, M2 and M4;
- X_{41}, X_{42}, X_{43} – for D4 on machines M1, M2, M3;
- $X_{51}, X_{52}, X_{53}, X_{55}$ – for D5 on machines M1, M2, M3 and M5;
- X_{61}, X_{63} – for D6 on machines M1 and M3;
- X_{73}, X_{74} – for D7 on machines M3 and M4;

and also variables for end details processing time moments

- $X_{1E}, X_{2E}, X_{3E}, X_{4E}, X_{5E}, X_{6E}, X_{7E}$ – for details D1, D2, D3, D4, D5, D6 and D7.

Using these variables and the technological data from Table I the formulation (1)–(4) transforms to a single objective linear optimization Task 1:

$$\min X_{7E} \quad (5)$$

subject to

- restrictions for operation sequence for each detail:

$$X_{12} - X_{11} \geq 8 - O_{12} \text{ waits end of } O_{11}, \quad (6)$$

$$X_{14} - X_{12} \geq 6 - O_{14} \text{ waits end of } O_{12}, \quad (7)$$

$$X_{1E} - X_{14} \geq 6 - \text{end of D1 processing}, \quad (8)$$

$$X_{22} - X_{21} \geq 8 - O_{22} \text{ waits end of } O_{21}, \quad (9)$$

$$X_{24} - X_{22} \geq 10 - O_{24} \text{ waits end of } O_{22}, \quad (10)$$

$$X_{2E} - X_{24} \geq 6 - \text{end of D2 processing}, \quad (11)$$

$$X_{33} - X_{31} \geq 8 - O_{12} \text{ waits end of } O_{11}, \quad (12)$$

$$X_{32} - X_{33} \geq 8 - O_{32} \text{ waits end of } O_{33}, \quad (13)$$

$$X_{34} - X_{32} \geq 8 - O_{34} \text{ waits end of } O_{32}, \quad (14)$$

$$X_{3E} - X_{34} \geq 4 - \text{end of D3 processing}, \quad (15)$$

$$X_{42} - X_{41} \geq 4 - O_{42} \text{ waits end of } O_{41}, \quad (16)$$

$$X_{43} - X_{42} \geq 1 - O_{43} \text{ waits end of } O_{42} \quad (17)$$

$$X_{4E} - X_{43} \geq 2 - \text{end of D4 processing}, \quad (18)$$

$$X_{52} - X_{51} \geq 4 - O_{52} \text{ waits end of } O_{51}, \quad (19)$$

$$X_{53} - X_{52} \geq 12 - O_{53} \text{ waits end of } O_{52}, \quad (20)$$

$$X_{55} - X_{53} \geq 4 - O_{55} \text{ waits end of } O_{53}, \quad (21)$$

$$X_{5E} - X_{55} \geq 8 - \text{end of D5 processing}, \quad (22)$$

$$X_{63} - X_{61} \geq 6 - O_{63} \text{ waits end of } O_{61}, \quad (23)$$

$$X_{6E} - X_{63} \geq 8 - \text{end of D6 processing}, \quad (24)$$

$$X_{74} - X_{73} \geq 6 - O_{74} \text{ waits end of } O_{73}, \quad (25)$$

$$X_{7E} - X_{74} \geq 8 - \text{end of D7 processing}, \quad (26)$$

$$X_{21} - X_{11} \geq 8 - \text{D2 waits D1 on M1}, \quad (27)$$

$$X_{31} - X_{21} \geq 8 - \text{D3 waits D2 on M1}, \quad (28)$$

$$X_{41} - X_{31} \geq 8 - \text{D4 waits D3 on M1}, \quad (29)$$

$$X_{51} - X_{41} \geq 4 - \text{D5 waits D4 on M1}, \quad (30)$$

$$X_{61} - X_{51} \geq 4 - \text{D6 waits D5 on M1}, \quad (31)$$

$$X_{22} - X_{11} \geq 6 - \text{D2 waits D1 on M2}, \quad (32)$$

$$X_{32} - X_{22} \geq 10 - \text{D3 waits D2 on M2}, \quad (33)$$

$$X_{42} - X_{31} \geq 8 - \text{D4 waits D3 on M2}, \quad (34)$$

$$X_{52} - X_{42} \geq 1 - \text{D5 waits D4 on M2}, \quad (35)$$

$$X_{43} - X_{33} \geq 8 - \text{D4 waits D3 on M3}, \quad (36)$$

$$X_{53} - X_{43} \geq 2 - \text{D5 waits D4 on M3}, \quad (37)$$

$$X_{63} - X_{53} \geq 4 - \text{D6 waits D5 on M3}, \quad (38)$$

$$X_{73} - X_{63} \geq 8 - \text{D7 waits D6 on M3}, \quad (39)$$

$$X_{24} - X_{14} \geq 6 - \text{D2 waits D1 on M4}, \quad (40)$$

$$X_{34} - X_{24} \geq 6 - \text{D3 waits D2 on M4}, \quad (41)$$

$$X_{74} - X_{34} \geq 4 - \text{D7 waits D3 on M4}. \quad (42)$$

$$X_{21} - X_{11} \geq 8 - \text{D2 waits D1 on M1}, \quad (27)$$

$$X_{31} - X_{21} \geq 8 - \text{D3 waits D2 on M1}, \quad (28)$$

$$X_{41} - X_{31} \geq 8 - \text{D4 waits D3 on M1}, \quad (29)$$

$$X_{51} - X_{41} \geq 4 - \text{D5 waits D4 on M1}, \quad (30)$$

$$X_{61} - X_{51} \geq 4 - \text{D6 waits D5 on M1}, \quad (31)$$

$$X_{22} - X_{11} \geq 6 - \text{D2 waits D1 on M2}, \quad (32)$$

$$X_{32} - X_{22} \geq 10 - \text{D3 waits D2 on M2}, \quad (33)$$

$$X_{42} - X_{31} \geq 8 - \text{D4 waits D3 on M2}, \quad (34)$$

$$X_{52} - X_{42} \geq 1 - \text{D5 waits D4 on M2}, \quad (35)$$

$$X_{43} - X_{33} \geq 8 - \text{D4 waits D3 on M3}, \quad (36)$$

$$X_{53} - X_{43} \geq 2 - \text{D5 waits D4 on M3}, \quad (37)$$

$$X_{63} - X_{53} \geq 4 - \text{D6 waits D5 on M3}, \quad (38)$$

$$X_{73} - X_{63} \geq 8 - \text{D7 waits D6 on M3}, \quad (39)$$

$$X_{24} - X_{14} \geq 6 - \text{D2 waits D1 on M4}, \quad (40)$$

$$X_{34} - X_{24} \geq 6 - \text{D3 waits D2 on M4}, \quad (41)$$

$$X_{74} - X_{34} \geq 4 - \text{D7 waits D3 on M4}. \quad (42)$$

$$\bullet \text{ and also to restrictions for non negativity of variables}$$

$$X_{ij} \geq 0 \text{ for } i=1,2,\dots,7 \text{ and } j=1,2,\dots,5. \quad (43)$$

The Task 1 solution is shown in Table II.

TABLE II
 THE TASK 1 SOLUTION DATA

Detail	Operation	Processing start time	Processing end time	Duration, hours
D1	O ₁₁	0	8	20
	O ₁₂	8	14	
	O ₁₄	14	20	
D2	O ₂₁	8	16	24
	O ₂₂	16	26	
	O ₂₄	26	32	
D3	O ₃₁	16	24	28
	O ₃₃	24	32	
	O ₃₄	32	40	
D4	O ₄₁	24	28	19
	O ₄₂	40	41	
	O ₄₃	41	43	
D5	O ₅₁	28	32	37
	O ₅₂	41	53	
	O ₅₃	53	57	
D6	O ₅₅	57	65	33
	O ₆₁	32	38	
	O ₆₃	57	65	
D7	O ₇₃	65	71	14
	O ₇₄	71	79	
Overall process duration:				79

Taking into account that the solution should result to a manufacturing schedule it would be better to visualize it graphically as shown on Fig. 1.

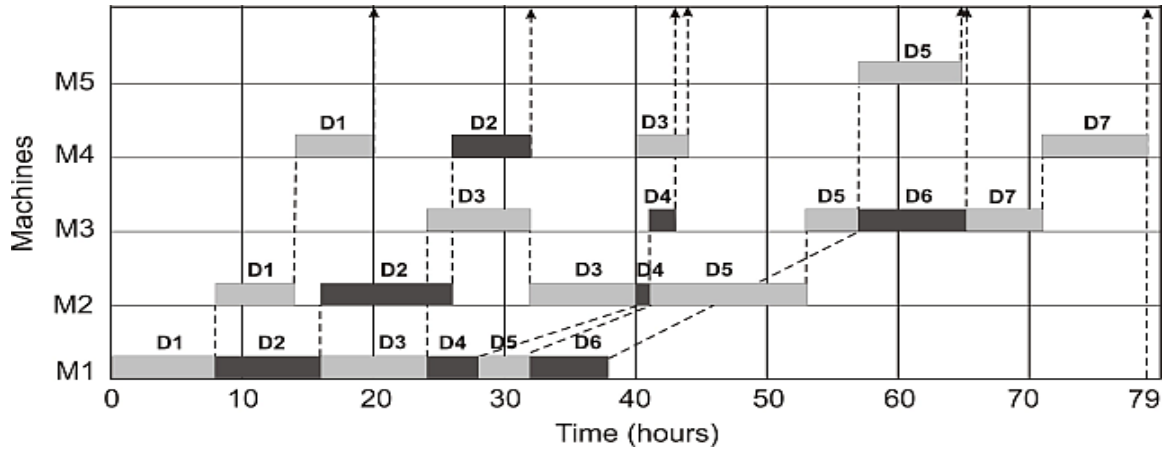


Fig. 1 processing schedule from the solution of the Task 1

The total processing duration time for all details is equal to the end processing moment of the last detail, i.e. 79 hours.

As it was pointed out the heuristics plays essential role in practical scheduling problem solving. For example, if it is possible to omit the requirement for detail D7 to be the last processed detail its machining could start earlier. That means omitting the restrictions (39) and (42) and solving of modified optimization Task 1a:

$$\min X_{7E} \quad (44)$$

subject to restrictions (6)-(38), (40)-(41) and (43).

The Task 1a solution is shown in Table III.

TABLE III
 THE TASK 1A SOLUTION DATA

Detail	Operation	Processing start time	Processing end time	Duration, hours
D1	O ₁₁	0	8	20
	O ₁₂	8	14	
	O ₁₄	14	20	
D2	O ₂₁	8	16	24
	O ₂₂	16	26	
	O ₂₄	26	32	
D3	O ₃₁	16	24	28
	O ₃₃	24	32	
	O ₃₂	32	40	
	O ₃₄	40	44	
D4	O ₄₁	24	28	19
	O ₄₂	40	41	
	O ₄₃	41	43	
D5	O ₅₁	28	32	37
	O ₅₂	41	53	
	O ₅₃	53	57	
	O ₅₅	57	65	
D6	O ₆₁	32	38	33
	O ₆₃	57	65	
D7	O ₇₃	0	6	14
	O ₇₄	6	8	
Overall process duration:				65

The Task 1a solution defines a new processing schedule shown on Fig. 2.

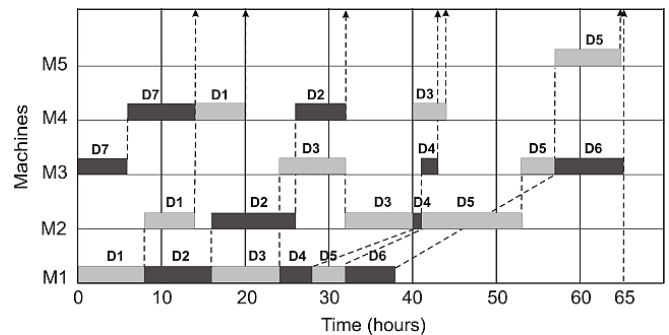


Fig. 2 processing schedule from the solution of the Task 1a

As it is seen from Fig. 2 the solution of the Task 1a provides a schedule requiring less processing time, 65 hours vs. 79 hours for schedule defined by Task 1.

V. NUMERICAL EXAMPLE DISCUSSION

The proposed scheduling optimization approach is based on realistic data and the used modelling technique can always find reasonably good solutions. Different type of heuristic assumptions could be taken into account and used for modification of the optimization model. For example, as it is seen from schedules on Fig. 1 and Fig. 2 the bottleneck of details processing is the capacity of machine M1. It would be interesting to modify the model considering more than one M1 machines. Adding additional machines increases the complexity of the combinatorial problem and corresponding optimization tasks.

The formulated optimization tasks could be solved by typical linear programming software optimization system. The LINGO v.11 [14] package was chosen for solution of the case study example as a well proven among researchers optimization system. The case study examples were solved on PC with 2.67GHz Intel processor, 1.24GB RAM under Windows XP platform. The solution times were less than a second but they obviously depend on the size of the

formulated tasks i.e. on the number of details, operations and machines, and on the available computational power.

VI. CONCLUSION

The core idea of the proposed scheduling optimization model is choosing of the decision variables as time moments. That gives the possibility to use straightforward variables restrictions to satisfy different technological requirements and to formulate easy to understand and solve optimization tasks for processing of multiple details on multiple machines. Further extensions considering availability of more than one of same type machine, defining of optimal number of needed machines, defining of schedules without fixed processing order will be investigated.

The proposed optimization approach is based on real manufacturing plant problem. Different variants of processing schedules for different technological restrictions were defined and implemented in the practice of Bulgarian company RAIS Ltd. The proposed approach could be generalized for other job shop scheduling problems for different applications.

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