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Encrypted Audio Transmission using Synchronized Nd:YAG Lasers

R.M. López-Gutiérrez, C. Cruz-Hernández, C. Posadas-Castillo, and E.E.García-Guerrero

Abstract—Encoded information based on synchronization of coupled chaotic Nd:YAG lasers in master-slave configuration is numerically studied. Encoding, transmission, and decoding of information in optical chaotic communication with a single channel is presented. We analyze the robustness of the encrypted audio transmission in a channel noise. In order to illustrate this synchronization robustness, we present two cases of study: synchronization and transmission with a single channel without and with noise in the channel.

Keywords—Encryption, Secure coomunication, Chaos, Synchronization, Complex networks, Nd:YAG laser.

I. Introduction

NOWADAYS, information transmission plays a crucial role, where an ever-growing capacity for communication services are required. Two of the major requirements in communication systems are privacy and security.

Synchronization of chaotic systems [1]-[9] has been greatly motivated by the possibility of encoding information by using a chaotic carrier. Firstly explored with electronic circuits [10]-[13], where a small signal (the confidential information) was added to a chaotic voltage and transmitted to a receiver circuit. If chaotic synchronization is achieved between transmitter and receiver circuits, then with the chaotic carrier itself, and subtraction of the synchronized signal from the transmitted signal (carrier plus information signal) results in the recovery of the information. Information transmission based on chaotic synchronization of lasers has been studied recently. For example, encoded information was studied in solid-state lasers [14], fibre ring lasers [15], semiconductor lasers [16], [17], and microchip lasers [18].

Chaotic optical communication is a promising technique to improve both privacy and security in communication networks. It needs chaotic synchronization between transmitter and receiver lasers to encode, transmit, and decode confidential information at the hardware level. The generated chaotic carrier at the transmitter laser is used to hide information which can only be extracted by using the authorized receiver laser. An alternative and simple way to improve security of encrypted information can be realized by additionally encoding at the physical layer using chaotic carriers generated by components operating in chaotic regime. For example, chaotic Nd:YAG (Neodymium doped: Yttium Aluminum Garnet) lasers are ideal candidates for the realization of these chaotic transmitter and receiver systems [19]. They are already inherently nonlinear devices that, under certain operating conditions, exhibit chaotic motion. Some authors investigated analytically and numerically the types of synchronous behavior that occur when solid-state Nd:YAG lasers are coupled [20], [21].

The aim of this paper is study the encoding, transmission, and decoding of confidential information, in particular, audio messages using a single channel with and without channel noise. This objective is achieved by using recent results from complex systems theory presented previously in [28]. We show that the proposed approach is indeed suitable to synchronize two chaotic Nd:YAG lasers in master-slave configuration, with and without channel noise and recovered information faithfully.

The rest of this paper is arranged as follows: in Section II a brief summary on synchronization of complex systems is given. In Section III, the Nd:YAG laser mathematical model is described. In Section IV the synchronization of two chaotic Nd:YAG is given. In Section V, an application to encrypt information audio signals is presented. Furthermore, is shown the robustness for audio messages recovery. Finally, in Section VI some concluding remarks are given.

II. Synchronization of complex systems

A. Complex systems

We consider a complex network composes of N identical nodes, linearly and diffusively coupled through the first state of each node. Each node constitutes a n-dimensional dynamical system, described by

$$\dot{\mathbf{x}}_i = f(\mathbf{x}_i) + u_i, \qquad i = 1, 2, \ldots, N, \tag{1}$$

where $\mathbf{x}_i = (x_{i1},\ x_{i2},\ ...,\ x_{in})^T \in \mathbb{R}^n$ is the state of the node i, $u_i = u_{i1} \in \mathbb{R}$ is the input signal of the node i, and is defined by

$$u_{i1} = c \sum_{j=1}^{N} a_{ij} \Gamma \mathbf{x}_j, \qquad i = 1, 2, \ldots, N, \tag{2}$$

the constant $c > 0$ represents the coupling strength in (1)-(2), and $\Gamma \in \mathbb{R}^{n \times n}$ is a constant 0-1 matrix linking coupled

Corresponding author: López-Gutiérrez is with Engineering Faculty (FIE), Baja California Autonomous University (UABC), Km. 103, Carretera Tijuana-Ensenada, 22860 Ensenada, B.C., México. roslopez@uabc.mx

Cruz-Hernández is with Electronics and Telecommunications Department, Scientific Research and Advanced Studies of Ensenada (CICESE), Km. 107, Carretera Tijuana-Ensenada, 22860 Ensenada, B.C. México.ccruz@cicese.mx

Posadas-Castillo is with Engineering Faculty (FIE), Baja California Autonomous University (UABC), Km. 103, Carretera Tijuana-Ensenada, 22860 Ensenada, B.C., México. And Engineering Mechanic and Electric Faculty (FIME), Nuevo León Autonomous University (UANL), Pedro de Alba, S.N., Cd. Universitaria, San Nicolas de los Garza, N.L., México. cposadas@fime.uanl.mx

García-Guerrero E.E. is with Engineering Faculty (FIE), Baja California Autonomous University (UABC), Km. 103, Carretera Tijuana-Ensenada, 22860 Ensenada, B.C., México. eegarcia@uabc.mx

states. Assume that, $\Gamma = \operatorname{diag}(r_1,\ r_2, \ldots, r_n)$ is a diagonal matrix with $r_i = 1$ for a particular i and $r_j = 0$ for $j \neq i$. This means that, two coupled nodes are linked through their $i - th$ state. $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{n\times n}$ is the coupling matrix, which represents the coupling topology in (1)-(2). If there is a connection between node i and node j, then $a_{ij} = 1$, otherwise $a_{ij} = 0$ for $i \neq j$. The diagonal elements of $\mathbf{A}$ are defined as

$$a_{ii} = -\sum_{j=1,\ j\neq i}^{N} a_{ij} = -\sum_{j=1,\ j\neq i}^{N} a_{ji}, \qquad i = 1, 2, \ldots, N. \tag{3}$$

Suppose that (1)-(2) is connected without isolated clusters. Then, $\mathbf{A}$ is a symmetric irreducible matrix. In this case, zero is an eigenvalue of $\mathbf{A}$ with multiplicity 1 and all the other eigenvalues are strictly negatives [37], [38]. Synchronization state of nodes in (1)-(2), can be characterized by the nonzero eigenvalues of $\mathbf{A}$. The complex network (1)-(2) is said to achieve (asymptotically) synchronization, if [38]:

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \ \ldots\ = \mathbf{x}_N(t), \quad \text{as} \quad t \to \infty. \tag{4}$$

The diffusive coupling condition (3) guarantees that the synchronization state is a solution, $\mathbf{s}(t) \in \mathbb{R}^n$, of an isolated node, that is

$$\dot{\mathbf{s}}(t) = f(\mathbf{s}(t)), \tag{5}$$

where $\mathbf{s}(t)$ can be an equilibrium point, a periodic orbit, or a chaotic attractor. Thus, stability of the synchronization state,

$$\mathbf{x}_1(t) = \mathbf{x}_2(t) = \ \ldots\ = \mathbf{x}_N(t) = \mathbf{s}(t), \tag{6}$$

of (1)-(2) is determined by the dynamics of an isolated node, i.e. the function f (and its solution $\mathbf{s}(t)$), the coupling strength c, the inner linking matrix Γ, and the coupling matrix $\mathbf{A}$. The following theorem give the conditions to achieve synchronization in (1)-(2), as established in (4).

III. Nd:YAG laser model

As in [28], we take the same model suggested in [24] for a single solid-state Nd:YAG laser with a sinusoidally modulated loss, described by the state equations

$$\begin{aligned} \dot{X} &= (F - (\alpha_0 + \alpha_1 \cos(\omega t)))\, X, \\ \dot{F} &= \gamma\left(A_0 - F - FX^2\right), \end{aligned} \tag{7}$$

where $X(t)$ and $F(t)$ constitute the states of the laser, physically represent the amplitude of the electronic field of the laser and its gain, respectively. The parameters α_0 and A_0 denotes the rates of intra cavity loss and pump strength, respectively. While α_1 represents the strength of modulation of the intra cavity loss at a frequency ω, and γ is a ratio of the time scale of light in the laser cavity, and the upper level spontaneous emission lifetime of the lasing media.

We performed our simulations using $\gamma = 10^{-2}$ to avoid stiffness problems that arise with smaller values. It is known that for suitable values of α_0 and α_1, the Nd:YAG laser (7) exhibits chaotic fluctuations, we select the following set for chaos: $\alpha_0 = 0.9$, $\alpha_1 = 0.2$, $A_0 = 1.2$, and $\gamma = 0.01$. For the particular case where all losses are modulated equally at the rate [21]; $0.9 + 0.2\cos(0.045t)$, the pump parameters were equal to 1.2. The laser is modulated with a depth α_1 relative to its mean losses α_0. In absence of modulation, the Nd:YAG laser is stable and exhibits damped oscillations to their fixed-point values.

IV. Synchronization of two chaotic Nd:YAG lasers

In this section, we show synchronization of two unidirectionally coupled chaotic solid-state Nd:YAG lasers. Such synchronization is achieved by using results from complex systems theory, for details, see previous paper [28]. The master-slave configuration is the laser arrays, where the coupling is purely via overlap of the electric field [21]. The lasers under consideration are class B, where only the field and gain variables need be considered. The lasers are subjected to identical periodic modulations of the loss and may become chaotic under the mentioned parameter values.

The arrangement of two chaotic Nd:YAG lasers proposed in [28], is described by

$$\begin{bmatrix} \dot{X}_{i1} \\ \dot{F}_{i2} \end{bmatrix} = \begin{bmatrix} (F_{i2} - (\alpha_0 + \alpha_1 \cos(\omega t)))\, X_{i1} + u_{i1} \\ \gamma\left(A_0 - F_{i2} - F_{i2} X_{i1}^2\right) \end{bmatrix},\ i = 1, 2, \tag{8}$$

where $(X_{i1},\ F_{i2})^T \in \mathbb{R}^2$ are the state variables of Nd:YAG lasers, $u_{i1} \in \mathbb{R}$ is the input signal of the lasers, and is defined by

$$u_{i1} = c\sum_{j=1}^{2} a_{ij}\Gamma\mathbf{x}_j, \qquad i = 1, 2, \tag{9}$$

the constant $c > 0$ represents the coupling strength of the lasers array, and $\Gamma \in \mathbb{R}^{2\times 2}$ is a constant 0-1 matrix linking coupled state variables. For simplicity, assume that $\Gamma = \operatorname{diag}(r_1,\ r_2)$ is a diagonal matrix with $r_i = 1$ for a particular i and $r_j = 0$ for $j \neq i$. This means that two coupled lasers are linked through their $i - th$ state variables. $\mathbf{A} = (a_{ij}) \in \mathbb{R}^{2\times 2}$ is the coupling matrix, which represents the coupling configuration of the lasers array. If there is a connection between laser i and laser j, then $a_{ij} = 1$; otherwise, $a_{ij} = 0$ for $i \neq j$. The diagonal elements of coupling matrix $\mathbf{A}$ are defined as

$$a_{ii} = -\sum_{j=1,\ j\neq i}^{2} a_{ij} = -\sum_{j=1,\ j\neq i}^{2} a_{ji}, \qquad i = 1, 2. \tag{10}$$

If the laser arrays is connected in the sense that there are no isolated clusters. Then, the coupling matrix $\mathbf{A}$ is a symmetric irreducible matrix. In this case, it can be shown that zero is an eigenvalue of $\mathbf{A}$ with multiplicity 1 and all the other eigenvalues of $\mathbf{A}$ are strictly negative [25], [27].

A. Synchronization conditions

Theorem 1 ([25],[27]): Consider the dynamical network (1)-(9). Let

$$0 = \lambda_1 > \lambda_2 \geq \lambda_3 \geq \cdots \geq \lambda_N \tag{11}$$

be the eigenvalues of its coupling matrix $\mathbf{A}$. *Suppose that there exists a* $n \times n$ *diagonal matrix* $\mathbf{D} > 0$ *and two constants* $\bar{d} < 0$ *and* $\tau > 0$*, such that*

$$\left[Df(\mathbf{s}(t)) + d\Gamma\right]^T \mathbf{D} + \mathbf{D}\left[Df(\mathbf{s}(t)) + d\Gamma\right] \leq -\tau\mathbf{I}_n \tag{12}$$

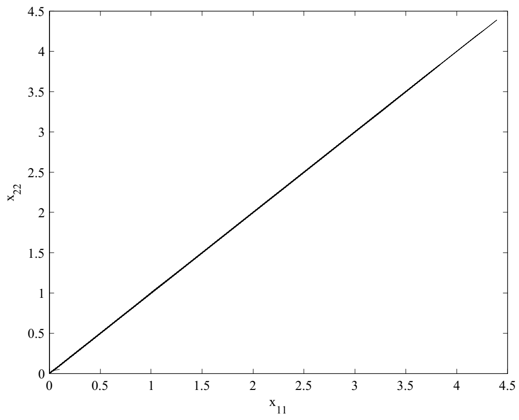

Fig. 1. Synchronization between master laser (15)-(16) and slave laser (17)-(18).

for all $d \leq \bar{d}$*, where* $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ *is an unit matrix. If, moreover,*

$$c\lambda_2 \leq \bar{d}, \tag{13}$$

then, the synchronization state (6) of dynamical network (1)-(9) is exponentially stable.

Since $\lambda_2 < 0$ and $\bar{d} < 0$, inequality (13) is equivalent to

$$c \geq \left|\frac{\bar{d}}{\lambda_2}\right|. \tag{14}$$

A small value of λ_2 corresponds to a large value of $|\lambda_2|$, which implies that dynamical network (1) can synchronize with a small coupling strength c. Therefore, synchronizability of dynamical network (1) with respect to a specific coupling configuration can be characterized by the second-largest eigenvalue of the corresponding coupling matrix $\mathbf{A}$.

In this array, the chaotic Nd:YAG laser (as master) is defined as

$$\begin{bmatrix}\dot{X}_{11}\\ \dot{F}_{12}\end{bmatrix} = \begin{bmatrix}(F_{12}-(\alpha_0+\alpha_1\cos(\omega t)))X_{11}+u_{11}\\ \gamma\left(A_0-F_{12}-F_{12}X_{11}^2\right)\end{bmatrix}, \tag{15}$$

$$u_{11} = c\left(a_{11}X_{11}+a_{12}X_{21}\right), \tag{16}$$

while the Nd:YAG laser (as slave) is designed as

$$\begin{bmatrix}\dot{X}_{21}\\ \dot{F}_{22}\end{bmatrix} = \begin{bmatrix}(F_{22}-(\alpha_0+\alpha_1\cos(\omega t)))X_{21}+u_{21}\\ \gamma\left(A_0-F_{22}-F_{22}X_{21}^2\right)\end{bmatrix}, \tag{17}$$

$$u_{21} = c\left(a_{21}X_{11}+a_{22}X_{21}\right), \tag{18}$$

the coupling matrix (14) is given by

$$A = \begin{bmatrix} 0 & 0 \\ 1 & -1 \end{bmatrix},$$

and the corresponding eigenvalues are $\lambda_1 = -1$ and $\lambda_2 = 0$, with a coupling value $c = 1$ obtained from (1), with initial conditions: $X_{11}(0) = 0.1$, $F_{12}(0) = 0.1$ and $X_{21}(0) = 0.05$, $F_{22}(0) = 0.05$. The synchronization between the transmitter and receiver in master-slave configuration is shown in Figure 1.

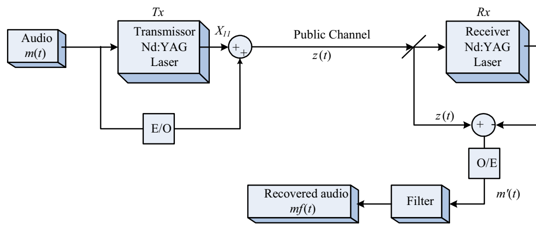

Fig. 2. Schematic audio transmission using a single channel.

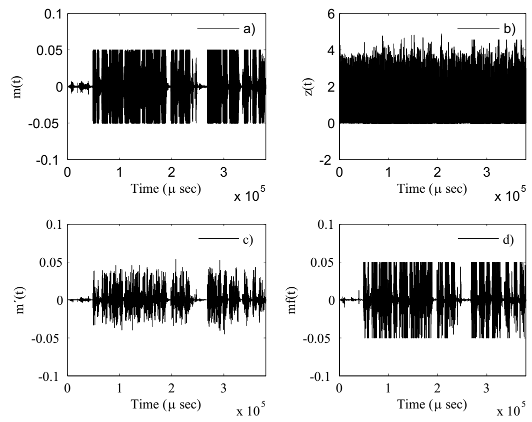

Fig. 3. Encrypted audio without channel noise.

V. Application to Encoding, Transmission, and Decoding

Synchronization of two chaotic Nd:YAG lasers allows us to design communication systems (see Figure 2), where the confidential information is hidden into the transmitter dynamics. The master and slave chaotic Nd:YAG lasers are the transmitter Tx and receiver Rx, respectively. The transmitted signal is a combination of the confidential information with the chaotic output signal of transmitter. The transmission of audio signal is encrypted using synchronized chaotic Nd:YAG lasers, shows in Figure 2. Audio message $m(t)$ (Figure 3a)) is hidden, as follows: information signal $m(t)$ with small amplitude is modulated in the parameter α_0 of the transmitter Tx, in such a way that the output of transmitter has implicit (hidden) the audio. The audio message is added to output the $X_{11}(t)$ in the trasmitter Tx again. The transmitted signal $z(t)$ (Figure 3b)) is received in the receiver end Rx, $z(t)$ synchronizes Tx with Rx. The signal $z(t)$ is subtracted to the output $X_{21}(t)$ from Rx to extract the information $m'(t)$.(see Figure 3c)). We need to apply a filtering stage for recovered the message mf (Figure 3 d)).

VI. Channel Noise

Now, let us show the robustness of the synchronization to additive channel noise, and their effects on the process of information decoding. Figure 4 shows numerical results of the process of encrypted transmission and recovery of confidential information $m(t)$, when the signal-to-noise ratio (SNR) is given by 32 dB.

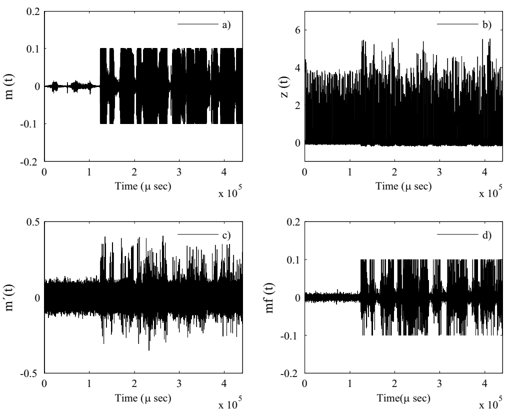

Fig. 4. Encrypted audio message with noise channel.

The public channel is corrupted with Gaussian noise. Figure 4a) shows the original audio message to be transmitted $m(t)$, Figure 4b) shows the chaotic transmitted signal $z(t) + noise$ through a public channel. Figure 4c) shows the recovered audio message $m'(t)$, but this signal is affected by the noise. For this case, we apply a filtering stage to obtain the message mf, shown in Figure 4 d).

VII. Conclusion

In this paper, we have presented the transmission of audio messages using synchronized chaotic Nd:YAG lasers. Recovery audio messages by using a single channel shown high quality with without noise. When noise was added, necessary a filtering stage by computer simulations is required. The importance of this study is for practical implementation of synchronized lasers. This work shown that the proposed chaotic communication schemes show a great potential for actual optical communication systems in which the encoding is required to be secure. In a forthcoming article we will be concerned with a physical implementation of the synchronization of two chaotic lasers in master-slave configuration. And application to private communication of image transmission.

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López-Gutiérrez R. M. was born on 10-11-1972. She is a Professor of Electronics Engineering in Baja California Autonomous University since 2001. She received her Master Science degree and Ph.D. degree in Electronics and Telecommunications from CICESE, Mexico in 1996 and 2003, respectively. Her research interests involve synchronization of complex systems and applications.

Cruz Hernández C. received the M.S. and Ph.D. degrees in electrical engineering from CINVESTAV, México, in 1991 and 1995, respectively. Since 1995, he is with the Department of Electronics and Telecommunications of the Scientific Research and Advanced Studies of Ensenada (CICESE), where, he is curren Professor of Automatic Control. His research interests include multimode oscillations of coupled oscillators, nonlinear systems analysis, and synchronization and control of complex dynamical systems

Posadas-Castillo C. was born in Poza Rica, Veracruz, México; in March 10 of 1973. He received the Engineer Degree in Control and Computation from the Faculty of Engineering Mechanic and Electrical of the Autonomous University of Nuevo León, Monterrey, México in 1997, and Master in Science Degree in Electronics and Telecommunications, specialty in Control; from Electronics and Telecommunications Department of Scientific Research and Advanced Studies of Ensenada; México in 2001. Since 1997, he has been Associated Professor of the University Autonomous of Nuevo León, Monterrey, México. His research interests include synchronization and control of complex systems, nonlinear systems analysis, and private communications.

Garcia-Guerrero E. E. studied physics engineering at the University Autonomous Metropolitana, Mexico, and received the PhD degree in optical physics from the Scientific Research and Advanced Studies Center of Ensenada, B.C, (CICESE) Mexico. He has been with the Engineering Faculty, Baja California Autonomous University (UABC) Mexico since 2004. His current interests are in the field of Optical Synchronization of Complex Systems