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Entropy Generation for Natural Convection in a Darcy – Brinkman Porous Cavity

Ali Mchirgui, Nejib Hidouri, Mourad Magherbi, and Ammar Ben Brahim

***Abstract*—**The paper provides a numerical investigation of the entropy generation analysis due to natural convection in an inclined square porous cavity. The coupled equations of mass, momentum, energy and species conservation are solved using the Control Volume Finite-Element Method. Effect of medium permeability and inclination angle on entropy generation is analysed. It was found that according to the Darcy number and the porous thermal Raleigh number values, the entropy generation could be mainly due to heat transfer or to fluid friction irreversibility and that entropy generation reaches extremum values for specific inclination angles.

***Keywords*—**Porous media, entropy generation, convection, numerical method.

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I. Introduction

THE fluids motion caused solely by temperature gradients, known as natural convection, occurs in many natural and industrial processes including wind, oceanic currents, movements within the Earth's mantle, in heating of homes, cooling of equipment, oil extraction, nuclear waste disposal, etc. because of its wide range of application, several studies dealing with natural convection due to thermal buoyancy forces have been reported during the last decades. Sen [1] investigated natural convection in a Brinkman porous rectangular cavity with differentially heated sidewalls. Laureat and Prasad [2] investigated the buoyancy effects on natural convection in a vertical enclosure using Brinkman-extended Darcy formulation. Natural convection with partial heating in a square cavity filled with porous media is was numerically investigated by Alam et al. [3] Selamat et al. [4] numerically studied natural convection in a square porous cavity using finite difference method. They found that the time taken to reach the steady state considerably depend on the Rayleigh number. Nader et al. [5] reported a numerical study about natural convection in air-filled 2D square enclosure heated with a constant source from below and cooled from above with variety of thermal boundary conditions at the top and sidewalls. The critical Rayleigh number at the onset of the natural convection in anisotropic horizontal porous layers with high porosity was determined by Shiina and Hishida [6]. They showed that the critical Rayleigh number decreases with the increase of the Darcy number and inversely, with the decrease of the effective thermal diffusivity ratio.

Natural convection includes the irreversible phenomena of heat transfer and viscous dissipation expressed by entropy generation that's why its study was often relied to the second low analysis. Baytas [7, 8] numerically studied the entropy generation in porous medium. He found that minimum of entropy generation is relied to the Rayleigh number and the enclosure inclination angle. Famouri and Hooman [9] numerically studied entropy generation in free convection in a partitioned cavity. Hidouri et al. [10, 11] investigated the influence of cros effects of Soret and Dufour on entropy generation in steady state of thermosolutal convection. Mukhopadhyay [12] carried out a numerical study about entropy generation in Two-dimensional steady state of natural convection, developed in a square enclosure heated by two discrete isoflux heat sources on the bottom wall. It was found that, for the studied case, heat transfer irreversibility predominates entropy generation due to the fluid friction. The dependence of entropy generation on the thermal boundary conditions of heated and cooled walls for the case of natural convection inside a porous enclosure was investigated by Zahmatkesh [13]. The focus of the present paper is on the numerical study of entropy generation encountered in natural convection in an inclined square porous cavity filled with a perfect gas mixture. The analysis was performed using Darcy – brinkman formulation with the Boussinesq approximation. Influence of medium permeability and inclination angle on the entropy generation due to heat transfer and viscous dissipation was investigated.

II. Mathematical Formulation

The geometry considered consists of a square cavity filled with a perfect gas mixture saturating a porous medium (Fig. 1). Left and right walls are submitted to different but uniform temperatures and concentrations. The two horizontal walls are insulated and adiabatic. The porous medium is isotropic, homogeneous and in thermodynamic equilibrium with the fluid. The flow in the cavity is laminar and two-dimensional . All physical properties of the fluid are assumed to be constant, except its density which satisfies the Boussinesq approximation such that:

$$\rho(C,T) = \rho_0[1 - \beta_T(T - T_0) - \beta_C(C - C_0)] \quad (1)$$

Ali Mchirgui, Nejib Hidouri and Ammar Ben Brahim are with Gabès University, Chemical and Process Engineering Department, Engineers National School of Gabès , Applied Thermodynamics Unit, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia (E-Mails: ali.mchirgui@voila.fr ; n_hidouri@yahoo.com ; ammar.benbrahim@enig.rnu.tn).

Mourad Magherbi is with Gabès University, Civil Engineering Department, High Institute of Applied Sciences and Technology, Omar Ibn El Khattab Street, 6029 Gabès, Tunisia (E-Mail: magherbim@yahoo.fr).

Author to whom correspondence should be adressed; (E-mail: ali.mchirgui@voila.fr)(Tel: +216 97 823 655).

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β_T and β_C are the thermal and the solutal expansion coefficients, respectively.

Under the foregoing assumptions and description of the problem, using the Darcy-Brinkman model, the conservation equations of mass, momentum, energy and chemical species can be written in dimensionless form as follow.

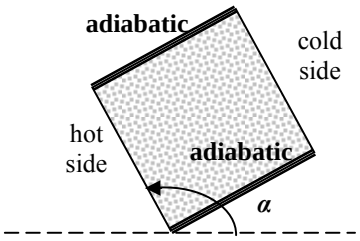


Fig. 1 Mathematical model

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2}$$

$$\frac{1}{\varepsilon}.\frac{\partial u}{\partial \tau}+\frac{1}{\varepsilon^2}(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y})=-\frac{\Pr}{D_A}u-\frac{\partial p}{\partial x}+ \Lambda.\Pr(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})+G_{rT}.\theta.\cos\alpha \tag{3}$$

$$\frac{1}{\varepsilon}.\frac{\partial v}{\partial \tau}+\frac{1}{\varepsilon^2}(u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y})=-\frac{\Pr}{D_A}v-\frac{\partial p}{\partial y}+ \Lambda.\Pr(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2})+G_{rT}.\theta.\sin\alpha \tag{4}$$

$$\sigma\frac{\partial \theta}{\partial \tau}+u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=R_k.(\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}) \tag{5}$$

The governing equations are obtained using the following dimensionless variables:

$$u=\frac{U}{W};v=\frac{V}{W};x=\frac{X}{h};y=\frac{Y}{h};\tau=\frac{t.W}{h};p=\frac{P-P_0}{\rho.W^2};$$

$$\theta=\frac{T-T_0}{\Delta T};\ \Delta T=T_h-T_c;\Delta C=C_h-C_c;\ Pr=\frac{\nu}{W.a};$$

$$\Lambda=\frac{\mu_e}{\mu};\ R_K=\frac{k_m}{k_f} \tag{6}$$

a and W are scales of length and viscosity, respectively. $G_{rT}=\frac{g\beta_T\,\Delta T a^3}{\upsilon^2}$ is the thermal Grashof numbers, respectively and N is the buoyancy ratio. $D_A=\frac{K}{a^2}$ is the Darcy number. Λ is the ratio of the viscosity in the Brinkman term to the fluid viscosity and R_k is the ratio of the thermophysical properties of the porous medium to the fluid thermal conductivity. The thermal porous Rayleigh number $Ra^*=Pr\,D_A G_{rT}$ will be used in the analysis.

The average heat transfer through the heated wall is given in dimensionless terms by Nusselt number as follow:

$$Nu=\int_0^1(-\frac{\partial\theta}{\partial x})dy\,. \tag{7}$$

The study of different irreversibilities competition is given by Bejan number defined as:

$$Be=\frac{N_\theta}{N_\theta+N_f} \tag{8}$$

The appropriate initial and boundary conditions of the problem are:

For the hole space, at

$\tau=0:\ u=v=0\,,\ p=0\,,\theta=0.5-x$ and $\phi=0.5-x\,.$

At $x=0\ :\ \phi=\theta=0.5$

At $x=1\ :\ \phi=\theta=-0.5$

At $y=0$ and $y=1:\ \frac{\partial\theta}{\partial y}=\frac{\partial\phi}{\partial y}=0$ (9)

For the considered problem, the volumetric entropy generation is therefore the sum of irreversibilities due to thermal gradients and viscous dissipation. Following Hidouri et al. [10] and Hooman et al. [14] local entropy generation is obtained by using the dimensionless variables previously listed and takes the following form:

$$N=N_\theta+N_f \tag{10}$$

N_θ and N_f are the dimensionless local entropy generation due to thermal gradients and the fluid friction respectively they are obtained by using the dimensionless variables previously listed:

$$N_\theta=\left[\left(\frac{\partial\theta}{\partial x}\right)^2+\left(\frac{\partial\theta}{\partial y}\right)^2\right] \tag{11}$$

$$Nf=Br^*\left\{u^2+v^2+D_A.\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]\right\} \tag{12}$$

$Br^*=\frac{Br}{\Omega}=\frac{\varphi_3}{D_A}$ is the modified Darcy-Brinkman number, φ_1, φ_2 and φ_3 are dimensionless irreversibility distribution ratios. They are given by:

$$\varphi_1 = \frac{D_e R}{k.\Omega}\left(\frac{\Omega'}{\Omega}\right)\Delta C \quad (13)$$

$$\varphi_2 = \frac{D_e R}{k.\Omega}.\Delta C \quad (14)$$

$$\varphi_3 = \frac{\mu T_0}{k_m}\left(\frac{W}{(\Delta T)}\right)^2 \quad (15)$$

The total dimensionless entropy generation is obtained by numerical integration, over the cavity volume A, of the dimensionless local entropy generation. It is given by:

$$s_T = \int_A s_{lT}.dA \quad (16)$$

III. Numerical Method

The purpose of using the numerical method is the determination of the temperature and the velocity scalar fields. From the known temperature, concentration and velocity fields, calculated at any time local entropy generation S_{lT} is then obtained. The total entropy generation is calculated by numerical integration. The numerical used method consists on the Control Volume Finite-Element Method (CVFEM) of Saabas and Baliga [15]. Standard-staggered grids were employed in order to calculate and store the velocity components. The pressure is obtained by using the finite element method classical grids. The grid of size 31x31 and 41x41 nodal points is found sufficiently enough to achieve the imposed global and local convergence criteria given respectively by:

$$\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right) \le 10^{-5},\ max\left|\frac{\Gamma^{\tau+\Delta\tau}-\Gamma^{\tau}}{\Gamma^{\tau+\Delta\tau}}\right| \le 10^{-5}$$

Γ is the dependent variable, $\Gamma = (u, v, \theta, \phi)$.

To test the accuracy of the present numerical study, the average values for the Nusselt number for natural convection are given in Table I and compared with those of Younsi et al. [16] and Nithiarasu et al. [17]. It is seen that the results are in good agreement with those given by the literature.

TABLE I
AVERAGE NUSSELT NUMBER
(N = 0, Pr = 1, DA =10^{-2})

Ra*	Present study	Ref [16]	Ref [17]
10	1.009	1.02	0.99
100	1.72	1.71	1.68
1000	4.26	4.26	4.24

IV. Results and Discussion

The present paper aims to investigate the influence of the Darcy number as well as the inclination angle on entropy generation. The considered medium is a square inclined porous cavity filled with a binary perfect gas mixture characterized by Pr = 0.71 and Le = 1.2. The operating parameters are in the following ranges: $10^{-6} \le D_A \le 10$, $10 \le Ra^* \le 10^3$, the following values of irreversibility coefficients are considered: $\varphi_1 = 0.5$, $\varphi_2 = 10^{-2}$, $\varphi_3 = 10^{-6}$. Due to large number of parameters, the porous medium proprieties are kept constant, they are given by: $\Lambda = 1$, $\sigma = 1$, $R_k = 1$.

Fig. 2 illustrates the variation of Bejan number versus Darcy number for different values of thermal porous Raleigh number.

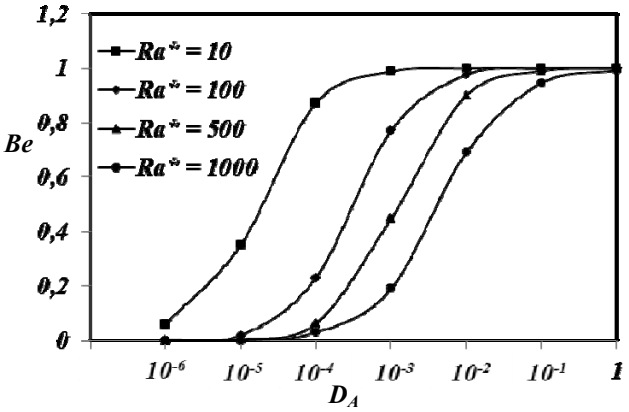


Fig. 2 Bejan number versus Darcy number

Results show that for higher values of Darcy number entropy generation is mainly due to heat transfer, when D_A decreases, fluid friction irreversibility begins to dominate the heat transfer entropy generation. The tow irreversibilities have the same intensity (Be = 0.5) for a specific value of Darcy number "D_{Ac}" depending on the thermal porous Raleigh number. The D_{Ac} increases when Ra^* is more important.

In fact, at low Darcy numbers, the Darcy term increases which indicates that the balance between the viscous force and the buoyancy force in the boundary layer is progressively replaced by a Darcy term versus buoyancy term balance, causing the increase of the velocity of convective motion which explain the dominance of fluid friction irreversibility at lower value of Darcy number. This can be illustrated by Fig. 3 showing the midsection stream function y-component at x = 0.5 for different values of Darcy number. It's seen that the flow velocity is intensified when the Darcy number decreases.

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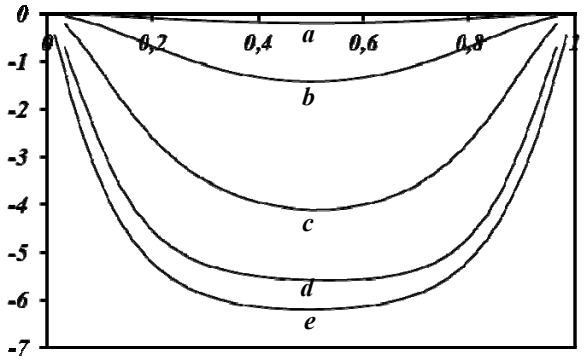


Fig. 3 Midsection stream function *y*- component
a: $D_A = 1$, *b:* $D_A = 10^{-1}$, *c:* $D_A = 10^{-2}$,*d:* $D_A = 10^{-3}$,*e:* $D_A = 10^{-4}$

The evolution of heat transfer, fluid friction irreversible and total entropy generation versus inclination angle of the cavity is depicted in Fig. 4 (a) and b for $D_A = 10^{-4}$, Ra* = 100and for $\phi_3 = 0.510^{-6}$ and $\phi_3 = 10^{-6}$

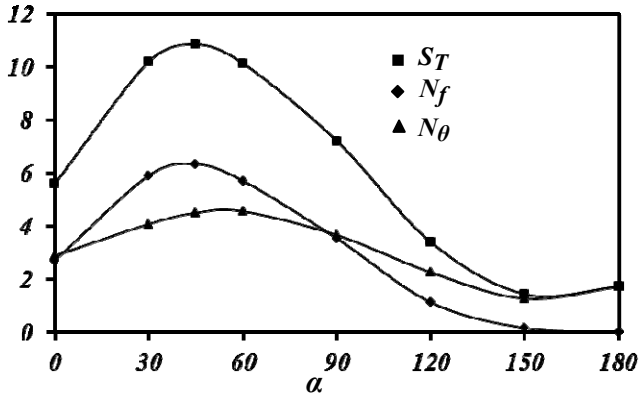


Fig. 4 (a) entropy generation versus inclination ($D_A = 10^{-4}$, φ_3=0,5.10^{-6})

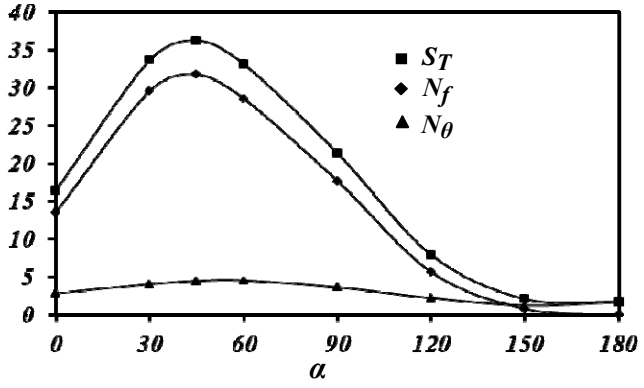


Fig. 4 (b) entropy generation versus inclination ($D_A = 10^{-4}$, φ_3=0,5.10^{-6})

A maximum of fluid friction entropy generation is obtained for α = 45° in fact, the heat transfer buoyancy force for these orientations acts in the directions of both active and adiabatic walls, thus the velocity of convective motion increases. For the same reason the maximum of total entropy generation is reached at the same inclination.

It's observed that the fluid friction irreversibility predominates entropy generation due to heat transfer for α < 90° if $\varphi_3 = 0.5.\ 10^{-6}$ and for α < 150° if $\varphi_3 = 10^{-6}$.

For the tow values of the irreversibility distribution ratio, a minimum value of entropy generation is obtained for α = 150° imposed by the absence of convection motion.

To explain the entropy generation behaviour, the effect on the inclination angle on the heat transfer and on the viscous dissipation is shown in fig. 5 and fig. 6, respectively.

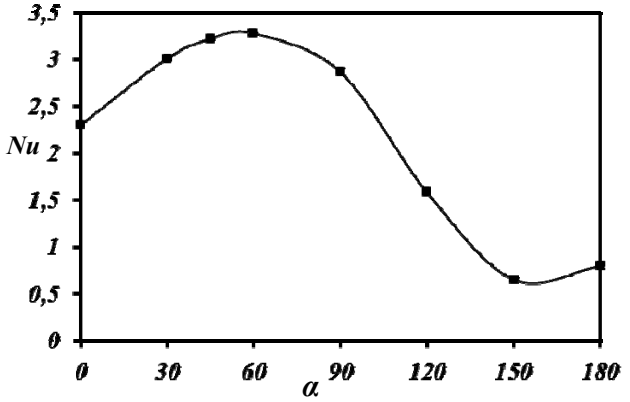


Fig. 5 Average Nusselt number versus inclination angle (*DA* $= 10^{-4}$, *Ra* =100*)

It is to note that the maximum of heat exchanges is for an inclination of about 60° when the thermal forces acts in a direction recovering the four walls, the minimum of heat transfer is for 150°<α < 180° added to that, the velocity of convection motion is minimized, thus the Flow is converted into conduction regime for the same inclination. This is due to the reason that buoyancy is no longer available between these angles.

The magnitude of the velocity is more important for α = 45° which explains the increase of fluid friction irreversibility and then the increase of total entropy generation.

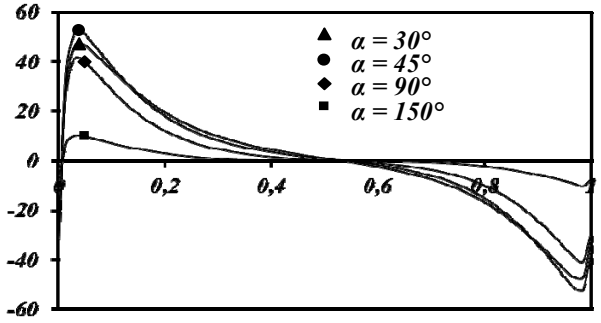


Fig. 6 velocity *y*-component at *x* = 0.5 (*Ra** = 100, $D_A = 10^{-4}$)

V. Conclusion

Entropy generation for natural convection in inclined porous cavity has been studied numerically by using Control

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Volume Finite-Element Method, the influence of the Darcy number and the inclination of the cavity on entropy generation behaviour was evaluated.

When fixing the thermal porous Raleigh number, it was obtained that, for high medium permeability, entropy generation is mainly due to heat transfer, when D_A decreases, fluid friction irreversibility begins to dominate the heat transfer entropy generation. Total entropy generation reaches a maximum value for α = 45° and a minimum for α = 150°. A competition between fluid friction irreversibility and heat transfer irreversibility depending on the irreversibility distribution ratio is observed.

Nomenclature:

C: concentration (mol· m^{-3}).
D_A: Darcy number.
G_{rT}: thermal Grashof number.
k: thermal conductivity ($W.m^{-1}.K^{-1}$).
K: permeability of the porous medium (m^2).
Nu: average Nusselt number
P: pressure ($kg{\cdot}m^{-1}.s^{-2}$).
p: dimensionless pressure.
Pr : Prandtl number.
R_K: thermal conductivity ratio (k_m/k_f).
Ra^*: thermal porous Rayleigh number.
T: temperature (K).
t : time (s)
u, v : dimensionless velocity components.
U, V: velocity components along X, Y directions ($m{\cdot}s^{-1}$).
x, y : dimensionless Coordinates.
X, Y : Cartesian coordinates (m).
W : characteristic Velocity ($m{\cdot}s^{-1}$).

Greek symbols:

α : thermal diffusivity (m^2s^{-1}).
β_T : thermal volumetric expansion coefficients (K^{-1}).
β_C : solutal volumetric expansion coefficients ($m^3.mol^{-1}$).
Λ : viscosity ratio (μ_{eff} / μ)
ε : porosity of the medium.
μ : fluid dynamic viscosity ($kg.m^{-1}.s^{-1}$).
μ_{eff} : viscosity in the Brinkman model ($kg.m^{-1}.s^{-1}$).
ν : kinematic viscosity ($m^2.s^{-1}$).
ϕ : dimensionless concentration.
ρ : fluid density ($kg.m^{-3}$).
σ : specific heat ratio [$(\rho c)_m / (\rho c)_f$].
θ : dimensionless temperature.

REFERENCES

[1] A. K. Sen , Natural convection in a shallow porous cavity-the Brinkman model, int. J. Heat Mess Tran., 5. pp. 855-868. May1987.
[2] Lauriat, G., Prasad, V., Natural convection in a vertical porous cavity: a numerical study for Brinkman-extended Darcy formulation. J. Heat. Trans. 109, pp 688–696 Aug. (1987).
[3] P. Alam, S. R. Ansari, and R. Kumar, Numerical study on natural convection in a porous cavity due to patial heating and cooling at vertical walls, Int. J. of Appl. Math. and Mech. Vol. 7(18): pp1-21, feb. 2011.
[4] M. S. Selamat, I. Hashim, M. K. Hasan, transient Natural Convection in Porous Square Cavity Heated and Cooled on Adjacent Walls, Math. Prob. in Eng.Vol 2012, jan. 2012.
[5] B.C. Nader , B.B. Brahim , L. Taieb, Influence of thermal boundary conditions on natural convection in a square enclosure partially heated from below. Int. Comm. in Heat and Mass Trans., vol. 34, pp. 369–379, Mar.2007.
[6] Shiina, Y., Hishida, M.: Critical Rayleigh number of natural convection in high porosity anisotropic horizontal porous layers, Int. J. of Heat and Mass Trans., Vol 53, pp 1507–1513 Mar. 2010.
[7] A. C. Baytas, optimisation in an inclined enclosure for minimum entropy generation in natural convection. j. Non-Equilib. Thermodyn. Vol 22, pp 145-155, Oct.1997.
[8] A. C.Baytas, Entropy generation for natural convection in an inclined porous cavity. Int. J. Heat and mass Trans. Vol 43, pp 2089-2099, Jun.2000.
[9] Famouri, M., Hooman, K.,Entropy generation for natural convection by heated partitions in a cavity. Int. Com. in Heat and Mass Trans. Vol 35, pp 492–502, Apr. 2008.
[10] N.Hidouri, H. abbassi, M. Magherbi, A. Ben Brahim, Entropy generation in double diffusive convection in presence of the Soret effect, progress in computational fluid dynamics, Vol. 7, 5 , pp 237 – 246, Jun.2007.
[11] Hidouri, N., Ben Brahim, A. Influence of Soret and Dufour effects on entropy generation in transient double diffusive convection, Int. J. Exergy, 7, 4, 454-481, Jun. 2010.
[12] A. Mukhopadhyay, Analysis of entropy generation due to natural convection in square enclosures with multiple discrete heat sources, Int. Comm. in Heat and Mass Transfer vol.37, pp867–872, Aug. 2010.
[13] I. Zahmatkesh, on the importance of thermal boundary conditions in heat transfer and entropy generation for natural convection inside a porous enclosure, Int. J. of Ther. Sci., 47, pp 339–346, Mar. 2008.
[14] K. Hooman, F. Hooman, S. R. Mohebpour, Entropy generation for forced convection in a porous channel with isoflux or isothermal walls. Int. J. Exergy. Vol.5, pp 78-96, Jan. 2008.
[15] H. J. Saabas, B.R. Baliga, Co-located equal-order control-volume finite-element method for multidimensional incompressible fluid flow. Part I: Formulation. Numer. Heat Transfer Pt. B-Fund.vol 26, 381–407, Jan. 1994.
[16] R. Younsi, A. Harkati, D. Kalache, Numerical simulation of double diffusive natural convection in a porous cavity: opposing flow. The Arabian J. for Sci. and Eng. Vol 27, pp181-194, Dec. 2002.
[17] P. Nithiarasu, K.N Seetharamu, T. Sundararajan, Double-diffusive natural convection in an enclosure filled with fluid- saturated porous medium: a generalized non-Darcy approach. Num. Heat Trans. Vol 30, pp 413– 426, Mar.1996.

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