

The Variable Step-Size Gauss-Seidel Pseudo Affine Projection Algorithm

F. Albu, C. Paleologu

Abstract—In this paper, a new pseudo affine projection (AP) algorithm based on Gauss-Seidel (GS) iterations is proposed for acoustic echo cancellation (AEC). It is shown that the algorithm is robust against near-end signal variations (including double-talk).

Keywords—pseudo affine projection algorithm, acoustic echo cancellation, double-talk.

I. INTRODUCTION

In echo cancellation systems, an adaptive filter algorithm is used to reduce the echo. The echo path is usually modeled by a linear filter. The well known normalized least-mean-square (NLMS) algorithm has been widely used, but converges very slowly. The affine projection algorithm (APA) can be considered as a generalization of the NLMS algorithm that provides a much improved convergence speed compared to LMS-type algorithms, although it is sensitive to high level of noise [1]. It has a performance that rivals with the more complex recursive least-squares (RLS) algorithms in many situations. However, the fast affine projection (FAP) algorithm proposed in [2,3] suffers from numerical instability when implemented with an embedded fast RLS algorithm. A key element in other proposed FAP algorithms is the approach to solve the encountered linear system. The choice of the approach (i.e., direct or iterative) determines the stability and robustness of the FAP algorithm. Several proposed FAP algorithms use an approximation that leads to simpler algorithms if the step size is $\mu = 1$ (non-relaxed case) or close to 1 (e.g. $0.7 < \mu \leq 1$) [4-9]. For such values, these algorithms have a fast convergence, but they exhibit a high sensitivity to noisy inputs. They have been applied in different application areas such as AEC [2-5] and [9], active noise control [10-11], hearing aid [6-7] etc. All these FAP versions use a fixed step size. Better performances in adverse conditions might be expected if a variable step size is used.

In [5] an efficient pseudo affine projection algorithm based on Gauss-Seidel method has been proposed for AEC systems. In [9] a robust GSPAP algorithm with variable step size, called VSS-PAP has been disclosed. Other variable step size solutions for the NLMS and AP algorithms have been proposed in [12] and [13], respectively. The solution for the

NLMS algorithm is a particular case of APA, for a projection order equal to 1. It was shown that it was a reliable solution in case of near-end signal variations, including double-talk. The same step size computational method used in VSS-NLMS can be adapted to the much less computational GSPAP algorithm.

The outline of the paper is as follows. The VSS-GSPAP algorithm is described in Section II. In Section III, the behavior of VSS-GSPAP algorithm for echo cancellation in single-talk and double-talk scenarios is examined. A comparison of the proposed algorithm with VSS-PAP is performed. Section IV concludes the paper.

II. THE VSS-GSPAP ALGORITHM

Let us follow the notation used in deriving the DCD-AP (see [6]): L is the filter length, K is the projection order, δ a regularization parameter, λ a forgetting factor; $x(n)$ is the input signal, $y(n)$ is the desired signal, $e(n)$ is the output error and $\bar{e}(n)$ is the normalized error.

$\mathbf{X}_n = [x(n), x(n-1), \dots, x(n-L+1)]^T$ and $\mathbf{R}(n)$ is the auto-correlation matrix of the signal. $\boldsymbol{\xi}_n = [\xi(n), \xi(n-1), \dots, \xi(n-K+1)]^T$, \mathbf{b} is an N vector with only one nonzero element that is unity at the top. $\mathbf{U}(n) = [u(n), \dots, u(n-L+1)]^T$ is the approximated decorrelated vector. $\mathbf{H}(n) = [h_1(n), \dots, h_L(n)]^T$ is the filter coefficient vector; and \mathbf{P} is an N length vector and $P_i, i = 0$ to $K-1$ is its i th element. $\mu(n)$ is the variable step size respectively at time instant n .

More details about the GSPAP algorithm can be found in [5]. The step sizes are computed as in [12] (see steps 6-8 from Table 1). The step-size equations of the proposed VSS-GSPAP do not depend explicitly on the near-end signal, although they were derived by taking into account its presence, and consequently, a robust behavior under near-end signal variations (e.g., background noise variations and double-talk) is expected. The total computational complexity of the VSS-PAP is $2L + K^2 + 5K + 12$ multiplications and divisions [9]. The VSS-GSPAP algorithm has $2L + K^2 + 3K + 10$ multiplications, 3 divisions and 1 square root. For typical projection orders, the numerical complexity of the considered algorithms is roughly similar.

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TABLE I THE VSS-GSPAP ALGORITHM

Step	Equation
	Initialization $\hat{\sigma}_e^2(-1) = 0$, $\hat{\sigma}_s^2(-1) = 0$, $\mathbf{X}(-1) = 0$, $\mathbf{U}(-1) = 0$, $\mathbf{H}(-1) = 0$, $\mathbf{P}(-1) = 0$, $\varepsilon = 10^{-8}$, and $\mathbf{R}(-1) = \delta \mathbf{I}$
	For $n = 0, 1, \dots$
1	$\mathbf{R}(n) = \mathbf{R}(n-1) + \xi(n)\xi^T(n) - \xi(n-L)\xi^T(n-L)$
2	Solve $\mathbf{R}(n)\mathbf{P}(n) = \mathbf{b}$ (using one GS iteration)
3	$\mathbf{U}(n) = \left[\xi^T(n)\mathbf{P}(n)\mathbf{P}(1) \bar{\mathbf{U}}(n-1) \right]$
4	$z(n) = \mathbf{X}^T(n)\mathbf{H}(n-1)$
5	$e(n) = y(n) - z(n)$
6	$\hat{\sigma}_s^2(n) = \lambda \hat{\sigma}_s^2(n-1) + (1-\lambda)(y^2(n) - z^2(n))$
7	$\hat{\sigma}_e^2(n) = \lambda \hat{\sigma}_e^2(n-1) + (1-\lambda)e^2(n)$
8	$\mu(n) = \left 1 - \sqrt{\frac{\hat{\sigma}_s^2(n)}{\varepsilon + \hat{\sigma}_e^2(n)}} \right $
8	$\bar{e}(n) = \frac{\mu(n)e(n)}{\mathbf{U}^T(n)\mathbf{U}(n) + \delta}$
9	$\mathbf{H}(n) = \mathbf{H}(n-1) + \mathbf{U}(n)\bar{e}(n)$,
Total: $2L + K^2 + 3K + 10$ Mults + 3 div + 1 sqrt	

III. SIMULATIONS

The simulations were performed in an AEC context and the VSS-PAP and VSS-GSPAP were compared. The length of the adaptive filter is set to 512 coefficients. The measured impulse response of the acoustic echo path is plotted in Fig. 1(a) (the sampling rate is 8 kHz); its entire length has 1024 coefficients. This length is truncated to the first 512 coefficients [before the dotted line in Fig. 1(a)] for a first set of experiments performed in an exact modeling case. Then, the entire length of the acoustic impulse response is used for a second set of experiments performed in the under-modeling case [13]. The value of the projection order for all simulations was $K = 4$.

The simulations are performed in an exact modeling scenario ($N = L = 512$), and in an under-modeling scenario, using the entire acoustic impulse response from Fig. 1(a), while the length of the adaptive filter remains the same ($N = 1024, L = 512$). The performance for the first scenario is evaluated in terms of the normalized misalignment (in dB), defined as $20 \log_{10}(\|\mathbf{h} - \mathbf{H}\| / \|\mathbf{h}\|)$. In the second scenario, the expression of the normalized misalignment is evaluated by padding the vector of the adaptive filter coefficients with $N - L$ zeros, i.e., $20 \log_{10}(\|\mathbf{h} - [\mathbf{H} \mathbf{0}_{N-L}^T]\| / \|\mathbf{h}\|)$. The forgetting factor λ is computed as in [14].

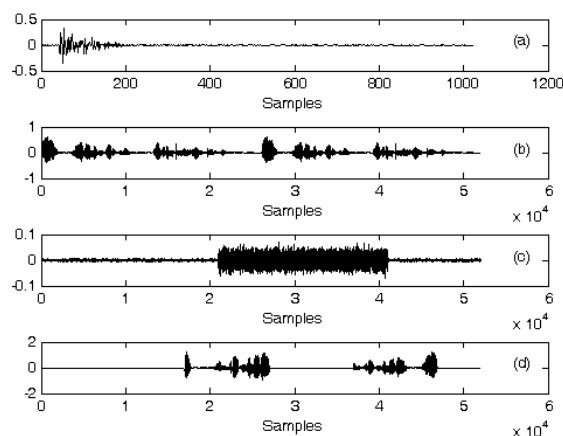


Fig. 1 (a) Measured room acoustic impulse response; (b) Far-end speech signal used in the experiments; (c) The used background noise with variable SNR; (d) Near-end speech signal used in the double-talk case

In the following experiments, in order to approach the context of typical AEC applications, only the speech sequence from Fig. 1(b) will be used as the far-end signal. Single-talk and double-talk scenarios are considered.

A. Single-talk scenario

For the first set of simulations we considered a variation of the background noise [9] (i.e., the SNR decreases from 20 dB to 10 dB after 14 seconds from the debut of the adaptive process, for a period of 14 seconds – shown in Fig. 1(c)) is considered in Fig. 2 and 3. The behavior of VSS-PAP and VSS-GSPAP is evaluated in the exact modeling case (Fig. 2) and the under-modeling case (Fig. 3). It can be noticed that the proposed algorithm is better than VSS-PAP algorithm in this situation.

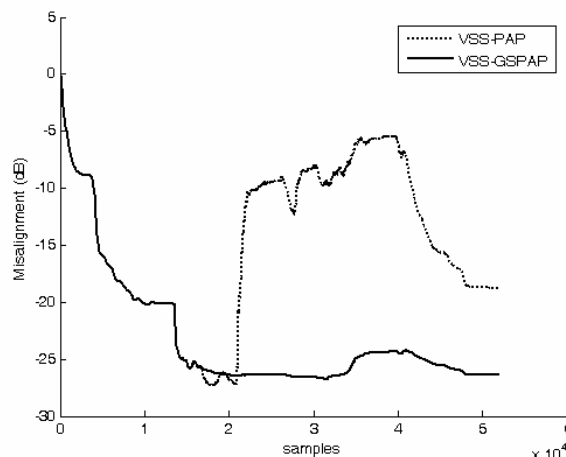


Fig. 2 Misalignments of VSS-PAP and VSS-GSPAP. Single-talk case, $L=512$, exact modeling scenario, variable background noise (SNR decreases from 30 dB to 10 dB for 20000 samples).

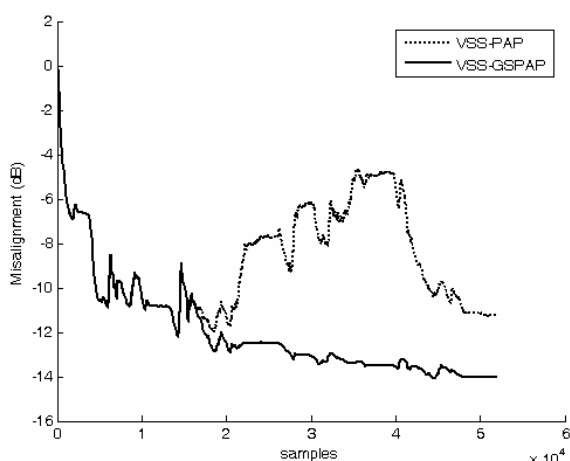


Fig. 3 Misalignments of VSS-PAP and VSS-GSPAP. Single-talk case, $L=512$, under-modeling scenario, variable background noise (SNR decreases from 30 dB to 10 dB for 20000 samples).

Fig. 4 shows the misalignment curve in case of exact modeling while Fig. 5 considers the under-modelling case and a sudden change of the acoustic path. The acoustic impulse response of Fig. 1a was shifted to the right by 12 samples after 26000 samples from the debut of the adaptive process.

The robustness of the VSS-GSPAP algorithm in the under-modelling case and its close performance to VSS-PAP is verified. It can be noticed the losses in performance in the under-modeling case for both algorithms. Their tracking capabilities are good.

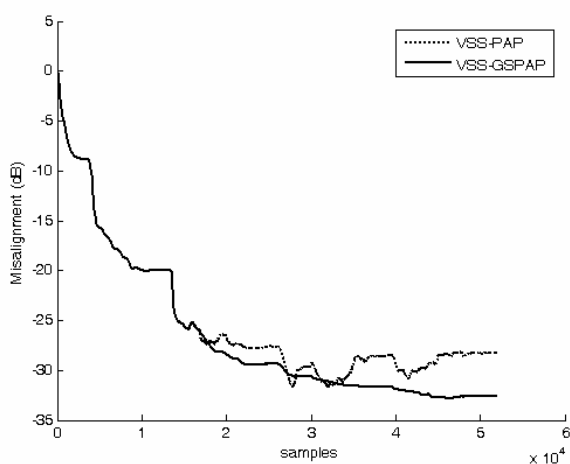


Fig. 4. Misalignments of VSS-PAP and VSS-GSPAP. Single-talk case, $L=512$, exact modeling scenario, SNR=30 dB.

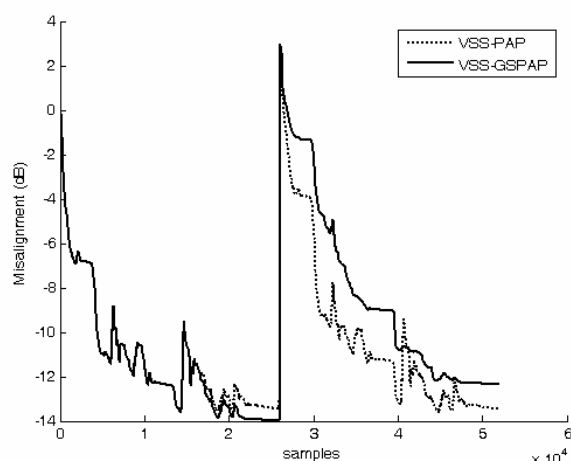


Fig. 5. Misalignments of VSS-PAP and VSS-GSPAP. Single-talk case, $L=512$, under-modeling scenario, SNR=30 dB, echo path change after 26000 samples.

B. Double-talk scenario

Perhaps the most challenging problem in echo cancellation is the double-talk situation. Such a scenario is considered in the simulations using the speech signals from Fig. 1(b) and Fig. 1(d). In Figs. 6 and 7, the VSS-PAP and VSS-GSPAP algorithms are involved. It can be noticed from Figs. 9 and 10 that the VSS-GSPAP algorithm is superior to VSS-PAP algorithm in both exact modeling and under-modeling cases.

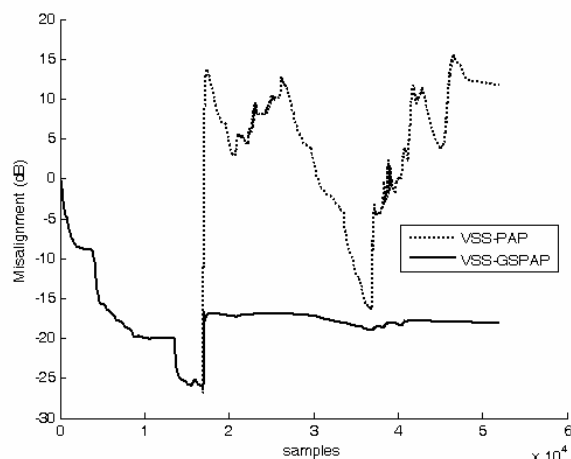


Fig. 6 Misalignments of VSS-PAP and VSS-GSPAP. Double-talk case, $L=512$, exact modeling scenario, SNR=30 dB.

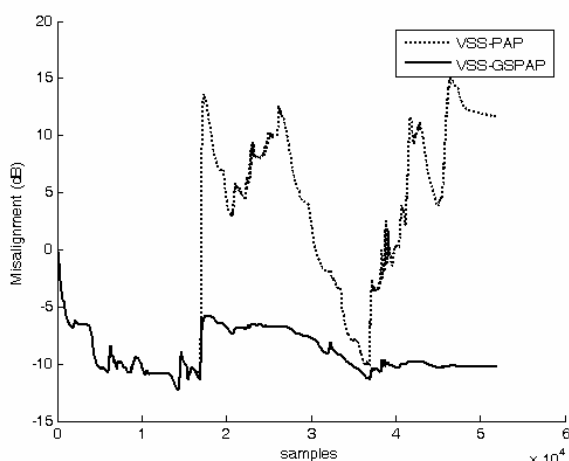


Fig. 7. Misalignments of VSS-PAP and VSS-GSPAP. Double-talk case, $L=512$, under-modeling scenario, SNR=30 dB.

IV. CONCLUSION

A VSS-GSPAP algorithm suitable for AEC applications has been proposed in this paper. A variable step size was used in order to take into account the existence and the non-stationarity nature of the near-end signal as well as the under-modeling noise. The simulation results performed in an AEC context showed its superior robustness to near-end signal variations like the increase of the background noise or double-talk than VSS-PAP algorithm.

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REFERENCES

- [1] K. Ozeki and T. Umeda, "An adaptive filtering algorithm using an orthogonal projection to an affine subspace and its properties," *Electronics and Communications in Japan*, vol. 67-A, no. 5, 1984.
- [2] S. L. Gay and S. Tavathia, "The fast affine projection algorithm," in *Proc. IEEE ICASSP*, pp. 3023 - 3026, Detroit, Michigan, U.S.A, May 1995
- [3] M. Tanaka, Y. Kaneda, S. Makino, and J. Kojima, "A fast projection algorithm for adaptive filtering," *IEICE Trans. Fundamentals*, vol. E78-A, no. 10, pp. 1355-1361, Oct. 1995.
- [4] F. Albu, J. Kadlec, N. Coleman, and A. Fagan, "The Gauss-Seidel fast affine projection algorithm," in *Proc. IEEE SIPS 2002*, pp. 109 - 114, San Diego, U.S.A, October 2002.
- [5] F. Albu, and H.K. Kwan, "Combined echo and noise cancellation based on Gauss-Seidel pseudo affine projection algorithm", *Proc. IEEE ISCAS 2004*, Vancouver, Canada, pp. 505-508
- [6] S. Lee, I. Kim, Y. Park, "Approximated affine projection algorithm for feedback cancellation in hearing aids" *Computer Methods and Programs in Biomedicine*, Volume 87, Issue 3 (September 2007) Pages 254-261
- [7] S. Lee, I. Kim, Y. Park, "An efficient feedback canceller for hearing aids based on affine projection", 2006, *Lecture notes in computer science*, 4115 LNBIII-III, pp. 711-720
- [8] P. Sheikhzadeh, H. Whyte, K.R.L. Brennan, "Partial Update Subband Implementation of Complex Pseudo-Affine Projection Algorithm on Oversampled Filterbanks", *ICASSP 2004*, vol. IV, pp. 373-376
- [9] Robust pseudo affine projection algorithm with variable step-size, Lee, L, Park, Y.-C., Youn, D.-H. 2008 *Electronics Letters* 44 (3), pp. 250-252

- [10] J.M. Wesselink, A.P. Berkhoff, "Fast affine projections and the regularized modified filtered-error algorithm in multichannel active noise control, - *The Journal of the Acoustical Society of America* -- August 2008 -- Volume 124, Issue 2, pp. 949-960
- [11] F. Albu, M. Bouchard, and Y. Zakharov, Pseudo-affine projection algorithms for multichannel active noise control, *IEEE Transactions on Audio, Speech and Language Processing*, Vol. 15 No. 3, March 2007, pp. 1044-1052
- [12] C. Paleologu, S. Ciochina, and J. Benesty, "Variable step-size NLMS algorithm for under-modelling acoustic echo cancellation", *IEEE Signal Process. Lett.* VOL.15, PP. 5-8, 2008
- [13] C. Paleologu, J. Benesty, and S. Ciochina, "Robust variable step-size affine projection algorithm suitable for acoustic echo cancellation", *Proc. Eusipco 2008*, Laussane, Switzerland
- [14] J. Benesty, H. Rey, L. Rey Vega, and S. Tressens, "A nonparametric VSS NLMS algorithm," *IEEE Signal Process. Lett.*, vol. 13, no. 10, pp. 581-584, Oct. 2006.