

# Using ANSYS to Realize a Semi-Analytical Method for Predicting Temperature Profile in Injection/Production Well

N. Tarom, and M.M. Hossain

**Abstract**—Determination of wellbore problems during a production/injection process might be evaluated through temperature log analysis. Other applications of this kind of log analysis may also include evaluation of fluid distribution analysis along the wellbore and identification of anomalies encountered during production/injection process. While the accuracy of such prediction is paramount, the common method of determination of a wellbore temperature log includes use of steady-state energy balance equations, which hardly describe the real conditions as observed in typical oil and gas flowing wells during production operation; and thus increase level of uncertainties. In this study, a practical method has been proposed through development of a simplified semi-analytical model to apply for predicting temperature profile along the wellbore. The developed model includes an overall heat transfer coefficient accounting all modes of heat transferring mechanism, which has been focused on the prediction of a temperature profile as a function of depth for the injection/production wells. The model has been validated with the results obtained from numerical simulation.

**Keywords**—Energy balance equation, reservoir and well performance, temperature log, overall heat transfer coefficient.

## I. INTRODUCTION

THE temperature log has been widely used to diagnose many injection/production related well problems reliably. It is also used to obtain a qualitative indication of the fluid distribution along the wellbore, and identify the root causes for many anomalies encountered during production/injection process. The quantitative knowledge of wellbore and reservoir heat transfer process to the surrounding rock formation is important for accurate interpretation and prediction of this temperature profile.

In order to predict temperature log along a wellbore, steady-state energy balance equations are the common procedure. Various mathematic models or tools are used to predict the temperature distribution along the wellbore. These models are mostly analytical or semi-analytical developed based on energy balance equations for steady state condition, which hardly describe the real conditions as observed in typical oil and gas flowing wells during production operation. There are hardly any works found to be reported in the literatures that deals with the process of transient heat transmission. Ramey [1] applied the energy balance equations to demonstrate an analytical solution to estimate the temperature of fluid, tubing

and casing in a hot water injection well as a function of depth and time. References [2] and [3] improved and revisited Ramy's heat transmission estimations for applying real gases and providing a graphical correlation estimating the temperature of transient period, respectively. Also, references [4] and [5] describe the application of general energy balance equations and heat flow equations comprehensively. Consequently, they indicated that the energy balance equations play a vital role for the prediction of temperature profile.

During production process, heat is usually conducted throughout surrounding formation, cement sheaths, casing, annulus and tubing(s). So, each of these media have individual thermal properties, which make the process of heat transmission prediction more complex. In addition, opening, shutting, restarting and changing the production schedule are often the normal and daily program of a well producing operation. Each of these operations may cause transient heat losses through surrounding media. References [6], [7], [8] and [9] discuss the complexity of heat transfer mechanism of transient conditions. Therefore, different conditions and mechanisms of heat loss from the wellbore to the surroundings result complex and cumbersome mathematical models to predict the temperature profile and require using expensive numerical simulators, which are often impractical for industry standard routine engineering calculations. In this study, a practical method has been proposed through development of a simplified semi-analytical model to apply for predicting temperature profile along the wellbore for such a complex condition. These estimations can be applied for transient and steady-state condition as well.

The developed model also includes an overall heat transfer coefficient accounting all modes of heat transferring process, which has been focused on the prediction of a temperature profile as a function of depth for the injection/production wells. A simple spread sheet calculation based program has developed, which can be used as a confident tool for industry standard routine engineering calculations. The model has been used to generate temperature profile for typical producing wells and compared with measured temperature profile to justify for potential field application. The predicted results have also shown good agreement with estimations by numerical simulator ANSYS Fluent. The results have been taken through ANSYS (Fig. 5 and 8) show the transient temperature profile around a wellbore. These figures explain that the transient time for this case study to reach the steady

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state condition is around 8 days, which is in a good agreement with Ramey's results[1] that the convergence transient time has presented on the order of one week for many transient heat-transmission reservoirs problem. The paper also presents step by step calculation procedures including detail mathematical formulations.

## II. SEMI-ANALYTICAL MODEL

As discussed earlier, during petroleum production process, heat is usually conducted throughout surrounding formation, cement sheaths, casing, annulus and tubing(s). In addition to heat conduction, the heat is also transferred into/from the flowing fluid by convection process to the innermost tubing. In case of annulus which is filled with fluid, radiation heat transfer mechanism will also result in addition of convection heat transfer mechanism [10] and [11]. To predict temperature profile accurately for this situation, it is necessary to evaluate the wellbore heat loss considering all different heat transfer mechanisms, which is always challenging issues faced by the petroleum engineers. To ease the calculations process, overall heat transfer coefficient concept has been applied to deal with this problem. The overall heat transfer coefficient has been calculated based on the heat resistivity of total layers around a wellbore that determines the overall rate of heat loss per unit area.

The calculation of wellbore heat loss and overall heat transfer coefficient has been discussed by many authors [12], [13], [14] and [15]. In a vertical fluid flow, the mechanism of convection is the process of heat transfer from the flowing fluid to the innermost pipe of the well. Moreover, the process of heat transfer is also dependent on the type of flowing fluid, and the physical and thermal properties of fluid have also filled the annulus.

The ratio of temperature difference between the borehole and the ground to the total thermal resistance is called the overall heat transfer through any unit section of desired well [10].

$$q = \Delta T_{overall} / R_{total} \quad (1)$$

In which the total thermal resistance is the summations of the thermal resistivities of all layers. Considering a well surrounded by different layers of tubing, annulus, casings, cement layers and the surrounding ground as illustrated in Fig. 1, which have different physical and thermal properties. So, the total thermal resistance per unit area of surface may be calculated by:

$$R_{total} = R_w + R_t + R_a + R_c + R_{cmt} + R_g \quad (2)$$

Further details for the estimations of thermal resistivity for each layer are provided in Appendix A.

By definition, overall heat transfer coefficient can be estimate by:

$$U_{overall} = \frac{1}{R_{total}} \quad (3)$$

where  $U_{overall}$  and  $R_{total}$  are overall heat transfer coefficient and total thermal resistance, respectively.

By neglecting the thermal resistivity of tubing and casing and assuming  $T_{ot}=T_w$ , then in accordance with (2), (A13) and (A15) the total thermal resistance between tubing and cement sheath can be expressed by:

$$R_{total} = \frac{1}{2\pi\Delta L} \left( \frac{1}{r_m h_{avg}} + \frac{\ln\left(\frac{r_{ocmt}}{r_{icmt}}\right)}{k_{cmt}} \right) \quad (4)$$

where  $r_{ocmt}$  and  $r_{icmt}$  are the outer and inner radius of cement sheath, respectively. Also  $k_{cmt}$  shows the thermal conductivity coefficient of cement layer, and  $h_{avg}$  explains the average heat transfer coefficient of annulus.

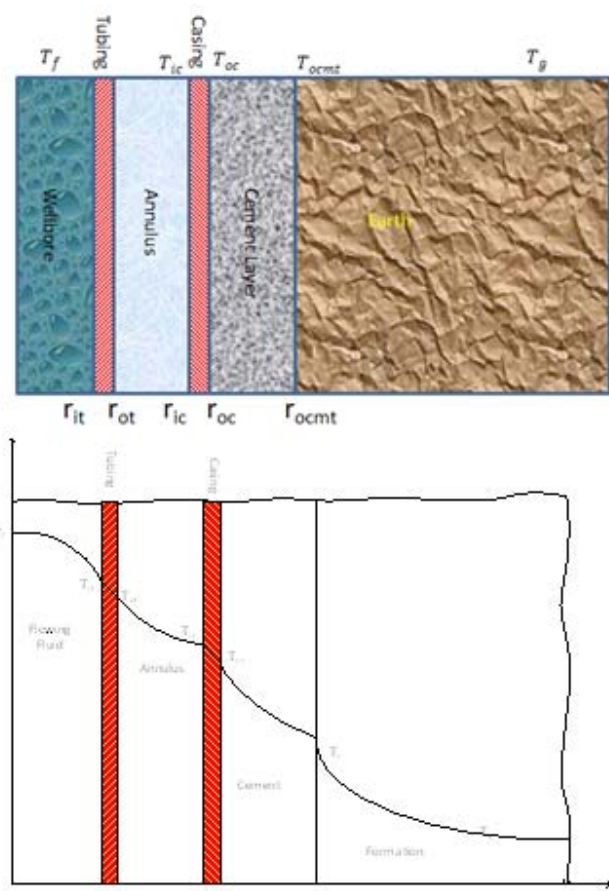


Fig 1 Heat distribution from wellbore through surrounding area

At steady state, the heat flow rate per unit length of wellbore, can be expressed in [7]:

$$Q = 2\pi r_{ot} U_{overall} (T_w - T_{ocmt}) \Delta L \quad (5)$$

where the terms  $T_w$  and  $T_{ocmt}$  explain the temperature of media at the wellbore and cement sheath, respectively.

Combining (1-5), the overall heat transfer coefficient can be expressed by:

$$U_{overall} = [R_{total}]^{-1} / 2\pi r_{ot} \Delta L = \left[ \frac{r_{ot}}{r_m h_{avg}} + \frac{r_{ot} \ln \left( \frac{r_{ocmt}}{r_{icmt}} \right)}{k_{cmt}} \right]^{-1} \quad (6)$$

where  $r_{ot}$  and  $r_m$  are defined as outer radius of tubing and mean radius of annulus, respectively.

### III. CALCULATION OF TEMPERATURE DISTRIBUTION AT TUBING AND CASING SURFACES

In (3-6), the knowledge of the tubing and casing temperature is required to calculate average heat transfer coefficient ( $h_{avg}$ ) of annulus. However, temperature of casing and cement-ground interface can be determined using following equations (7-13) by assuming that the temperature of tubing is known (i.e.  $T_{ot}=T_w$ ).

Overall:

$$Q = \frac{2\pi k_{overall}}{\ln \left( \frac{r_{ocmt}}{r_{it}} \right)} (T_w - T_{ocmt}) \Delta L \quad (7)$$

Casing:

$$Q = \frac{2\pi k_c}{\ln \left( \frac{r_{oc}}{r_{ic}} \right)} (T_{ic} - T_{oc}) \Delta L \quad (8)$$

Cement:

$$Q = \frac{2\pi k_{cmt}}{\ln \left( \frac{r_{ocmt}}{r_{icmt}} \right)} (T_{icmt} - T_{ocmt}) \Delta L \quad (9)$$

where, at the following, there are the definitions of all symbols used in (7-9).

- $Q$  and  $\Delta L$  are heat flow rate and length increment.
- $k_{overall}$ ,  $k_c$  and  $k_{cmt}$  are thermal conductivity coefficient of overall, casing and cement sheath.
- $r_{it}$ ,  $r_{ic}$ , and  $r_{icmt}$  show the inner radius of tubing, casing and cement sheath.
- $r_{oc}$ ,  $r_{ocmt}$  explain the outer radius of casing and cement sheath.
- $T_w$  is the temperature of wellbore fluid.
- $T_{ic}$  and  $T_{icmt}$  define the temperatures at inner surface of casing and cement sheath, and  $T_{oc}$  and  $T_{ocmt}$  also define the temperatures at outer radius of casing and cement sheath.

Since heat flow through all layers,  $Q$  is constant, after rearranging (7-9) yields:

$$T_{ic} = T_{ocmt} + \left( \frac{k_{overall}}{\ln \left( \frac{r_{ocmt}}{r_{it}} \right)} \right) \left( \frac{\ln \left( \frac{r_{ocmt}}{r_{oc}} \right)}{k_{cmt}} + \frac{\ln \left( \frac{r_{oc}}{r_{ic}} \right)}{k_c} \right) (T_w - T_{ocmt}) \quad (10)$$

Since the thermal resistance of casing is negligible due to its physical properties, (10) becomes, reduces to the following form.

$$T_{ic} = T_{ocmt} + \left( \frac{k_{overall}}{\ln \left( \frac{r_{ocmt}}{r_{it}} \right)} \right) \left( \frac{\ln \left( \frac{r_{ocmt}}{r_{oc}} \right)}{k_{cmt}} \right) (T_w - T_{ocmt}) \quad (11)$$

In (11), the unknown term,  $T_{ocmt}$ , may be calculated following the Ramy's procedure [1] and [16]. Therefore;

$$Q = \frac{2\pi k_g (T_{ocmt} - T_g) \Delta z}{f(t)} \quad (12)$$

Equating the overall heat flow in the well (7) with the radial heat flow through the ground (12), and considering  $\Delta z = \Delta L$  will result the following equation.

$$T_{ocmt} = \frac{\left( \frac{k_{overall}}{\ln \left( \frac{r_{ocmt}}{r_{it}} \right)} \right) T_w + \left( \frac{k_g}{f(t)} \right) T_g}{\left( \frac{k_{overall}}{\ln \left( \frac{r_{ocmt}}{r_{it}} \right)} \right) + \left( \frac{k_g}{f(t)} \right)} \quad (13)$$

The logarithmic term in (13), makes it non-linear, which is required to be solved iteratively in order to calculate the total thermal resistance. Following iterative steps can be followed to calculate the total thermal resistance.

1. Guess a value of  $k_{overall}$ .
2. Determine  $f(t)$ .

For the production time more than 7 days [1]:

$$f(t) = \ln \frac{2\sqrt{\alpha t}}{r_{ocmt}} - 0.29 \quad (14)$$

In other cases:

- Without annulus: using Fig. 1 of [1]
  - With annulus: using Table I.
3. Calculate  $T_{ocmt}$  using (13).
  4. Calculate  $T_{ic}$  using (11).
  5. Estimate  $Q_r$  and  $Q_{cv}$  using (A6) and (A8), respectively.
  6. Estimate  $Q_a = Q_r + Q_{cv}$ .
  7. Estimate  $Q_{overall}$  using (7).
  8. If  $Q_a = Q_{overall}$ , the calculation will be finished. Otherwise, guess a new value for the  $k_{overall}$  and repeat the procedure until  $Q_{overall} = Q_a$ .

An Excel Spread sheet calculation based program was developed employing this mathematical model. The algorithm of this program for estimation the total thermal resistance and the calculation of temperatures at casing and cement sheaths are also provided in the form of block diagram in Fig. 2.

#### IV. NUMERICAL SIMULATION

To validate the program developed based on proposed simple semi-analytical model, numerical simulation study has been carried out using multipurpose widely excepted commercial flow modeling software package ANSYS Fluent.

The purpose of this study is not only to validate the accuracy of the proposed model; it is also to justify how effectively the proposed model can be used to solve similar problems by saving long computational time which is not desirable by the industry for a routine engineering calculations.

#### V. INTRODUCTION TO ANSYS FLUENT

ANSYS Fluent numerical simulator has broad capabilities to model flow, turbulence, heat transfer, etc. for wide range of industrial applications including flow of fluid and heat flow through wellbore and its surroundings. Main module of this simulator that deals with flow of fluid and heat flow related problems is Fluent, which has been embedded within ANSYS Fluent package. However, Fluent module was used to simulate heat loss (gain) through (from) wellbore surrounding area and realize the semi-analytical method which is developed by this work.

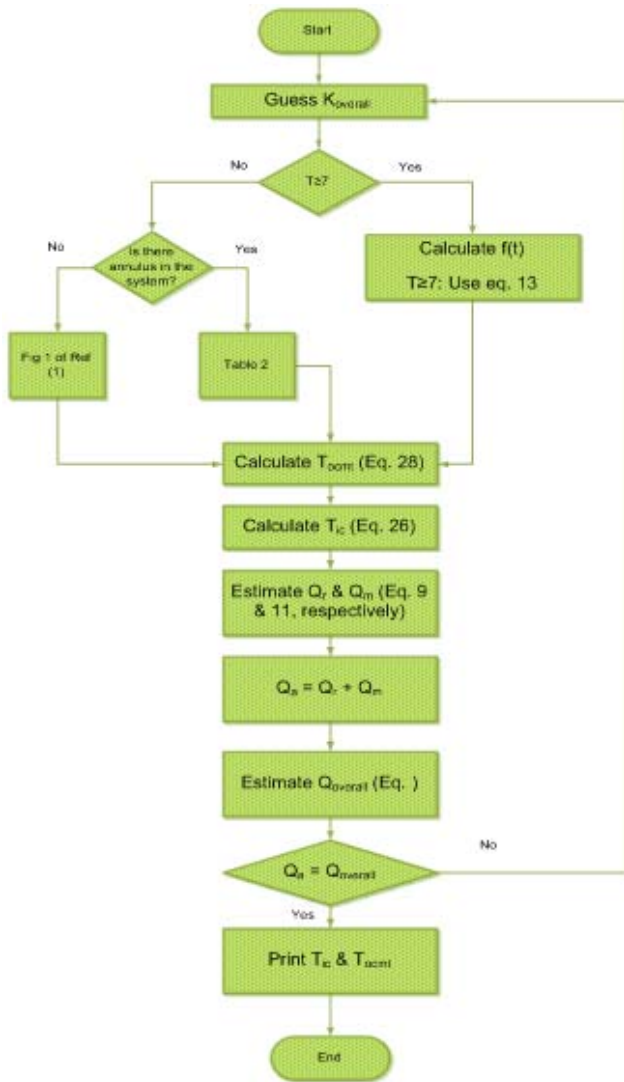


Fig. 2 Heat transfer calculation algorithm

TABLE I  
 TIME FUNCTION F(T) FOR THE RADIATION BOUNDARY CONDITION MODEL

$\frac{r_{to} U_{to}}{K_g} \frac{at}{r_{cmt}^2}$	0.01	0.02	0.05	0.1	0.2	0.5	1	2	5	10	20	50	100	$\infty$
0.1	0.313	0.313	0.314	0.316	0.318	0.323	0.33	0.345	0.373	0.396	0.417	0.433	0.438	0.445
0.2	0.423	0.423	0.424	0.427	0.43	0.439	0.452	0.473	0.511	0.538	0.568	0.572	0.578	0.588
0.5	0.616	0.617	0.619	0.623	0.629	0.644	0.666	0.698	0.745	0.772	0.79	0.802	0.806	0.811
1	0.802	0.803	0.806	0.811	0.82	0.842	0.872	0.91	0.958	0.984	1	1.01	1.01	1.02
2	1.02	1.02	1.03	1.04	1.05	1.08	1.11	1.15	1.2	1.22	1.24	1.24	1.25	1.25
5	1.36	1.37	1.37	1.38	1.4	1.44	1.48	1.52	1.56	1.57	1.58	1.59	1.59	1.59
10	1.65	1.66	1.66	1.67	1.69	1.73	1.77	1.81	1.84	1.86	1.86	1.87	1.87	1.88
20	1.96	1.97	1.97	1.99	2	2.05	2.09	2.15	2.15	2.16	2.16	2.17	2.17	2.17
50	2.39	2.39	2.4	2.42	2.44	2.48	2.51	2.56	2.56	2.57	2.57	2.58	2.58	2.58
100	2.73	2.73	2.74	2.75	2.77	2.81	2.84	2.88	2.88	2.89	2.89	2.89	2.89	2.9

The ANSYS Fluent also provides complete mesh types including 2D and 3D; and mesh flexibilities including the ability to solve the heat and flow problems. Moreover, natural, forced and mixed heat convection mechanism; conjugate (fluid/solid) heat transfer; radiation heat transfer mechanism; transient and steady-state heat transfer conditions are some of the capabilities of the heat transmission problem solving of the Fluent software. Consequently, the ANSYS Fluent can be a powerful and reliable tool to validate the proposed semi-analytical model.

At the following, it can be seen that the general energy equation (12) used in ANSYS Fluent to solve different conditions of energy flow [18]. Section 5.2.1 of ANSYS manual [18] comprehensively describes the heat transfer theory used by ANSYS Fluent simulator including wide range of various form of energy terms such as pressure work, kinetic energy, viscous dissipation, diffusion, reaction, radiation, anisotropy conductivity, interphase energy source and energy equation in solid region. In this work, it is supposed that there are fluids into wellbore and annulus section and the other parts are solids. Therefore the solution is for the mixing of fluid and solid including different properties for each section.

$$\frac{\partial}{\partial t}(\rho E) + \nabla \cdot (\vec{\vartheta}(\rho E + P)) = \nabla \cdot (k_{eff} \nabla T - \sum_j h_j \vec{J}_j + (\overline{\tau_{eff} \cdot \vec{\vartheta}})) + S_h \quad (12)$$

where terms  $k_{eff}$ ,  $h_j$ ,  $J_j$  and  $v$  show effective conductivity, enthalpy, diffusion flux and kinematic viscosity of the desired control system, respectively. Also, the energy transfer due to conduction, diffusion and viscous dissipation are explained by the first three terms of right hand side of (12). In this equation, the term  $S_h$  describes any heat exchange due to chemical reaction and other volumetric heat sources.

## VI. NUMERICAL MODELING

Specification of geometry properties, material definition and meshing are conducted at the pre-processor level in ANSYS / Design Modular. In this stage of work, a two dimensional (2D) geometry has created and meshed. Fig.3 shows a schematic designed and meshed by Design Modular for wellbore heat calculation including material properties of each layer. Also, all definitions which are necessary to define boundary conditions can be seen in Fig. 4. The proposed model and definitions of which are considered for ANSYS Fluent simulation are the same as those for semi-analytical model. Moreover, ANSYS Fluent post-processing tools can also be used as easy tools to create meaningful graphics and reports. For further data analysis, case and data file can be read by other software. Therefore as it can be seen on the Figs. 5 – 11, the result of the semi-analytical spread sheet developed by this model and the fluent software are compared easily.

TABLE II  
 CALCULATION DATA

Tubing	$r_{it} =$	0.14	Ft
	$r_{ot} =$	0.146	Ft
Casing	$r_{ic} =$	0.355	Ft
	$r_{oc} =$	0.4	Ft
Cement	$r_{ocmt} =$	0.5	Ft
Cement thermal conductivity	$K_{cmt} =$	0.2	Btu/hrsqt F/ft
Earth thermal conductivity	$K_g =$	1	Btu/hrsqt F/ft
Earth thermal diffusivity	$\alpha =$	0.0286	sqft/hr
Tubing surface emissivity	$\epsilon_{tbg} =$	0.9	
Casing surface emissivity	$\epsilon_{csg} =$	0.9	
Heat capacity of annulus fluid	$C_p =$	0.245	Btu/lb F
Fluid density of annulus fluid	$\rho_a =$	0.0388	lb/cu ft
Fluid viscosity of annulus fluid	$\mu_a =$	6.90E-02	lb mass/fthr
Thermal conductivity of annulus fluid	$K_a =$	0.0255	Btu/hrsqt F/ft

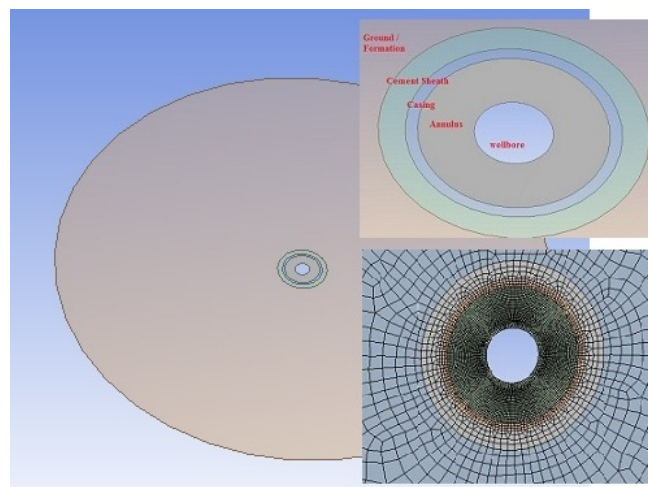


Fig. 3 Wellbore schematic which is designed and meshed by ANSYS/Design Modular

## VII. CALCULATION

When a mesh has been read into Fluent, all operations such as setting boundary conditions, defining fluid properties, executing the solution, refining the mesh, post-processing and viewing the results are executed within Fluent. Optional inputs also allow the user to specify different sources or fixed values such as temperature, mass, flow and so on. In this work, the temperatures at tubing ( $T_t$ ) and ground/formation ( $T_g$ ) are known, and selected as boundary condition. The thermal and physical properties of each layer have been defined based on the data provided in Table II. The results of different runs of



this work by ANSYS Fluent can be seen in the Figs. 3 – 11. Fig. 5 and 8 also show the transient heat transmission profile around the desired wellbore.

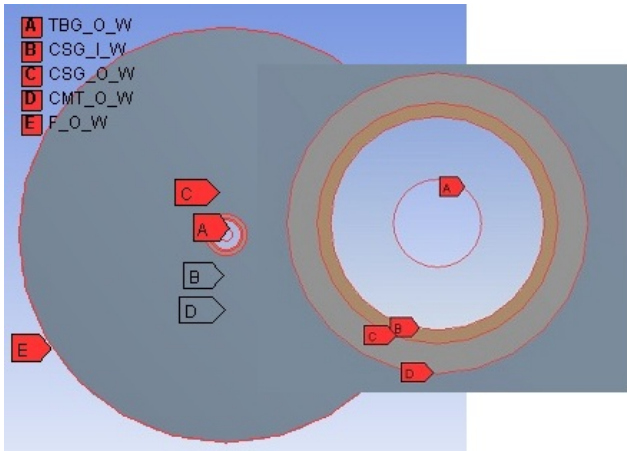


Fig. 4 Wellbore schematic including boundaries definition

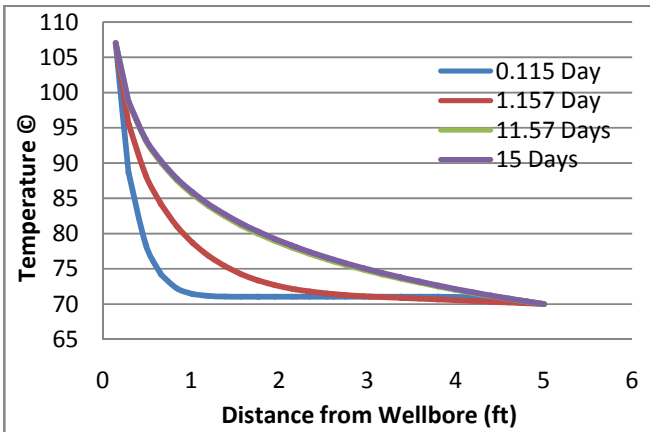


Fig. 5 Transient temperature profile around a wellbore for a simple case study

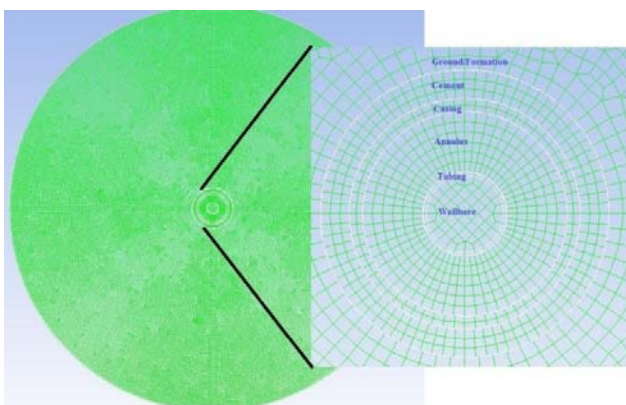


Fig. 6 Wellbore schematic surrounded by different layers

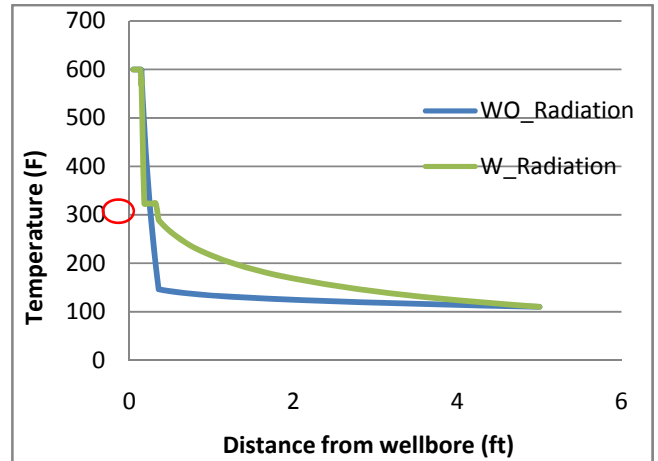


Fig. 7 The comparison of heat loss around a wellbore with- and without- the application of radiation heat transfer mechanism through annulus

### VIII. VALIDATION

For the purpose of studying the temperature loss (gain) to the media around a wellbore, several simulation runs were performed. In order to validate the proposed semi-analytical model, a series of the spread sheet results has compared with the ANSYS Fluent results which are designed for a wellbore surrounded with a multi-layer media including tubing, annulus, casing, cement and earth/formation. So, analysis of the results of semi-analytical model and numerical simulation using ANSYS Fluent are lead to the following discussion and validation.

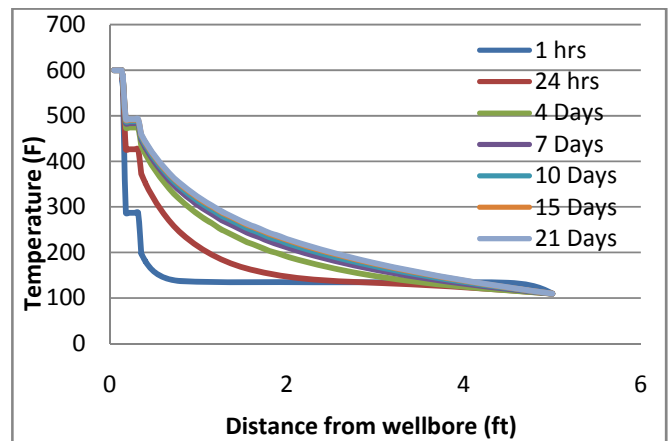


Fig. 8 Transient temperature profile around a wellbore

Fig. 5 demonstrates the recalculations using ANSYS Fluent for the case studied in [17], and extend it for transient temperature profile estimations. As it can be seen, the heat loss mechanism reaches steady state condition after around 8 days producing from a well. The figure also shows some additional calculations made after 0.115 and 1.157 days production to predict the transient temperature profiles.

In a real case, the wellbore might be surrounded by different layers of tubing, casing, cement sheaths and ground/formation

(Fig. 3). In such a complex case, there is an annulus filled with a fluid that makes complex process of heat loss around a wellbore. For example, radiation heat transfer mechanism is added to the convection heat transfer mechanism. Fig. 7 predicts the effect of radiation heat transfer mechanism using one dimensional steady-state condition. The red circle on this figure shows the temperature of annulus fluid which is the same at a distance of 0.18 - 0.285ft far from the center of wellbore.

Starting, shutting in and starting again of a producing/injecting well might be a frequent procedure of production process. So, transient time specifies the processes of heat loss (gain) reached to the steady-state condition. Regarding this matter, Fig. 8 predicts transient temperature profiles, which provide the process of heat loss from wellbore to the surrounding media from the production starting time to the steady-state condition. In this Figure, all estimations are included the radiation effect through the annulus section.

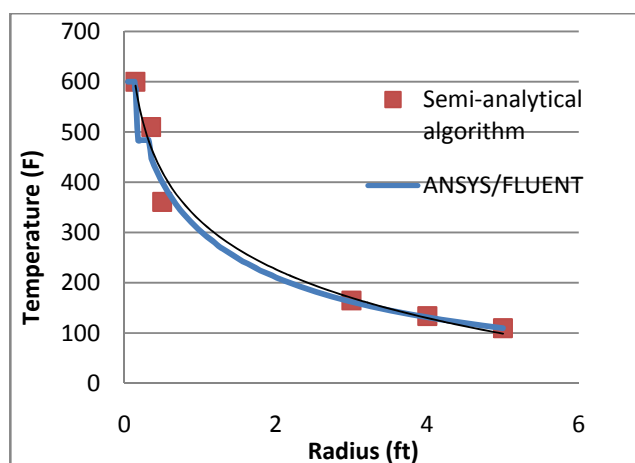


Fig. 9 The comparison of temperature profile of this work and FLUENT results after 7 Days producing

Finally, to validate the developed semi-analytical spread sheet results, Fig. 9 – 11 compare the estimations with the ANSYS Fluent results. The boundary conditions for both methods of calculations were kept the same. As it can be seen, the results obtained from the semi-analytical algorithm have shown a close match with results obtained by ANSYS Fluent simulator. But the computation time and required memory for semi-analytical method are extremely less as compared to simulation required by ANSYS Fluent simulator. In addition to computation time, physical modeling part is also utterly time consuming, and requires very cumbersome skilled efforts, which restricts the use of such numerical simulator for routine industry works. Therefore, the developed semi-analytical method can be applied to predict the temperature profile around a well surrounded with different complex layers with reasonable accuracy without going through cumbersome and time consuming physical modeling works.

The calculation was presented for single point in the wellbore to demonstrate the process of calculation procedure. This procedure can be repeated at different point within the

wellbore to obtain the temperature distribution of a producing and injection well, and so thus to generate the temperature log for a given reservoir.

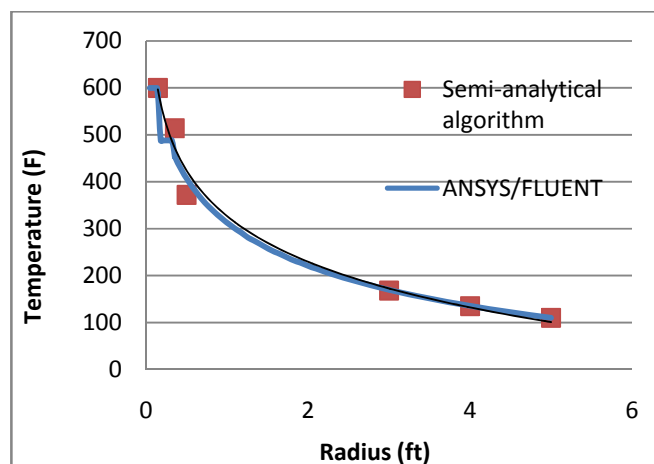


Fig. 10 The comparison of temperature profile of this work and FLUENT results after 10 Days producing

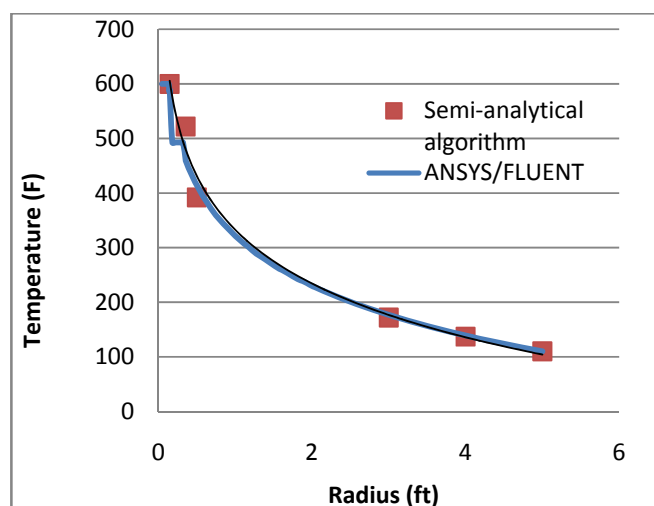


Fig. 11 The comparison of temperature profile calculated by this work and FLUENT results after 21 days producing

## IX. CONCLUSION

The temperature log has been used to diagnose injection/production problems; to obtain a qualitative indication of the fluid distribution along the wellbore; and to identify the root causes for many anomalies encountered during production/injection process. In this work, a simple semi-analytical solution spread sheet based programming is developed as a tool to predict temperature profile along a wellbore. The developed model includes an overall heat transfer coefficient accounting all modes of heat transferring process. For the purpose of studying the temperature loss (gain) to the media around a wellbore, several simulation runs were performed. It is demonstrated that proposed model can be used as a powerful tool to predict temperature profile along a production/injection wellbore surrounded with a multi-layer

media including tubing, annulus, casing, cement and earth/formation.

#### NOMENCLATURE

$A_{ic}$  = Inside surface area of casing, ft<sup>2</sup>  
 $A_{ot}$  = Outside surface area of tubing, ft<sup>2</sup>  
 $C_p$  = Heat capacity, Btu/(lb. F)  
 $f(t)$  = Ramey's transient time function, dimensionless  
 $h_{avg}$  = Average heat transfer coefficient, Btu/(hr. ft<sup>2</sup>. °F)  
 $h_{cv}$  = Convection heat transfer coefficient of annulus fluid, Btu/(hr. ft<sup>2</sup>. °F)  
 $h_j$  = Enthalpy  
 $h_w$  = Convection heat transfer coefficient of wellbore fluid, Btu/(hr. ft<sup>2</sup>. °F)  
 $J_j$  = Diffusion flux  
 $k$  = Thermal conductivity coefficient of pipe, btu/(hr. ft. °F)  
 $k_a$  = Thermal conductivity coefficient of annulus fluid, Btu/(hr. ft. °F)  
 $k_c$  = Thermal conductivity coefficient of casing, Btu/(hr. ft. °F)  
 $k_{cmt}$  = Thermal conductivity coefficient of cement sheath, Btu/(hr. ft. °F)  
 $K_{eff}$  = Effective conductivity  
 $k_g$  = Thermal conductivity coefficient of ground/formation, btu/(hr. ft. °F)  
 $k_{overall}$  = Overall thermal conductivity coefficient, Btu/(hr. ft. °F)  
 $k_p$  = Thermal conductivity coefficient of pipe, btu/(hr. ft. °F)  
 $k_t$  = Thermal conductivity coefficient of tubing, Btu/(hr. ft. °F)  
 $P$  = Pressure, psi  
 $Q$  = Heat flow rate, Btu/hr  
 $Q_r$  = Radiation heat flow rate in annulus, Btu/hr  
 $Q_{cv}$  = Convection heat flow rate in annulus, Btu/hr  
 $q$  = Heat flow rate, Btu/hr  
 $R_a$  = Thermal resistance of annulus, 1/k<sub>a</sub>  
 $R_c$  = Thermal resistance of casing, 1/k<sub>c</sub>  
 $R_{cmt}$  = Thermal resistance of cement sheath, 1/k<sub>cmt</sub>  
 $R_g$  = Thermal resistance of ground/formation, 1/k<sub>g</sub>  
 $R_{total}$  = Total thermal resistance, 1/k<sub>overall</sub>  
 $R_t$  = Thermal resistivity coefficient of tubing, 1/k<sub>t</sub>  
 $R_w$  = Thermal resistance of wellbore, 1/k<sub>w</sub>  
 $r_{ic}$  = Inside radius of casing, ft  
 $r_{icmt}$  = Inside radius of cement sheath surface, ft  
 $r_{ip}$  = Inside radius of pipe, ft  
 $r_{it}$  = Inside radius of tubing, ft  
 $r_{oc}$  = Outside radius of casing, ft  
 $r_{ocmt}$  = Outside radius of cement sheath surface, ft  
 $r_{op}$  = Outside radius of pipe, ft  
 $r_{ot}$  = Outside radius of tubing, ft  
 $r_m$  = Mean radius of annulus, ft  
 $S_h$  = Heat exchange due to chemical reaction  
 $T_g$  = Ground/formation temperature, °F  
 $T_{ic}$  = Temperature at inside of casing surface, °F  
 $T_{icmt}$  = Temperature at inside of cement sheath surface, °F  
 $T_{it}$  = Temperature at inside of tubing surface, °F  
 $T_{ocmt}$  = Temperature at outside of cement sheath surface, °F  
 $T_{ot}$  = Temperature at outside of tubing surface, °F  
 $T_w$  = wellbore temperature, °F

$\Delta T_{overall}$  = Overall temperature difference, °F  
 $t$  = Time, hr  
 $U_{overall}$  = Overall heat transfer coefficient, °F  
 $\epsilon_c$  = Casing emissivity, dimensionless  
 $\epsilon_t$  = Tubing emissivity, dimensionless  
 $\sigma$  = Stefan-Boltzmann constant, Btu/(hr. ft<sup>2</sup>. °R)  
 $\delta$  = Increment length, ft  
 $\mu$  = Viscosity, lb mass/(ft. hr)  
 $\nu$  = Kinetic viscosity, ft<sup>2</sup>/s  
 $\beta$  = Thermal volumetric expansion coefficient of annulus fluid, 1/°R  
 $\rho_a$  = Density of annulus fluid, lb/ft<sup>3</sup>  
 $g$  = gravity, ft/hr<sup>2</sup>  
 $\Delta L$  = Increment length, ft  
 $\Delta z$  = Increment depth, ft  
 $Nu_\delta$  = Nusselt number, dimensionless  
 $Ra$  = Rayleigh number, dimensionless  
 $Pr$  = Prandtl number, dimensionless  
 $Gr$  = Grashof number, dimensionless

#### APPENDIX A

##### A. Convective Heat Transfer between Flowing Fluid and Innermost Pipe

The heat transferred between flowing fluid and inside tubing wall is given by the following heat convection equation.

$$Q = 2\pi r_{it} h_w (T_f - T_{it}) \Delta L \quad (A1)$$

Therefore;

$$R_w = \frac{1}{2\pi r_{it} \Delta L h_w} \quad (A2)$$

The term  $h_w$  is the heat transfer film coefficient which is a function of the flowing fluid properties. For a two phase flow, the term  $h_w$  is dependent to the flow regime.

##### B. Conductive Heat Transfer Equation for Pipes

The steady-state, radial and one-dimensional heat conduction equation for pipes is expressed by:

$$R_p = \frac{\ln\left(\frac{r_{op}}{r_{ip}}\right)}{2\pi \Delta L k_p} \quad (A3)$$

Tubing resistivity modifications are required to account for the build-up of scale or paraffin on the pipe wall.

##### C. Heat Transfer Mechanism through Annulus

The total heat flux through annulus (between outside of tubing and inside of casing surfaces) may explain by:

$$Q = Q_r + Q_{cv} = 2\pi r_m h_{avg} (T_{ot} - T_{ic}) \Delta L \quad (A4)$$

At the following the step by step procedure of  $Q_r$  and  $Q_{cv}$  estimations can be seen.



### C1. Radiation Heat Transfer Mechanism ( $Q_r$ )

In case of annulus around a wellbore, because of tubing and casing physical properties, radiation heat transfer mechanism may apply. Therefore, the radiation heat exchange between tubing and casing may calculate in [10]:

$$Q_r = \frac{\sigma A_{ot}(T_{ot}^4 - T_{ic}^4)}{\frac{1}{\epsilon_t} + \left(\frac{A_{ot}}{A_{ic}}\right)\left(\frac{1}{\epsilon_c} - 1\right)} \quad (A5)$$

The area ratio  $\frac{A_{ot}}{A_{ic}}$  of (8) may be replaced by the radius ratio  $\frac{r_{ot}}{r_{ic}}$ . Therefore, the heat exchange per unit area of pipe may rewrite as following:

$$Q_r = \frac{\sigma(T_{ot}^4 - T_{ic}^4)}{\frac{1}{\epsilon_t} + \left(\frac{r_{ot}}{r_{ic}}\right)\left(\frac{1}{\epsilon_c} - 1\right)} \quad (A6)$$

where the term  $\sigma$  is the Stefan-Boltzmann constant. The terms  $\epsilon_t$  and  $\epsilon_c$  refer to the emissivity of tubing and casing, respectively.

$$\sigma = 5.669 \times 10^{-8} \text{ (W/m}^2 \cdot \text{K}^4) = 0.1714 \times 10^{-8} \text{ (Btu/h.ft}^2 \cdot \text{R}^4) \quad (A7)$$

### C2. Convection Heat Transfer mechanism ( $Q_{cv}$ )

The heat transfer through annulus is a function of temperature difference between tubing and casing, the gap width and height of annulus and the fluid properties such as viscosity, thermal capacity and thermal conductivity (8). Therefore:

$$Q_{cv} = 2\pi r_m h_{cv}(T_{ot} - T_{ic})\Delta L \quad (A8)$$

The convection heat transfer coefficient is explained by:

$$h_{cv} = \frac{K}{\delta} Nu_{\delta} \quad (A9)$$

$Nu_{\delta}$  is the Nusselt number which is defined by:

$$Nu_{\delta} = 0.049 Ra^{0.333} Pr^{0.074} \quad (A10)$$

where (A10) is valid for the range of  $5 \times 10^4 < Ra < 7.17 \times 10^4$

$Ra$ ,  $Pr$  and  $Gr$  are the Rayleigh, Prandtl and Grashof numbers, respectively.

$$\begin{aligned} Ra &= Gr \cdot Pr \\ Pr &= \frac{c_p \mu}{k} \\ Gr &= \frac{g \beta (T_{ot} - T_{ic}) \delta^3}{\nu^2} \end{aligned} \quad (A11)$$

where  $\beta$  is the volume coefficient of expansion,  $c_p$  is the thermal capacity,  $\mu$  is the dynamic viscosity and  $\nu$  is the kinematic viscosity of the annulus fluid.

Also,  $r_m$  is the mean area for cylindrical annulus defined by:

$$r_m = \frac{r_{ic} - r_{ot}}{\ln(r_{ic}/r_{ot})} \quad (A12)$$

Therefore, as a result of (A4–A12), the resistance to radiation, natural convection and conduction heat transfer through the annulus may explain by:

$$R_a = \frac{1}{2\pi r_m \Delta L h_{avg}} \quad (A13)$$

### D. Heat Transfer Mechanism Through cement Sheath

The steady-state, radial and one-dimensional heat transferred per unit surface area between the outer surface of the last casing and the cement sheath may apply by:

$$Q = \frac{2\pi k_{cmt}}{\ln\left(\frac{r_{ocmt}}{r_{oc}}\right)} (T_{ocmt} - T_{oc}) \quad (A14)$$

Therefore, the resistance of conductive heat transfer through the ground may explain by:

$$R_{cmt} = \frac{\ln\left(\frac{r_{ocmt}}{r_{oc}}\right)}{2\pi \Delta L k_g} \quad (A15)$$

### E. Heat Transfer Mechanism Through Ground

The process of heat transfer through ground may be the same as for cement sheath. Therefore:

$$Q = \frac{2\pi k_g}{\ln\left(\frac{r_g}{r_{ocmt}}\right)} (T_{ocmt} - T_g) \quad (A16)$$

and:

$$R_g = \frac{\ln\left(\frac{r_g}{r_{ocmt}}\right)}{2\pi \Delta L k_g} \quad (A17)$$

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