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Estimating Correlation Dimension on Japanese Candlestick, Application to FOREX Time Series

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***Abstract*—**Recognizing behavioral patterns of financial markets is essential for traders. Japanese candlestick chart is a common tool to visualize and analyze such patterns in an economic time series. Since the world was introduced to Japanese candlestick charting, traders saw how combining this tool with intelligent technical approaches creates a powerful formula for the savvy investors.

This paper propose a generalization to box counting method of Grassberger-Procaccia, which is based on computing the correlation dimension of Japanese candlesticks instead commonly used 'close' points. The results of this method applied on several foreign exchange rates vs. IRR (Iranian Rial). Satisfactorily show lower chaotic dimension of Japanese candlesticks series than regular Grassberger-Procaccia method applied merely on close points of these same candles. This means there is some valuable information inside candlesticks.

***Keyword*—**Chaos, Japanese candlestick, generalized box counting, strange attractor.

I. Introduction

DURING the last 25 years time series analysis has become one of the most important and widely used branches of mathematical statistics.

To simply visualize and analyze the behavior of an economic time series a common tool is the Japanese candlestick chart which is widely applied in modern technical analysis methods, what traders refer to as the candlestick patterns implicitly reveals the existence of some dynamical system behind all economic time series [1].

In order to test for chaos, two quantities may be derived from a time series. Firstly, one can estimate the correlation dimension measuring the fractal nature of a possibly underlying strange attractor. The main classic method in this approach is the method of Grassberger and Procaccia [2].

Secondly, one can estimate the largest Lyapunov exponent which, when found to be positive, measures the sensitive dependence on initial conditions so characteristic of a chaotic system [3].

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This paper introduces an extension to this method, which computes the correlation dimension of Japanese candlesticks instead of correlation dimension of commonly used 'close' points of the economic time series, the results show lower chaotic dimension which implies a less complex dynamical system behind the tested macro-economical time series, a lower dimension in a dynamical system is satisfying. The rest of this paper is organized as follow: Section II explains the economic dynamic, section III is about specification of chaotic processes. Section IV talks about Foreign exchange rate section V introduces Japanese candlesticks. Section VI explains attractor dimension. Section VII demonstrates the new method suggested by this paper, results are presented in section VIII. Remarks, conclusion, and consequently references are taken at the end.

II. Economic Dynamics

Historically, economists have, whenever possible, used linear equations to model economic phenomena, because they are easy to manipulate and usually yield unique solutions. However, as the mathematical and statistical tools available to economists have become more sophisticated, it has become impossible to ignore the fact that many important and interesting phenomena are not amenable to such treatment.

There is a strong support in economics for both the significance of linear models, and the advantages of nonlinear models. But nonlinear models clearly outperform linear models. Clearly the economic world is nonlinear, so it would appear that focusing on linear dynamics is of limited interest. However economists have typically found nonlinear models to be so difficult and intractable that they have adopted the technique of linearization to deal with them.

Important phenomena for which linear models are not appropriate include depressions and recessionary periods, stock market price bubbles and corresponding crashes, persistent exchange rate movements and the occurrence of regular and irregular business cycles. Therefore, economic theorists are turning to the study of non-linear dynamics and chaos theory as possible tools to model these and other phenomena [4].

The most exciting feature of nonlinear systems is their ability to display chaotic dynamics. Much economic data has this random-like behavior, but it comes from agents and markets that are presumably rational and deterministic. Random-like data that economists often encounter might not be coming from a random system. The generating system could be deterministic and perhaps the economy can be explained by a relatively simple nonlinear system.

Chaos is widely found in the fields of physics and other natural sciences. However, the existence of chaos in economic data is still an open question. Since the mid eighties several economists have tried to test for nonlinearity and in particular for chaos in economic and financial time series [5]. One route toward finding a nonlinear underlying system in the economy would be to show that the data itself demonstrates nonlinear or chaotic properties Researchers developed tests for chaos and nonlinearity in data. There are two major classes of tests for chaos within data. The first is ways to look at the paths or trajectories of the data when the system's initial conditions are adjusted slightly. This can be done by estimating a Lyaponuv exponent.

The Lyaponuv exponents are a measure of the average divergence (or convergence) between experimental data trajectories generated by systems with infinitesimally small changes in their initial conditions. If the data points deviate exponentially when there is a very small tweak on a deterministic model, it will have a positive Lyaponuv exponent. If the paths converge back to a steady state, then the Lyaponuv exponent will be negative. A positive exponent signals that the system must have sensitive dependence to initial conditions and therefore it is chaotic [6]. The second type of test for chaos examines the dimensionality of the system. It may seem easy to explain that a square has two dimensions, and a line has one, but it is significantly more complicated for chaotic systems since they have non-integer dimensionality. The "fractional" dimensionality is what coined the term "fractal" for shapes generated by chaotic data. Dimensionality analysis becomes extremely complicated with realistic chaotic systems. Chaos exists in many different fractal dimensions. Unfortunately, the analysis gets more and more complex the larger the fractal dimension being searched in, and the tests for chaos become weaker. Unless data displays low-dimensional chaos, it may be undetectable to current tests. This is a significant obstacle to chaotic-economic theory, and one of the main reasons the literature has not reached a consensus on the existence of chaotic dynamics in data [7].

III. Specifications of Choatic Processes

Chaos theory include bases like butterfly effect and strange attractors.

A. Butterfly Effect

Edward Lorenz professor of meteorology in MIT university in 1973 published results of calculations of differential equations apparatus comprising 3 equations of non-linear and definite related to thermo displacement in atmosphere and observed that in a defined vicinity of equations factors, without interference of random elements or entrance of exogenous shocks, some kind of irregular fluctuations in reply of system is revealed. He in continuance of his survey surprisingly reached to this conclusion that a minor change in primary conditions of equation for forecasting atmosphere condition will culminate in fluctuations in system reply and drastic changes in obtained results from them. To this concept that for example if a butterfly files in Beijing it is possible that due to flying of this butterfly a cloud would move and a storm in New York will come to existence. He called this entity as butterfly effect [3]. Butterfly effect in effect disapproves linear relations between cause and effect and confirms non-linear relations amongst entities and systems. This means that a minor change in primary conditions could possibly ended to vast and unpredictable results in system output. This is the milestone of chaos theory. In chaos theory or disorder it is believed that in all entities, there exist points that a slight change in them would cause extreme changes and in this connection economical, political, sociological and organizational systems like atmospherical systems have butterfly effect and analysts should analyze and organize related issues with knowledge of this important point.

B. Strange Attractors

Consider a particle that is moving in space based on a definite law. If we follow the course of this particle we may see four types of different behaviors:

1. The particle ultimately will stop in a special place.
2. The particle ultimately will have an oscillating movement between 2 or more points.
3. The particle in its future motion will have a random behavior.
4. The last possibility is that the particle will continue its irregular and unpredictable movement. But it will remain in a limited part of space. In this condition a slight carelessness in measuring current proper position of particle culminates in big mistakes in foreseeing its next course. This is a clue of chaotic movement. Geometrical description of ultimate condition and behavior of the particle in above example idiomatically is called attractor. In 1st condition attractor is a point. In 2nd condition attractor could be a circle, a square, an oval or an irregular complicated circuit. But considering that the part of space in witch 4th condition is used is highly sophisticated, it is called strange attractor. When a particle is attracted to a strange attractor, there is no room for escaping and although particle's movement inside attractor is specified with precise regulations but particle behaves in a way that it has random motion [4]. It is possible to think that for making such complicated movements, sophisticated equations are necessary. But on the contrary new insights that have been acquired in recent years shows that chaotic motion mostly abides by simple regulations [2].

IV. Foreign Exchange Rate

An exchange rate represents the value of one currency in another. An exchange rate between two currencies fluctuates over time.

The foreign exchange market is the largest and most liquid of the financial markets. Foreign exchange rates are amongst the most important economic indices in the international monetary markets. The forecasting of them poses many theoretical and experimental challenges.

Foreign exchange rates are affected by many highly correlated economic, political and even psychological factors. The interaction of these factors is in a very complex fashion. Therefore, to forecast the changes of foreign exchange rates is generally very difficult. Researchers and practitioners have been striving for an explanation of the movement of exchange rates. Thus, various kinds of forecasting methods have been developed by many researchers and experts. Technical and fundamental analyses are the basic and major forecasting methodologies which are in popular use in financial forecasting. Like many other economic time series, forex has its own trend, cycle, season, and irregularity. Thus to identify, model, extrapolate and recombine these patterns and to give forex forecasting is the major challenge.

Foreign exchange rates were only determined by the balance of payments at the very beginning. The balance of payments was merely a way of listing receipts and payments in international transactions for a country. Payments involve a supply of the domestic currency and a demand for foreign currencies. Receipts involve a demand for the domestic currency and a supply of foreign currencies. The balance was determined mainly by the import and export of goods. Thus, the prediction of the exchange rates was not very difficult at that time. Unfortunately, interest rates and other demand} supply factors had become more relevant to each currency later on. On top of this the fixed foreign exchange rates was abandoned and a floating exchange rate system was implemented by industrialized countries in 1973. Recently, proposals towards further liberalization of trades are discussed in General Agreement on Trade and Tariffs. Increased Forex trading, and hence speculation due to liquidity and bonds, had also contributed to the difficulty of forecasting Forex [8].

Generally, there are three schools of thought in terms of the ability to profit from the financial market. The first school believes that no investor can achieve above average trading advantages based on the historical and present information. The major theory includes the Random Walk Hypothesis and Efficient Market Hypothesis. The second school's view is that of fundamental analysis. It looks in depth at the financial condition of each country and studies the effects of supply and demand on each currency.

Technical analysis belongs to the third school of thought who assumes that the exchange rates move in trends and these trends can be captured and used for forecasting. It uses such tools as charting patterns, technical indicators and specialized techniques like Gann lines, Elliot waves and Fibonacci series [9].

V. Japanese Candlestick

Candlestick charts are said to have been developed in the 17th century by legendary Japanese rice trader Munehisa Honma. The charts gave Honma and others an overview of open, high, low, and close market prices over a certain period. This style of charting is very popular due to the level of ease in reading and understanding the graphs. Since the 17th century, there has been a lot of effort to relate chart patterns to the likely future behavior of a market. This method of charting prices proved to be particularly interesting, due to the ability to display four data points instead of one. Candlestick technical analysis was undiscovered in the western world until 1991 when Steve Nilsson published ‘‘Japanese Candlestick Charting Techniques: A Contemporary Guide to the Ancient Investment Techniques of the Far East’’. Since the western world was introduced to Japanese candlestick charting, traders saw how candlesticks could give them an edge by combining them with western technical analysis methods. Recognizing patterns in the stock market is a critical resource for today's trader. Combining candlestick charting techniques with traditional technical approaches creates a powerful formula for the savvy investor.

In the Japanese candlestick, one of the major elements is the body of the candlestick. The difference between the open and close prices makes a box which we call real body. A black body means the close is lower than the open, and a white body means the close is higher than the open(Figure 1). Usually there are extensions lines come out from the ends of the real bodies, they are called shadows. Those lines represent the high and low prices for the trading days [1].

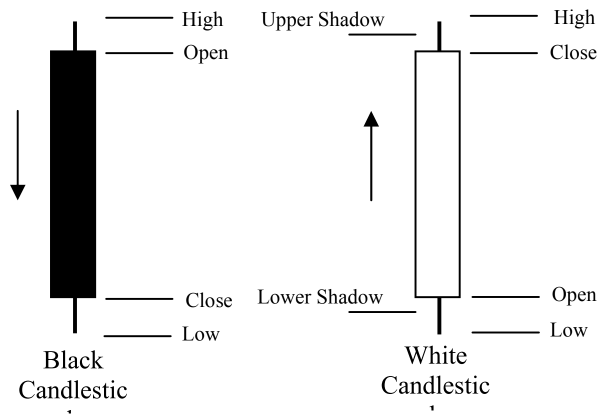

Fig. 1 Black and White Candlestick

While candle terminology may still sound like bizarre to some analysts, candlestick analysis is much more commonplace now than it was in the mid-1980s. In recent years, Western technicians, including stock traders and analysts, have come to appreciate the additional insights these visually distinct price charts can bring to market analysis. They believe that there are many similarities in candlestick patterns, and candlestick is now known as an important weapon and tool in trading arsenal. Figure 2 shows an instance of this candlesticks and corresponding closing values in one pass of Forex time series of USD vs. 10RIAL used in this research. The Iranian Rial (IRR) is the currency of Iran. Although not an official currency since 1932, the Toman (10 Rial) is frequently used to express amounts of money. It enjoys wide usage among Iranians today as an amount of ten Rial.

The length of the real body and whether the close is higher or lower than the open reveal some patterns for technical analyzers. These facts about the power of candlestick are main motivations to use a candle instead of merely single points in computing chaotic dimension in economic time series.

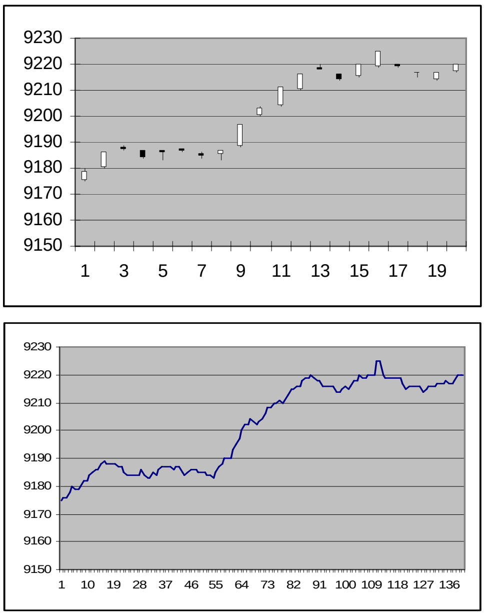

Fig. 2 Japanese candle stick chart for 140 day of used data and corresponding close values

VI. Attractor Dimension Test

This test is based on one of the special specifications of random process in comparison with chaotic process. Random processes include unlimited dimensions. But a chaotic process has more limited dimensions. It means that it includes a complex of points that time series would result in them. Therefore by calculating dimensions of a series, it is possible to understand its making process. According to this method if series domain was high, it would show a random process, otherwise, it would be a chaotic process. Attractor dimension by using a variable called integration correlation that was introduced by procaccia and grassberger in 1983 is calculated as follows [7].

Dimension is nominated as low bound of necessary independent variable's quantity for describing the model. Attractor is the developed concept of all equilibrium paths in phase space like equilibrium points and limit circles in stable systems that have got correct dimension. In contrast, chaotic systems attractor has fractal dimension and are called strange attractor. In most primitive method of designating of fractal dimension, M(L) are considered as quantity of ultra cubes with the dimension of M and length of the line "L" that covers attractor, based on this:

$$M(l) \sim l^{-D} \qquad (1)$$

And "D" that is fractal dimension is obtained:

$$D = \lim_{l \to 0} \frac{-\log[M(l)]}{\log l} \qquad (2)$$

According to definition, point, line and plate in 2 dimension space have dimensions of 0, 1 and 2 respectively. This definition of fractal has practical limits and solely attractor's geometrical structure has been considered in it.

Correlation dimension is the most common estimation of attractor dimension that is simply calculated by procaccia and grassberger method. Based on this method, vectors "m" are part of X_i condition of X(t) time series that are made with the length of "N".

$$X_i = [x(t_i), \quad x(t_i+1) \quad \cdots \quad x(t_i+m)] \qquad (3)$$

Correlation integral for "N" vector with the distance less than "r" from each other is calculated like this:

$$C(r,m) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \left\| X_i - X_j \right\|\right) \qquad (4)$$

In which C(r,m) is an estimation of a probability that 2 vectors of time series with length of N, have a distance less than "r" from each other. "I" also is a function of heavy side and is defined as below:

$$I(x,y) = \begin{cases} 0 & \|x-y\| > r \\ 1 & \|x-y\| < r \end{cases} \qquad (5)$$

In case quantities of "N" points are big enough, distribution is Exponential function

$$C(r;m) \sim r^{\upsilon} \qquad (6)$$

That υ is correlation dimension

$$\upsilon = \lim_{r \to 0} \frac{\log C(r,m)}{\log r} \qquad (7)$$

This test is based on this fact that chaotic maps do not fill the space in big dimensions, but random data are not like this. When the chaotic process is more complicated, it is necessary that data should be considered in bigger and higher dimensions. A chaotic process can fill a space with "n" dimensions but it leaves big holes in "n+1" dimension. It is clear that this method is not practical in big dimension graphically.

VII. Extending Grassberger & Procaccia's Method to Japanese Candlesticks

When the chaotic process becomes more complex, we need to look at the data in higher dimensions. A chaotic process can fill up the first n dimensions, but leave large 'holes' in the (n+1)th dimension. This section offers an extension to Grassberger and Procaccia's method. Say we are given data of a set of N quadric tuple candlesticks $z_1, z_2, \ldots, z_n$ on the attractor. The state vector Z_i is a m dimensional vector and is made of lagged candlesticks' of the time series:

$$Z_i = [z_{t_i}, z_{t_i+1}, \ldots, z_{t_i+m}] \qquad (8)$$

We want to estimate the natural measure of the attractor. Correlation Integral provides such an estimate:

$$C(r,m) = \lim_{N \to \infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \left\| Z_i - Z_j \right\|\right) \qquad (9)$$

Where U(0) is the heavy side step function, N is large and candlesticks are sampled at some fixed interval ΔT . In these conditions υ is exponentially proportional to correlation integral, i.e. $C(r,m) \propto r^{\upsilon}$. υ is the correlation dimension:

$$\upsilon = \lim_{r\to 0} \frac{\log C(r,m)}{\log r} \tag{10}$$

By increasing the controlling parameter m, υ converges to its actual value. Now we should define the norm $\| Z_i - Z_j \|$. In proposed method, we interpolate a candlestick with a cubic polynomial as (Fig. 3):

$$z_i \equiv f_i(t) = p_3 t^3 + p_2 t^2 + p_1 t + p_0 \tag{11}$$

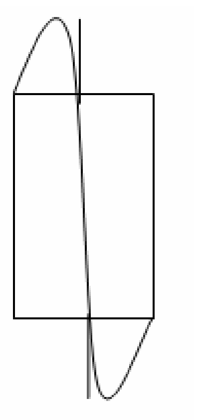

Fig. 3 Interpolate a candlestick with a cubic polynomial

Attention that this interpolation does not assure fixed t location of high and low of the candlestick but guarantees the maximum of the curve to be in 'high' and minimum to be in 'low'. Now we compute the Euclidian distance between two candles Z_i and Z_j by the following integral:

$$| z_i - z_j | = \sqrt{\int_0^{\Delta T} (f_i(t) - f_j(t))^2 dt} \tag{12}$$

Now the norm of $\| Z_i - Z_j \|$ is computed as following:

$$\| Z_i - Z_j \| = \sqrt{\sum_{k=0}^{m-1} \left(\int_0^{\Delta T} (f_{i+k}(t) - f_{j+k}(t))^2 dt \right)} \tag{13}$$

The kernel integral can be parametrically solved which can reduce the volume of computations.

$$C(r,m) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \left\| Z_i - Z_j \right\|\right) \tag{14}$$

Then we will have general equation:

$$C(r,m) = \lim_{N\to\infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \sqrt[Q]{\sum_{k=0}^{m-1} \left(\int_0^{\Delta T} (f_{i+k}(t) - f_{j+k}(t))^Q dt \right)} \right) \tag{15}$$

So, the correlation dimension is:

$$\lim_{r\to 0} \frac{\log \lim_{N\to\infty} \frac{1}{N^2} \sum_{i=1}^{N} \sum_{j=1}^{N} I\left(r - \sqrt[Q]{\sum_{k=0}^{m-1} \left(\int_0^{\Delta T} (f_{i+k}(t) - f_{j+k}(t))^Q dt \right)} \right)}{\log r} \tag{16}$$

VIII. RESULTS

This method is applied on foreign exchange rates data of day time slice sampled of some foreign exchange rate vs. 10RIAL, since March 5, 2002 to May 23, 2007. The results show lower fractal dimension of Japanese candlesticks series than regular Grassberger-Procaccia method applied merely on 'close' points of these same candles. Table I shows the results, a lower dimension of chaotic attractor means that we have less complex dynamics behind these macroeconomic systems.

TABLE I
COMPUTED DIMENSIONS

EUR	CHF	CAD	SEK	USD
9.56	8.84	9.28	7.36	8.3
8.44	8.56	8.4	6.95	7.91

AED	AUD	JPY100	NOK	GBP
7.04	9.2	8.12	7.28	9.84
6.98	8.72	6.78	6.99	9.65

IX. CONCLUSION AND FUTURE WORKS

In this paper, an improved method based on Japanese candlesticks for the estimation of embedding dimension is proposed. The more accurate estimation of embedding dimension is, better we can analyze the system and the lower the dimension is, the simpler model of system can be achieved. There are still some open issues ahead: some other neighborhood models can be defined for economic time series components to make the dimension estimator system intelligently detect the similarity between temporal patterns happening in different historical situations. This may also exclude the force of fixed size historical dependency.

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