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Multi-objective Optimization of Vehicle Passive Suspension with a Two-Terminal Mass Using Chebyshev Goal Programming

Chuan Li, Ming Liang and Qibing Yu

Abstract**—**To improve the dynamics response of the vehicle passive suspension, a two-terminal mass is suggested to connect in parallel with the suspension strut. Three performance criteria, tire grip, ride comfort and suspension deflection, are taken into consideration to optimize the suspension parameters. However, the three criteria are conflicting and non-commensurable. For this reason, the Chebyshev goal programming method is applied to find the best tradeoff among the three objectives. A simulation case is presented to describe the multi-objective optimization procedure. For comparison, the Chebyshev method is also employed to optimize the design of a conventional passive suspension. The effectiveness of the proposed design method has been clearly demonstrated by the result. It is also shown that the suspension with a two-terminal mass in parallel has better performance in terms of the three objectives.

Keywords**—**Vehicle, passive suspension, two-terminal mass, optimization, Chebyshev goal programming

I. Introduction

SUSPENSION is one of the important vehicle components and influences significantly on comfort, safety and maneuverability of the modern vehicles [1-3]. As a dynamics transmission system between the body and the tire, the conventional vehicle passive suspension can be regarded as a parallel combination of a damper and a spring. Though the mass is also one of the three basic vibration components, it has only one genuine manipulation terminal [4], and thus cannot be directly embedded into a passive suspension where two terminals are required to connect the body and the tire of the vehicle.

Mass components play important roles in dynamical systems including the suspension. Efforts have been made to improve the performance of the passive suspension exploiting inertial (mass) force. For example, a dynamic vibration absorber including a damper, a spring and a mass was mounted on the French subcompact Citroen 2 CV to reduce the wheel resonance without jeopardizing ride comfort [5]. However, the large added mass has limited the use of such absorbers in other types of cars. To reduce the gravitational mass while improving the inertial mass in a single-DOF passive vibration isolation system, Rivin proposed a screw transmission flywheel which is named as motion transformer [6]. A similar system, i.e., an inerter, was introduced by Smith et al. to resolve the synthesis issue of mechanical networks [7-9]. The motion transformer and the inerter are similar in that they both have two manipulation terminals for a flywheel. Later research on two-terminal mass has shown that other rectilinear or rotary mass components such as mass blocks could also realize a mass system with two manipulation terminals [10].

A two-terminal mass differs from traditional mass components such as mass block and flywheel because it has two manipulation terminals [11]. In addition, the two-terminal mass has much greater inertial mass compared to its gravitational mass. Hence it is suitable for practical applications. The two-terminal mass has been introduced for vibration isolation of the passive suspension and numerical simulation result shows that it contributes to better isolation performance [12]. The vibration isolation index, however, is a non-mainstream measure related to the vehicle suspension performance. Tire grip, passenger comfort and suspension deflection are three commonly used criteria to evaluate the suspension performance [13-15]. However, the parametric optimization results obtained separately based on one of the three criteria are often conflicting. Yet, as the three criteria are non-commensurable, it is impropriate or at least subjective to "optimize" the design based on the weighted sum of the three associated objective functions. The Chebyshev goal programming method is an effective multi-objective optimization approach [16, 17] in dealing with conflicting and non-commensurable objectives. It is therefore adopted in this study to find the optimal tradeoff of the three design objectives.

The rest of this paper is organized as follows. Section II briefly introduces the two-terminal mass and its application to the passive suspension. The dynamics model of the suspension is also proposed in this section. Section III proposes the state-space model of the suspension governing equation for the solution of the three single-objective issues. The Chebyshev method for multi-objective suspension optimization is described in detail in this section. The comparison between the

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proposed and the conventional designs is reported in section IV. Conclusions are given in section V.

II. Influence of Suspension Parameters on Dynamical Performances

A. Two-Terminal Mass Basics

The two-terminal mass is a mass component or device characterized by the facts that [11]: a) it contains two free, genuine and controllable terminals and none of which is necessary to be fixed to the ground; b) the relative force between two terminals is proportional to the second-order time derivative of the relative displacement between the two terminals of the component; and c) the inertial mass of the component is usually much greater than the gravitational mass of the mass core.

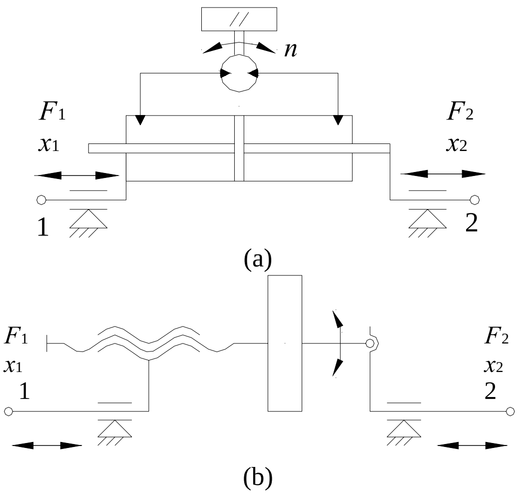


Fig. 1 Kinetic sketches of two kinds of two-terminal masses: (a) a hydraulic flywheel type two-terminal mass, and (b) a screw flywheel type component

Fig. 1 displays two types of the two-terminal mass devices designed and manufactured by the Engineering Laboratory of Detection, Control and Integrated System, Chongqing Technology and Business University. The first one is a hydraulic flywheel prototype shown in Figure 1(a). Letting S denote the cross sectional area of the cylinder (rod not included), n the displacement of the hydraulic motor, r the radius of the cylindrical flywheel, m gravitation mass, and F relative force, one has

$$F = m\left(\frac{rS}{\sqrt{2}n}\right)^2 \tag{1}$$

With reference to the second two-terminal mass device shown in Fig. 1(b), let D denote the pitch diameter of the screw, and α, the helix angle. The relative force between two terminals of a screw flywheel is [10]

$$F = m\left(\frac{\sqrt{2}r}{D\tan\alpha}\right)^2 \ddot{x} \tag{2}$$

Comparing (1) and (2) with $F = m\ddot{x}$ described by Newton's Second Law indicates that the gravitational mass m of the flywheel is magnified to inertial masses $m_t = m(rS/(\sqrt{2}n))^2$ and $m_t = m(\sqrt{2}r/(D\tan\alpha))^2$ respectively, due to the transforms of the hydraulic and screw transmissions. Due to this mass magnification effect, the two-terminal mass is suggested to connect in parallel with the suspension strut, which will be introduced in the following subsection.

B. Passive Suspension with a Two-terminal Mass in Parallel

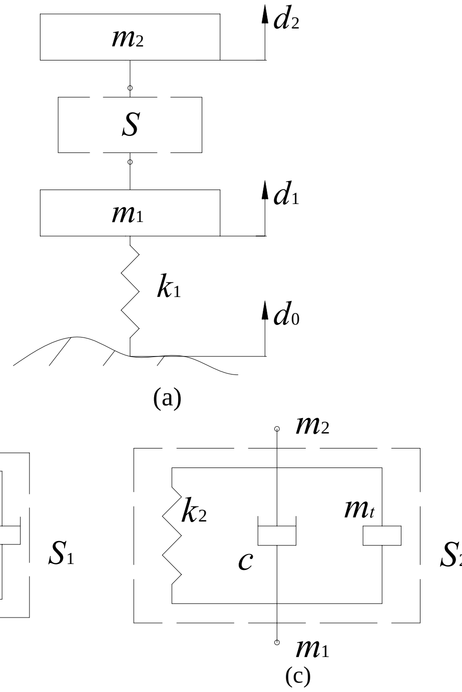


Fig.2 (a) 1/4 car model, (b) the conventional suspension S_1, and (c) the proposed suspension S_2 with a two-terminal mass in parallel

Fig. 2(a) displays the conventional 1/4 car model where m_1 denotes unsprung mass, m_2 sprung mass, d_0 road surface excitation, d_1 tire deformation, d_2 body displacement, k_1 tire stiffness and S the suspension (S=S_1 or S_2 respectively for the conventional and proposed suspensions). As shown in Fig. 2(b), a conventional suspension S_1 is composed of a suspension stiffness k_2 and a suspension damping c. Having a two-terminal mass m_t in parallel, the proposed suspension S_2 is shown in Fig. 2(c).The governing equations in the Laplace domain for the 1/4 car model shown in Fig. 2(a) are given by

$$\begin{cases} m_2 s^2 \hat{d}_2 + G(s)(\hat{d}_2 - \hat{d}_1) = 0 \\ m_1 s^2 \hat{d}_1 + G(s)(\hat{d}_1 - \hat{d}_2) + k_1\left(\hat{d}_1 - \hat{d}_0\right) = 0 \end{cases} \tag{3}$$

where, '^' represents the Laplace transform of the corresponding variables and $G(s)$ the transfer function. $G(s) = G_1(s)$ for S_1 and $G(s) = G_2(s)$ for S_2 respectively. According to Figs. 2(b) and (c), one has

$$\begin{cases} G_1(s) = k_2 + cs \\ G_2(s) = k_2 + cs + m_t s^2 \end{cases} \tag{4}$$

A filtered white noise is used to represent the vibration excitation of the road surface, i.e. [18]

$$\ddot{d}_0(t) = -2\pi f_0 d_0(t) + 2\pi\sqrt{G_0 v}\,w(t) \tag{5}$$

where f_0 is the lower cut-off frequency (Hz), G_0 the road roughness coefficient (m^3/cycle), v the vehicle speed (m/s) and $w(t)$ the zero mean Gaussian noise.

III. Suspension Parameter Optimization

A. Influence of Suspension Parameters on the Performance

As mentioned before, there are three commonly used criteria to evaluate suspension performance: tire grip (d_1- d_0), ride comfort ($\ddot{d}_2$) and suspension deflection (d_2- d_1). To calculate the three performance measures, the Laplace domain governing equations of the proposed suspension S_2 are converted into a state-space model. The state-space model of the suspension S_2 can be expressed as

$$\begin{cases} \dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \\ \mathbf{Y} = \mathbf{CX} \end{cases} \tag{6}$$

where **X, U**, and **Y** represent respectively the state, input, and output vectors; **A**, **B** and **C** are the associated coefficient matrixes. According to (3) and (4), we define

$$\begin{cases} \mathbf{X} = [x_a \quad x_b \quad x_c \quad x_d]^T \\ \mathbf{Y} = [y_a \quad y_b \quad y_c]^T \end{cases} \tag{7}$$

where $x_a=d_2-d_1$, $x_b=\dot{d}_2$, $x_c=d_1-d_0$, $x_d=\dot{d}_1$, $y_a = d_1 - d_0 = x_c$, $y_a = \ddot{d}_2 = \dot{x}_b$, $y_b = d_2 - d_1$ and $y_c = d_1 - d_0 = x_c$.

Combing (5) and (6) gives

$$\mathbf{U} = \dot{d}_0 \leftrightarrow \hat{\dot{d}}_0 = \frac{2\pi\sqrt{G_0 v}\,s}{s^2 + 2\pi f_0}\hat{w} \tag{8}$$

For the proposed suspension S_2, combining (3), (4), (6) and (7) leads to

$$\begin{cases} \mathbf{A} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ \frac{-k_2 m_1}{\Delta} & \frac{-cm_1}{\Delta} & \frac{-m_t k_1}{\Delta} & \frac{cm_1}{\Delta} \\ 0 & 0 & 0 & 1 \\ \frac{k_2 m_2}{\Delta} & \frac{cm_2}{\Delta} & \frac{-k_1 m_2 - m_t k_1}{\Delta} & \frac{-cm_2}{\Delta} \end{bmatrix} \\ \mathbf{B} = [0 \quad 0 \quad -1 \quad 0]^T \\ \mathbf{C} = \begin{bmatrix} \frac{-k_2 m_1}{\Delta} & \frac{-cm_1}{\Delta} & \frac{-m_t k_1}{\Delta} & \frac{cm_1}{\Delta} \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{cases} \tag{9}$$

where $\Delta = (m_1 + m_t)(m_2 + m_t) - m_t^2$.

The aforementioned equations show that the three performance indices are given by the three components of the output vector **Y** of the state-space model. With any given road input shown in (5), one may calculate the tire grip, the ride comfort and the suspension deflection from (6) to (9). The optimal suspension parameters (k_2, c, m_t) should ensure the following are achieved:

$$\begin{cases} \min\ J_1 = \frac{1}{T}\int_0^T \sqrt{y_a^2}\,dt \\ \min\ J_2 = \frac{1}{T}\int_0^T \sqrt{y_b^2}\,dt \\ \min\ J_3 = \frac{1}{T}\int_0^T \sqrt{y_c^2}\,dt \end{cases} \tag{10}$$

where minJ_1 denotes the optimal tire grip, minJ_2 the optimal rid comfort, and minJ_3 the optimal suspension deflection.

In the actual design of the vehicle suspension, the stiffness is usually specified first according to the use of the vehicle. Then the other dynamics parameters of the suspension are optimized afterwards.In our research, suppose the 1/4 car model parameters are [19]: m_1=60kg, m_2=375kg, and k_1=200kN/m. Let k_2 =80kN/m, the road excitation parameters $G_0=8\times10^{-6}m^3$/cycle, v=25m/s, f_0=0.2Hz and $w(t)$ be 20dB zero-mean white noise.

It should be noted that minJ_3 is zero if $c\to\infty$. To achieve a meaningful design results, we specify the c and m_t ranges as $c \in [1,20]$ kN.s/m and $m_t \in [20,500]$ kg. Simulating 2500m of the road length yields the running time 100s with the given vehicle speed v. The sampling frequency is set as 1kHz. Calculating from (6)-(9), the optimal suspension parameters in terms of the three objectives are respectively displayed in Table I.

TABLE I
Single-Objective Parametric Optimization Results of S_2

Objective	c_{opt}(kN.s/m)	m_{topt}(kg)	minJ_i
Tire grip (J_1)	5.108	135	2.3282×10^{-4}m
Ride comfort (J_2)	5.262	183	$0.1071m^2/s$
Suspension deflection (J_3)	20	20	3.240×10^{-4}m

Note: a) Subscript 'opt' means the optimal parameter; and b) i=1, 2, 3 respectively for minJ_i.

Table I reveals that it is impossible to simultaneously obtain minJ_1, min J_2 and min J_3 with a fixed combination of suspension parameters. The parametric optimization results in terms of the three single criteria are noncommensureable and conflicting. Optimizing one objective separately may jeopardize the other two. To achieve the optimization design tradeoff with respect to all the three objectives, the Chebyshev goal programming method is adopted to find the preferred compromise between these objectives as detailed in the following.

B. Multi-objective optimization using the Chebyshev goal programming method

The multi-objective suspension parameter optimization problem is expressed as

$$\min\ Q = f(J_1, J_2, J_2) \tag{10}$$

Besides the best single-objective performances minJ_1, min J_2 and min J_3 resulting from the optimal parameters, there are also the worst performances expressed as maxJ_1, maxJ_2 and maxJ_3 for the suspension in the given parameter ranges. With the best and worst single objective values, the Chebyshev goal programming model is formulated as follows:

$$\mathrm{Min}Q$$

subject to

$$\begin{cases} Q \geq \dfrac{J_{1mop} - \min J_1}{\max J_1 - \min J_1} \\ Q \geq \dfrac{J_{2mop} - \min J_2}{\max J_2 - \min J_2} \\ Q \geq \dfrac{J_{3mop} - \min J_3}{\max J_3 - \min J_3} \end{cases} \tag{11}$$

where min$Q$ is the multi-objective performance index incorporating the three single criteria, and subscript 'mop' represents the multi-objective optimum. Solving the above model yields an overall optimal solution in terms of all these three performance criteria.

Since we have already found out three optimal performance with respect to optimal parameters c_{opt} and $m_{t\text{opt}}$. Calculating in the same way described in last subsection, we hereafter present the worst performance resulting from the 'worst' parameters c_{wor} and $m_{t\text{wor}}$ (subscript 'wor' represents the worst parameter) in Table II.

TABLE II
WORST PERFORMANCES IN THE GIVEN PARAMETER RANGES

Objectives	c_{wor}(kN.s/m)	$m_{t\text{wor}}$(kg)	maxJ_i
Tire grip (J_1)	1	500	5.771×10^{-4}m
Ride comfort (J_2)	1	500	0.2424m^2/s
Suspension deflection (J_3)	1	385	9.639×10^{-4}m

Substituting the parameters shown in Table I and II into (11) yields the multi-objective optimized parameters c_{mop} = 11.220kN.s/m and $m_{t\text{mop}}$ = 177kg. The corresponding suspension performances are summarized in Table III. The relationship between the multi-objective performance and the suspension parameters in the given ranges is shown in Fig. 3.

TABLE III
MULTI-OBJECTIVE OPTIMIZATION RESULTS FOR S_2

$J_{1\text{mop}}$ (m)	$J_{2\text{mop}}$ (m^2/s)	$J_{3\text{mop}}$ (m)	min$Q$
2.5516×10^{-4}	0.1170	3.7096×10^{-4}	0.0734

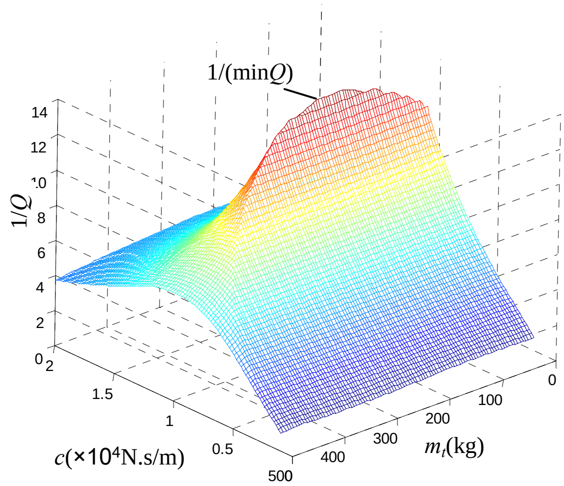


Fig. 3 Influence of the suspension parameters on the multi-objective performance of the proposed suspension S.

IV. COMPARISON WITH THE CONVENTIONAL SUSPENSION

As a comparison, the Chebyshev goal programming method is also applied for parametric optimization design of the conventional suspension S_1 shown in Fig. 2(b). With the same given parameters, the design optimization results are shown in Table IV. The influence of the damping on the multi-objective performance of the suspension S_1 in the simulation case is shown in Fig. 4.

TABLE IV
MULTI-OBJECTIVE OPTIMIZATION RESULTS FOR THE CONVENTIONAL SUSPENSION

Objective	Performance		c(kN.s/m)
Single-	minJ_1 (m)	2.4432×10^{-4}	5.736
	minJ_2 (m^2/s)	0.1163	6.551
	minJ_3 (m)	3.239×10^{-4}	20
	maxJ_1 (m)	3.666×10^{-4}	1
	maxJ_2 (m^2/s)	0.1830	1
	maxJ_3 (m)	8.485×10^{-4}	1
Multi-	$J_{1\text{mop}}$ (m)	2.5651×10^{-4}	10.579
	$J_{2\text{mop}}$ (m^2/s)	0.1201	
	$J_{3\text{mop}}$ (m)	3.7619×10^{-4}	
	min$Q$	0.0997	

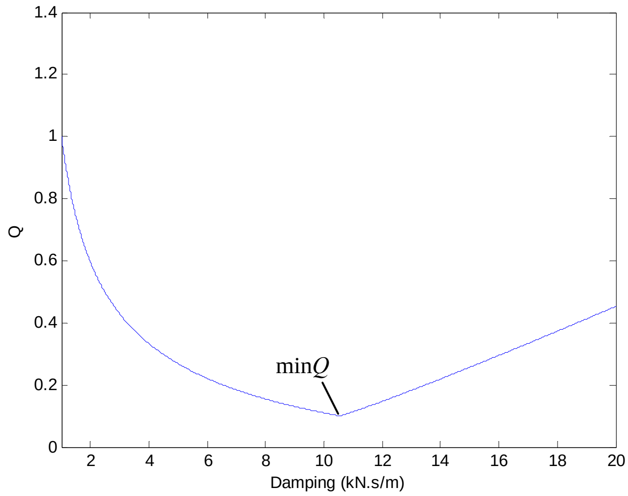
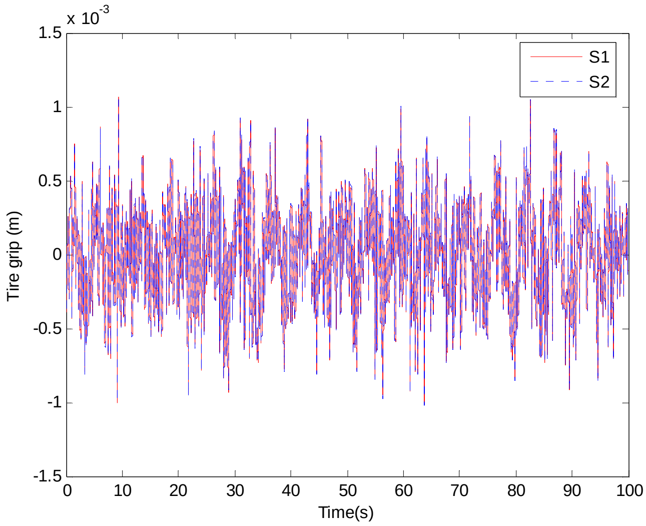
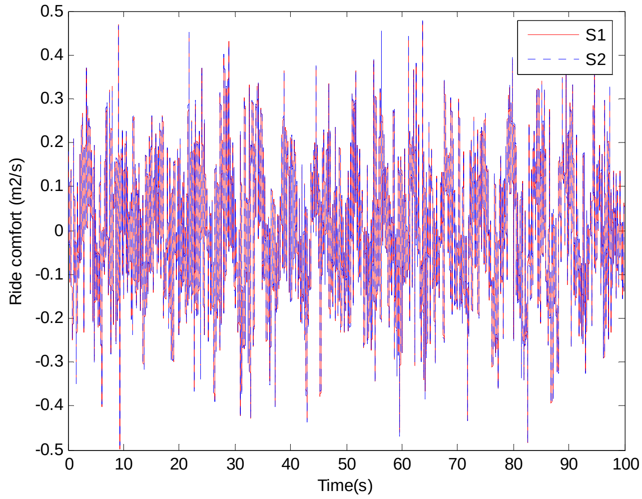


Fig. 4 Multi-objective performance variety resulting from the damping change of the conventional suspension S_1

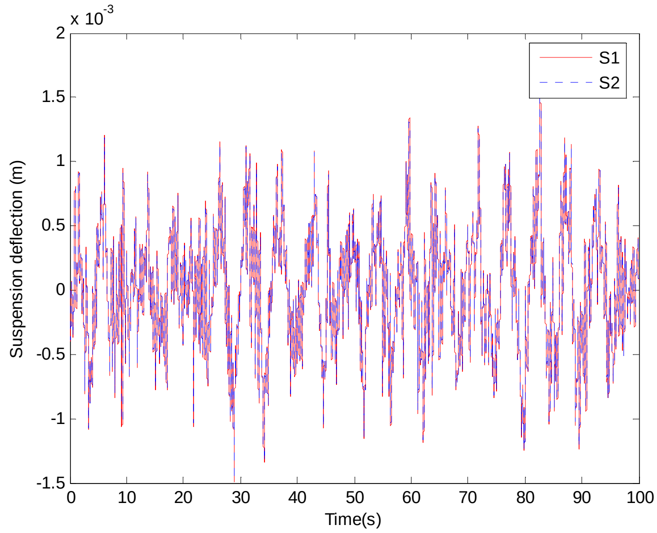
Fig. 5 displays the comparison between the two suspensions, S_1 and S_2, in terms of these three performance criteria. As compared to the conventional design, the proposed suspension S_2 has led to improvements of 0.53% [=(2.5651×10^{-4}-2.5516×10^{-4})/(2.5651×10^{-4})], 2.58% [=(0.1201-0.1170)/0.1201], and 1.39% [=(3.7619×10^{-4}-3.7096×10^{-4})/(3.7619×10^{-4})] in terms of tire grip, ride comfort, and suspension deflection, respectively. For this example case, the proposed design has also improved the overall multi-objective performance by 26.38% [=(0.0997-0.0734)/0.0997] over the conventional design S_1.



(a)



(b)



(c)

Fig. 5 Suspension comparison in terms of three indices: (a) tire grip; (b) ride comfort; and (c) suspension deflection. Solid line -- S_1, dashed line -- S_2.

V. Conclusion

In this paper, we examined the performance of a passive vehicle suspension with a two-terminal mass in parallel in terms of tire grip, ride comfort, and suspension deflection. As the three performance measures are non-commensurable and often conflicting, the Chebyshev goal programming method is applied to solving this multi-objective problem. For our example case, the proposed new design has outperformed the conventional one in all the three criteria, leading to 26.38% overall improvement.

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