World Academy of Science, Engineering and Technology
International Journal of Industrial and Manufacturing Engineering
Vol:6, No:12, 2012

A Study of Replacement Policies for Warranty Products with Different Failure Rate

Wen Liang Chang

Open Science Index, Industrial and Manufacturing Engineering Vol:6, No:12, 2012 publications.waset.org/8180/pdf

Abstract**—**This paper provides a replacement policy for warranty products with different failure rate from the consumer's viewpoint. Assume that the product is replaced once within a finite planning horizon, and the failure rate of the second product is lower than the failure rate of the first product. Within warranty period (WP), the failed product is corrected by minimal repair without any cost to the consumers. After WP, the failed product is repaired with a fixed repair cost to the consumers. However, each failure incurs a fixed downtime cost to the consumers over a finite planning horizon. In this paper, we derive the model of the expected total disbursement cost within a finite planning horizon and some properties of the optimal replacement policy under some reasonable conditions are obtained. Finally, numerical examples are given to illustrate the features of the optimal replacement policy under various maintenance costs.

Keywords**—**Planning horizon; Free-repair warranty; Minimal repair; Replacement.

I. Introduction

DUE to the rapid advance in functional innovation of the products, the consumers plan to use the product, and the product is replaced once under a finite planning horizon. It would be important to steady the operational efficiency of the product for consumers. This paper investigates single-replacement policy for warranty products with different failure rate under a finite planning horizon. For warranty products, the theories of replacement and maintenance policies are illustrated in Barlow and Proschan [1]. In 1982, Boland and Proschan [2] studied the periodic replacement policy which minimizes the total expected cost of repair and replacement over a fixed time horizon and the total expected cost per unit time over an infinite time horizon. Nakagawa and Kowada [3] proposed the replacement model for a system with minimal repair and derived some probability quantities and reliability properties. Since then, various issues associated with replacement and maintenance policies have been extensively published [4]-[8].

In general, the product is sold with a length of warranty. In developing a warranty model, FRW is the most commonly used for repairable products. The concept of warranty cost analysis is described in Blischke and Murthy [9]. Under FRW, the failed product is corrected using minimal repair without any cost to the consumers. After minimal repair, the failure rate of the product remains unchanged. Various maintenance models involving minimal repair can be found in the literatures [10]-[12]. Wang and Sheu [13] investigated a discrete general shift distribution to provide an optimal lot size so that the long run total cost of the setup, inventory holding, and warranty is minimized. In 2009, Nakagawa and Mizutani [14] converted the three usual models of the periodic replacement with minimal repair, block replacement and simple replacement to finite replacement models. Furthermore, the optimal policies for each model are analytically derived. As mentioned above, the past literatures considered that the failed product is replaced by a new product in the maintenance model from the seller's viewpoint. In this paper, we consider that the warranty product is replaced once within a finite planning horizon from the consumer's viewpoint, and the failure rate of the second product is lower than the failure rate of the first product. Furthermore, the mathematical model is derived and the optimal replacement time is obtained for the warranty product.

This paper is organized as follows. The mathematical model for the warranty product is derived in Section II. The optimal replacement time under some reasonable conditions is obtained in Section III. In Section IV, the performance of the optimal replacement policy is evaluated through numerical examples. Finally, some conclusions are drawn in Section V.

II. Mathematical Formulation

We used the following mathematical symbols in this paper:

T a planning horizon of using the product
w the length of warranty of the ith product, $i = 1, 2$
S the replacement time over a planning horizon T
V_i the purchase price of the ith product, $i = 1, 2$
$f_i(t)$ probability density function of the lifetime of ith product, $i = 1, 2$
$h_i(t)$ failure rate function of the ith product, $i = 1, 2$
$H_i(t)$ cumulative failure rate function of the ith product
C_{mi} minimal repair cost of the ith product, $i = 1, 2$
C_d downtime cost of the ith product, $i = 1, 2$

From the consumer's viewpoint, consider that a new product is sold with a length of warranty w. Suppose that the consumers plan to use the product under a planning horizon T. During the planning horizon, the product is replaced once at time S by a new product and $2w \le T$. Suppose that the failure rate of the first and second products are $h_1(t)$ and $h_2(t)$, respectively. Consider that $h_i(t), i = 1, 2$ is a strictly increasing

W.L. Chang is with Department of Information Management, Cardinal Tien College of Healthcare & Management, Taiwan, ROC (e-mail: chang@ctcn.edu.tw).

Open Science Index, Industrial and Manufacturing Engineering Vol:6, No:12, 2012 publications.waset.org/8180/pdf

function over time t with $h_1(0) = h_2(0) = 0$ and $h_1(t) > h_2(t) \geq 0, \forall t \geq 0$. The cumulative failure rate function of the product is $H_i(t) = \int_0^t h_i(u)\, du, i = 1, 2$.

Suppose that the purchase prices of the first and second products are V_1 and V_2, respectively. Within the planning horizon, each failure incurs a fixed downtime cost C_d to the consumers. Within WP, the failed product is corrected by minimal repair without any cost to the consumers. After WP, the ith failed product is repaired with a fixed cost $C_{mi}, i = 1, 2$ to the consumers. After minimal repair, the product is normally operating; however, its failure rate remains the same as that just prior to failure. Since the failed products are rectified using minimal repair, the failure process of the ith product in the interval $[0, T]$ is a non-homogeneous Poisson process (NHPP) with intensity function $h_i(t), i = 1, 2$. Assume that the time required for minimal repair is negligible comparing to T. Since the warranty product is replaced once at time S over T, the model of replacement police is considered: $0 \leq S \leq w$. Then, the model is discussed as follows.

From Fig. 1, the expected total disbursement cost (purchase price and downtime cost) of the first product is:

$$C_1 = V_1 + C_d \int_0^S h_1(t)\, dt = V_1 + C_d H_1(S).$$

The expected total disbursement cost (purchase price, downtime cost and repair cost) of the second product is:

$$C_2 = V_2 + C_d \int_0^w h_2(t)\, dt + (C_{m2} + C_d) \int_w^{T-S} h_2(t)\, dt$$
$$= V_2 + (C_{m2} + C_d) H_2(T - S) - C_{m2} H_2(w).$$

Therefore, the expected total disbursement cost within the planning horizon T is:

$$E(S) = C_1 + C_2$$
$$= V_1 + C_d H_1(S) + V_2 + (C_{m2} + C_d) H_2(T - S) - C_{m2} H_2(w). \quad (1)$$

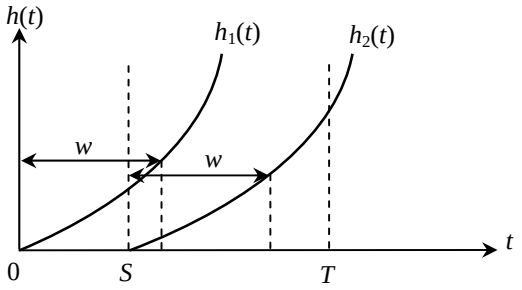


Fig. 1 Replacement policy under $0 \leq S \leq w$

The objective of this paper is to find an optimal replacement time S of the product such that the expected total disbursement cost in Eq. (1) is minimized. In the next section, the properties of the optimal replacement time are derived.

III. Optimal Replacement Time

To investigate the properties of the optimal replacement time, we take the first and second derivatives of Eq. (1) with respect to S, we have:

$$E'(S) = C_d h_1(S) - (C_{m2} + C_d) h_2(T - S) \quad (2)$$

and

$$E''(S) = C_d h_1'(S) + (C_{m2} + C_d) h_2'(T - S). \quad (3)$$

Since $h_i(t), i = 1, 2$ is a strictly increasing function, we have $E''(S) > 0$ in Eq. (3). This implies that $E'(S)$ is an increasing function of S in the interval $[0, T]$ and $E(S)$ is a convex function. Observing Eq. (2), the following theorem holds.

Theorem 1. *When $h_i(t) > 0, i = 1, 2, \forall t \geq 0$, the following results hold.*

(a) *If $C_d h_1(w) - (C_{m2} + C_d) h_2(T - w) \leq 0$, then the optimal replacement time $S^* = w$.*

(b) *If $C_d h_1(w) - (C_{m2} + C_d) h_2(T - w) > 0$, then there exists a unique solution $S^* \in [0, w)$ such that $E_1'(S)\big|_{S=S^*} = 0$ and minimizes $E_1(S)$.*

Proof. According to the boundary condition $0 \leq S \leq w$, substituting $S = 0$ and $S = w$ into Eq. (2), respectively, we have

$$E'(S)\big|_{S=0} = -(C_{m2} + C_d) h_2(T) \quad (4)$$

And

$$E'(S)\big|_{S=w} = C_d h_1(w) - (C_{m2} + C_d) h_2(T - w). \quad (5)$$

when $h_2(t) > 0, \forall t \geq 0$, we have $E'(S)\big|_{S=0} < 0$ in Eq. (4). Since $E'(S)$ is an increasing function of S and $h_i(t) > 0, i = 1, 2, \forall t \geq 0$, the following results hold. (a) If $C_d h_1(w) - (C_{m2} + C_d) h_2(T - w) \leq 0$, then $E'(S)\big|_{S=w} \leq 0$ in Eq. (5). This implies that $E(S)$ is a decreasing function of S in the interval $[0, w]$. Therefore, the optimal replacement time is $S^* = w$. On the other hand, (b) If $C_d h_1(w) - (C_{m2} + C_d) h_2(T - w) > 0$, then $E'(S)\big|_{S=w} > 0$ in Eq. (5). Since $E'(S)$ is an increasing function of S, $E'(S)$ changes its sign exactly once in the interval $[0, w)$. This implies that there exists a unique solution $S^* \in [0, w)$ such that $E'(S)\big|_{S=S^*} = 0$ and minimizes $E(S)$.

Theorem 1 shows that the optimal replacement time S^* is unique under the boundary condition $0 \leq S \leq w$ when $h_i(t) > 0, i = 1, 2, \forall t \geq 0$, and the optimal replacement time $S^* \in [0, w)$ can be obtained by solving the equation $E'(S) = C_d h_1(S) - (C_{m2} + C_d) h_2(T - S) = 0$.

Open Science Index, Industrial and Manufacturing Engineering Vol:6, No:12, 2012 publications.waset.org/8180/pdf

IV. Numerical Examples

Consider that the consumer plans to use the product under the planning horizon $T = 10$ and a new product is sold with a length of warranty w. During the planning horizon, the product is replaced once and $2w \le T$. Suppose that the lifetime distribution of the ith product follows a two-parameter Weibull distribution $f_i(t) = \alpha_i \beta_i (\alpha_i t)^{\beta_i - 1} e^{-(\alpha_i t)^{\beta_i}}$, $i = 1, 2$ for $t \ge 0$, where α_i is the scale parameter and β_i is the shape parameter. According to the definition of a failure rate function, the failure rate function of the Weibull distribution for the ith product is $h_i(t) = \alpha_i \beta_i (\alpha_i t)^{\beta_i - 1}$, $i = 1, 2$ and the failure rate of the first product is higher than the failure rate of the second product, that is, $h_1(t) > h_2(t) \ge 0, \forall t \ge 0$. $h_i(t), i = 1, 2$ increases in t if $\beta_i > 1$. When $\beta_i = 2$, $h_i(t), i = 1, 2$ is a linearly increasing function. $h_i(t), i = 1, 2$ is a convex and increasing function if $\beta_i > 2$, and a concave and increasing function in t if $\beta_i < 2$. Suppose that the initial purchase prices of the first and second products are $V_1 = 1000$ and $V_2 = 2000$, respectively. For the repairable product within WP, the failed product is corrected by minimal repair without any cost to the consumer. However, after WP, the ith failed product is repaired with a fixed cost $C_{mi}, i = 1, 2$ to the consumer. Within the planning horizon, each failure incurs a fixed downtime cost $C_d = 30$ to the consumer. For the model $0 \le S \le w$, when $w = 4$, Table I summarizes the numerical results for various C_{m2}. For example, when $(\alpha_1, \alpha_2, \beta_1, \beta_2, C_{m1}) = (0.2, 0.05, 2.1, 2.5, 50)$ and $C_{m2} = 130$, the optimal replacement time is $S^* = 2.1$ and the expected total disbursement cost is $\min E(S) = 3018$. Furthermore, from Table I, we have the following observations:

1. When the repair cost C_{m2} of the second product increases, the replacement time S and the expected total disbursement cost increases.
2. When $\beta_i, i = 1, 2$ increases, the replacement time and the expected total disbursement cost $\min E(S)$ decreases.

TABLE I
Optimal Replacement Time S^* for various C_{m2}

$w = 4$ $\alpha_1 = 0.2$ $\alpha_2 = 0.05$ $\beta_1 = 1.2$ $\beta_2 = 1.5$			
C_{m1}	C_{m2}	S^*	$\min E(S)$
50	100	2.2	3033
	110	2.7	3035
	120	3.2	3036
	130	3.7	3037
$w = 4$ $\alpha_1 = 0.2$ $\alpha_2 = 0.05$ $\beta_1 = 1.2$ $\beta_2 = 2.5$			
C_{m1}	C_{m2}	S^*	$\min E(S)$
50	100	1.9	3015
	130	2.1	3018
	160	2.4	3020
	190	2.6	3022

V. Conclusion

In this paper, from the consumer's viewpoint, the replacement of the product is investigated before WP. In the numerical examples, we have some conclusions as follows. (1) As the repair cost of the second product is higher, the product should be replaced late. (2) When the failure rate of the product is a convex and increasing function, the expected total disbursement cost is lower. Furthermore, some generalizations such as residual value of the product, renewing free-replacement warranty, non-renewing free-replacement warranty, or pro-rata warranty, are extended issues for future study in this area.

References

[1] R.E. Barlow and F. Proschan, *Mathematical Theory of Reliability*, Wiley, New York, 1965.
[2] P.J. Boland and F. Proschan, "Periodic replacement with increasing minimal repair costs at failure," *Operations Research*, vol. 30, pp. 1183–1189, 1982.
[3] T. Nakagawa and M. Kowada, "Analysis of a system with minimal repair and its application to replacement policy," *European Journal of Operational Research*, vol. 12, pp. 176–182, 1983.
[4] J.K. Chan and L. Shaw, "Modeling repairable systems with failure rates dependent on age and maintenance," *IEEE Transactions on Reliability*, vol. 42, pp. 566–570, 1993.
[5] Y.H. Chien, "Optimal age for preventive replacement under a combined fully renewable free replacement with a pro-rata warranty," *International Journal of Production Economics*, vol. 124, pp. 198–205, 2010.
[6] Y.H. Chien and J.A. Chen, "Optimal age-replacement policy for repairable products under renewing free-replacement warranty," *International Journal of Systems Science*, vol. 38, pp. 759–769, 2007.
[7] K.J. Chung, "Optimal repair-cost limit for a consumer following expiry of a warranty," *Microelectronics and Reliability*, vol. 34, pp. 1689–1692, 1994.
[8] A.N. Das and S.P. Sarmah, "Preventive replacement models: an overview and their application in process industries," *European Journal of Industrial Engineering*, vol. 4, pp. 280–307, 2010.
[9] W.R. Blischke and D.N.P. Murthy, *Warranty Cost Analysis*, Dekker, New York, 1994.
[10] T. Aven, "Optimal replacement under a minimal repair strategy–a general failure model," *Advances in Applied Probability*, vol. 15, pp.198–211, 1983.
[11] J. Jaturonnateea, D.N.P. Murthy and R. Boondiskulchoka, "Optimal preventive maintenance of leased equipment with corrective minimal repairs," *European Journal of Operational Research*, vol. 174, pp. 201–215, 2006.
[12] J.P. Jhang, "A study of the optimal use period and number of minimal repairs of a repairable product after the warranty expires," *International Journal of Systems Science*, vol. 36, pp. 697–704, 2005.
[13] C.H. Wang and S.H. Sheu, "Optimal lot sizing products sold under a free-repair warranty," *European Journal of Operational Research*, vol. pp. 141, 2003.
[14] T. Nakagawa and S. Mizutani, "A summary of maintenance policies for a finite interval," *Reliability Engineering and System Safety*, vol. 94, pp. 89–96, 2009.