World Academy of Science, Engineering and Technology
International Journal of Computer and Information Engineering
Vol:2, No:4, 2008

A Symbol by Symbol Clustering Based Blind Equalizer

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Abstract**—** A new blind symbol by symbol equalizer is proposed. The operation of the proposed equalizer is based on the geometric properties of the two dimensional data constellation. An unsupervised clustering technique is used to locate the clusters formed by the received data. The symmetric properties of the clusters labels are subsequently utilized in order to label the clusters. Following this step, the received data are compared to clusters and decisions are made on a symbol by symbol basis, by assigning to each data the label of the nearest cluster. The operation of the equalizer is investigated both in linear and nonlinear channels. The performance of the proposed equalizer is compared to the performance of a CMA-based blind equalizer.

Keywords**—** Blind equalization, channel equalization, cluster based equalisers

I. Introduction

INTERSYMBOL Interference (ISI) is a major impairment in today's high bit rate communication systems [1]. Channel equalizers used in the receiver part aim to suppress the effect of ISI. In most of the cases the communication channel is unknown and the design of the equalizer is performed on the basis of a known training sequence of information bits. However, there are many cases that the transmission of a training sequence is not possible or desirable. This mode of equalizer design is known as *blind*.

Blind channel equalization is a challenging task and has been the focus of intense research effort. Recently, an interest has risen on approaches based on data clustering techniques [2], [3], [4], [5].

In this paper a novel blind cluster based symbol by symbol equalizer is proposed. The equalizer extracts the information needed to perform data detection from the clusters formed by the received data. The whole process involves a simple symbol by symbol decision procedure. The cluster based blind channel estimation algorithm consists of two steps: a) data clusters estimation via an unsupervised learning technique and b) labeling of the estimated clusters by unraveling the information hidden in the geometry of the clusters constellation in the two dimensional space. That is, for data generated by bipolar alphabets (assumed in this paper) the clusters are arranged in pairs of clusters with the right sided cluster labeled as $+1$ and the left sided cluster labeled as -1. This is the property of symmetric labels and it is used in order to label the clusters. Determination of the pair of clusters is obtained, in the linear case, by using the results of [6] concerning the properties of the convex hull of the clusters constellation.

Manuscript received July 15, 2006.
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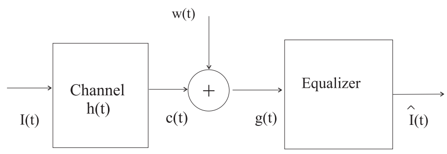

Fig. 1. The system diagram.

However, in contrast to [6], in this paper the estimation of the magnitude of the channel taps (absolute values) provide sufficient information for the design of the equalizer and therefore, there is no need to estimate the specific permutation of the channel taps.

When channel estimation is completed the received data are compared to clusters and a closest neighbor rule [7] is utilized to achieve data detection on a symbol by symbol basis. That is, the currently observed data is classified by assigning to it the label associated with the nearest cluster. The algorithm is applied to linear channels. The extension of the algorithm in nonlinear channels is also investigated.

The paper is organized as follows. Section II presents the system description and the properties of the two dimensional clusters constellation when the channel under consideration is linear. Section III describes the proposed symbol by symbol blind equalizer. Next, the paper is concerned with the extension of the algorithm to the nonlinear channel case. Section IV.A describes the clusters constellation properties for nonlinear channels and in Section IV.B the respective equalizer follows. In Section V simulation results are given and finally, in Section VI conclusions are drawn.

II. Clusters constellation properties

The channel equalization set up adopted in this paper is illustrated in Fig. 1.

The received signal $g(t)$ of an ISI and noise impaired linear system is written as:

$$g(t) = \sum_{i=0}^{L} h(i)I(t-i) + w(t), \quad (1)$$

where $I(t)$ is an equiprobable sequence of transmitted data taken from a binary alphabet, i.e., $I(t) \in \{\pm 1\}$, $h(i)$ is the channel impulse response and $w(t)$ is an Additive White Gaussian Noise (AWGN) sequence. Equation (1) can also be written as:

$$g(t) = c(t) + w(t), \quad (2)$$

where $c(t)$ is the noiseless channel output sequence which is a discrete values signal with 2^{L+1} different elements.

In this paper, the necessary information for data detection is extracted from the geometric structure created by the received data in the two dimensional space. Consider the 2 x 1 vector of successively received samples:

$$\mathbf{g}(t) = [g(t)\ g(t-1)]^T. \quad (3)$$

In the absence of noise, $\mathbf{g}(t)$ is associated with $Q = 2^{L+2}$ points in the two dimensional space. Each point corresponds to one of the 2^{L+2} possible realizations of the sequence of transmitted bits: $(I(t), ..., I(t-L-1))$. If the received data is corrupted by AWGN, then the randomness of noise leads to the formation of a *cluster* around each point. Each cluster is represented by a suitably chosen *representative*, which corresponds to the noiseless channel response vector in the two dimensional space, i.e.,

$$\mathbf{c}(t) = [c(t)\ c(t-1)]^T$$

where $\mathbf{c}(t) \in \{\mathbf{c}_k = [c_{k1}\ c_{k2}]^T, k = 1, ..., Q\}$. Each cluster representative, $\mathbf{c}_k$, corresponds to a specific sequence of transmitted data denoted as: $(I_{k0}I_{k1}, ..., I_{kL}I_{k(L+1)})$. The two components of a clusters representative, $\mathbf{c}_k$, are written as:

$$c_{k1} = \sum_{l=0}^{L} I_{kl}h(l), \quad c_{k2} = \sum_{l=1}^{L+1} I_{kl}h(l-1). \quad (4)$$

Each cluster is characterized by a *label*, X_k, which is defined as the value of the corresponding emitted data, i.e., $X_k = I_{kd} = I(t-d)$. Parameter d is an appropriately chosen delay and $X_k \in \{\pm 1\}$.

For a linear channel, the edges E_i, $i = 1, .., 2L+4$, of the convex hull, H, of the two dimensional data constellation contain information related to the channel taps [6]. That is, every edge E_i of H is parallel to some vector $\mathbf{u}_i$, where:

$$\mathbf{u}_0 = [h(0)\ 0]^T, \mathbf{u}_1 = [h(1)\ h(0)]^T, ..., \mathbf{u}_{L+1} = [0\ h(L)]^T.$$

The length of E_i is equal to $2|\mathbf{u}_i|$. Actually, there are two edges parallel to each vector $\mathbf{u}_i$.

Moreover, for the edges of the convex hull the following Theorem is shown.

Theorem 1: For each unique edge, E_i, (i=1,...,L+2) of the convex hull there are $Q/2$ pairs of clusters such that each cluster of a pair defines the endpoint of a line segment parallel to the edge E_i. The length of each line segment is equal to the length of E_i.

Proof: Let us consider two clusters: $\mathbf{c}_k = [c_{k1}\ c_{k2}]^T$ and $\mathbf{c}_j = [c_{j1}\ c_{j2}]^T$ which correspond to the transmitted sequences: $(I_{k0}....I_{k(L+1)})$ and $(I_{j0}...I_{j(L+1)})$, respectively, where

$$I_{kl} = \begin{cases} I_{jl} & l \neq i \\ -I_{jl} & l = i \end{cases}$$

for $l = 0, ..., L+1$ and $i \in \{0, ..., L+1\}$. Using (4) we get:

$$|c_{k1} - c_{j1}| = 2|h(i)| \quad (5)$$

and

$$|c_{k2} - c_{j2}| = 2|h(i-1)|.$$

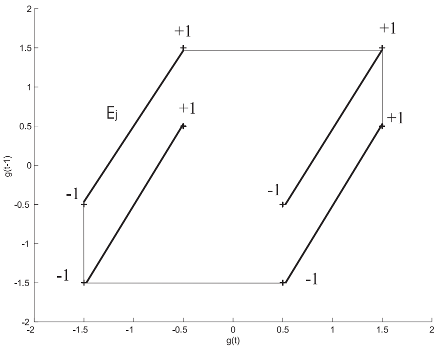

Fig. 2. Clusters formed by the linear channel $H(z) = 1 + 0.5z^{-1}$ and binary data. Convex hull of the two dimensional data constellation and line segments parallel to edge E_j formed by the corresponding pair of clusters.

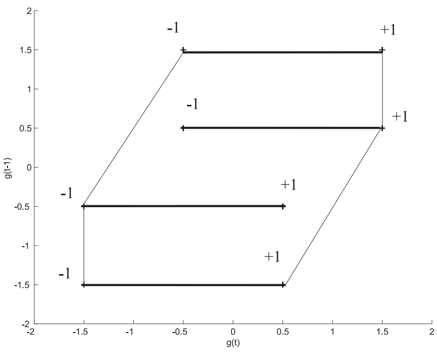

Fig. 3. Clusters formed by the linear channel $H(z) = 1 + 0.5z^{-1}$ and binary data. Labeling of clusters over the bit $I(t-1)$.

As a consequence, for each specific i, $i \in \{0, ..., L+1\}$, there are $Q/2$ such pairs of clusters, in the two dimensional constellation being separated by a distance equal to $2|\mathbf{u}_i|$. The corresponding transmitted sequences of the two clusters of each pair are the same except from the value of data I_{ki} and I_{ji} (i.e., $I(t-i)$).

■

The concept of Theorem 1 is graphically illustrated in Fig. 2. The figure represents the clusters constellation formed by a simple linear channel $H(z) = 1 + 0.5z^{-1}$ and Signal to Noise Ratio (SNR) equal to 40dB. In the figure also appear the convex hull of the two dimensional data constellation as well as a set of 4 pair of clusters parallel to the edge E_j.

Definition: By now, we will call *a pair of clusters* two clusters, $\mathbf{c}_k$ and $\mathbf{c}_j$, sharing the same data except from the value of I_{ki} and I_{ji} respectively ($i \in 0, ..., L+1$). In the linear channel case, the two clusters of a pair are separated by distance equal to $2|\mathbf{u}_i|$ (Theorem 1).

Definition: We call *a set of pairs of clusters* the $Q/2$ pairs of clusters satisfying Theorem 1, for a specific edge, E_i. Actually, for a channel with length $L+1$, there are $L+2$ such sets of pairs.

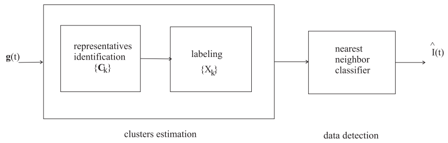

Fig. 4. The proposed blind symbol by symbol equalizer.

For the labels of the pairs of clusters belonging to a specific set the following Theorem holds.

Theorem 2: (Property of labels symmetry) Let us consider a) an edge, E_i, $(i = 1, ..., L + 2)$ of the convex hull of the two dimensional data constellation and b) the set of pairs of clusters corresponding to this edge. All the right-hand side clusters of all the pairs of the set correspond to a transmitted bit $I(t-i) = +1$ and all the left-hand side clusters correspond to a transmitted bit $I(t - i) = -1$ (or vice versa).

Proof: Consider the clusters $\mathbf{c}_k$ and $\mathbf{c}_j$ which form a pair of clusters of the specific set and let

$$c_{k1} = I_{ki}h(i) + \sum_{l \neq i} I_{kl}h(l), \quad c_{j1} = I_{ji}h(i) + \sum_{l \neq i} I_{jl}h(l).$$

Suppose that

$$c_{k1} < c_{j1}.$$

This simply implies that the corresponding labels of the clusters are ordered as:

$$I_{ki} < I_{ji}.$$

This is true since:

$$\sum_{l \neq i} I_{kl}h(l) = \sum_{l \neq i} I_{jl}h(l).$$

That is, the right sided cluster has label $I(t - i) = +1$ and the left cluster has label $I(t - i) = -1$. This ordering of the labels is valid under the assumption that $h(i) > 0$. If $h(i) < 0$ then the opposite ordering takes place, i.e., if $c_{k1} < c_{j1}$ then $I_{ki} > I_{ji}$. ■

Fig. 3 represents, for the same channel as in the previous example, a set of four pairs of clusters which form line segments parallel to the horizontal edge. It can be observed that for all the pair of clusters: the right hand side cluster has label $I(t) = +1$ and the left-hand side has label $I(t) = -1$. The corresponding labeling over the bit $I(t - 1)$ appears in Fig. 2.

III. Symbol by Symbol Blind Clustering Equalizer

In this Section, a novel symbol by symbol blind equalizer is proposed based on the clusters constellation properties developed in Section 2. The block diagram of the proposed equalizer appears in Fig. 4. First, clusters estimation takes place and then the signal detection procedure follows.

A cluster-based blind channel estimation algorithm consists of two steps [4]:

a) clusters representatives estimation via an unsupervised learning technique and

b) labeling of the estimated clusters.

In the proposed equalizer the two dimensional clusters representatives are identified by means of the Neural Gas algorithm [8].

The proposed labeling algorithm aims to the characterization of each specific cluster according to the respective value of the transmitted data $I(t-d)$, where d is an unknown delay. It is known that a nonzero lag, d, permits a better equalization performance [9]. In the proposed algorithm, it is chosen the delay which corresponds to the maximum tap of the channel impulse response. From (5) it is observed that the maximum channel tap corresponds to the maximum horizontal distance between two clusters. That is, that way, we impose the biggest separation among the two classes (+1, -1). For many channels this leads to two separable decision regions. This is important for a symbol by symbol equalizer as it makes its decisions much more robust to the errors. For example, for the simple channel $H(z) = 1+0.5z^{-1}$, the biggest separation between the two classes is obtained by choosing the first channel coefficient (Fig. 2 and Fig. 3) and consequently, delay $d = 0$.

According to Section 2, the clusters labels in the two dimensional space have a symmetric distribution. Thus, clusters labeling can be achieved by identifying the position of each cluster in the two dimensional constellation.

The proposed labeling algorithm is summarized as follows. First, the convex hull is estimated and the absolute values of the channel taps are subsequently extracted using [6]. The maximum channel tap, $|h(d)|$, is chosen. Then, all the clusters pairs that are separated by horizontal and vertical distance equal to: $[2h(d)\ 2h(d-1)]$ are determined. Note, that if $d = 0$ then the pair's horizontal - vertical distance is $[2h(0)\ 0]$. Then, each cluster of a pair, lying to the left is labeled as -1 and the other cluster is labeled as +1 (the ambiguity in the descending or ascending order of the labels is solved by using differential encoding [10]).

Once labeling is completed, a simple decision rule is adopted for the detection of the received data. Given the received data vector $\mathbf{g}(t)$, its distance from each cluster is calculated, i.e.,

$$r_i = |\mathbf{g}(t) - \mathbf{c}_i)|, \ i = 1, ..., Q.$$

The label of the closest cluster determines the decision for the currently observed data $g(t)$.

The algorithm for linear channels appears in Table 1.

IV. The nonlinear channel case

In the nonlinear channel case the convex hull of the clusters in the two dimensional space does not contain the necessary information for estimating the channel taps any more. Moreover, the property of the symmetric labels stated, for linear channels, in Theorem 2, is not valid in all cases of nonlinear channels. When the symmetry of the labels exists a variation of the linear case algorithm can be adopted. This Section deals with a) the clusters constellation properties for nonlinear channels and the cases under which the symmetry of the labels exists and b) the extension of the proposed algorithm in the nonlinear channel case.

TABLE I
SYMBOL BY SYMBOL CLUSTERING BASED ALGORITHM - LINEAR CASE

Symbol by Symbol Blind Equalizer for Linear Channels
A. Estimation of the two dimensional clusters
A1. Estimation of the clusters representatives, $\mathbf{c}_k$, $k = 1, ..., Q$
Unsupervised learning
A2. Labeling of clusters
a) Find the convex hull, H, and determine $\lvert h(i)\rvert$, $i = 0, ..., L$ b) Choose the max tap $\lvert h(d)\rvert$ c) Find the couples of clusters with distance $[\lvert 2h(d)\rvert\ \lvert 2h(d-1)\rvert]$ between them (for $d = 0$ the distance is $[\lvert 2h(0)\rvert\ 0]$) d) Label the (pair of) clusters: left cluster $I(t-d) = -1$, right cluster $I(t-d) = +1$
B. Data detection
Nearest neighbor rule.

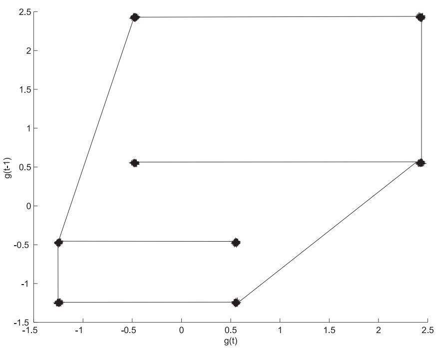

Fig. 5. Clusters formed by the linear filter $H(z) = 1 + 0.5z^{-1}$ and the nonlinearity $f(t) = c(t) + 0.15c(t)^2 + 0.1c(t)^3 + 0.05c(t)^4$.

A. Clusters constellation properties

When the channel under consideration is nonlinear then the received signal can be written as:

$$g(t) = f(I(t), I(t-1), ..., I(t-L)) + w(t)$$

where $f()$ is the nonlinear function representing the channel action. The model of the adopted nonlinear channel is a cascade of a linear filter and a memoryless polynomial nonlinearity.

The clusters representatives correspond to the noiseless channel response and are noted as:

$$\mathbf{f}_k = [f_{k1}\ f_{k2}]^T, k = 1, ..., Q.$$

Each cluster, $\mathbf{f}_k$, is related with a specific sequence of transmitted data: $(I_{k0}I_{k1}...I_{kL+1})$ and it is labeled by the bit $X_k = I_{kd}$.

As in the linear case, there are $L + 2$ sets of $Q/2$ pairs of clusters such that the two clusters of each pair, say $\mathbf{f}_k$ and $\mathbf{f}_j$, share the same transmitted data except one, i.e., $I_{kl} = I_{jl}$, for $l = 0, ..., L$ and $l \neq i$, $i \in \{0, ..., L\}$ and $I_{ki} = -I_{ji}$. We denote the horizontal and vertical distance between the two clusters of a pair as

$$\mathbf{l}_n(k) = [l_{n1}(k)\ l_{n2}(k)]^T,$$

with k ($k = 0, ..., L+1$) corresponding to the $L+2$ different sets and n ($n = 1, ..., Q/2$) corresponding to the $Q/2$ pairs of a set. In the linear case, all the $Q/2$ distances characterizing the $Q/2$ pairs of a set are the same (equal to $2|\mathbf{u}_i|$). However, in the nonlinear case, these distances are not equal any more.

A typical two dimensional constellation for the nonlinear channel case is illustrated in Fig. 5, where, the clusters formed by the linear filter $H(z) = 1 + 0.5z^{-1}$ and the nonlinear function $f(t) = c(t) + 0.15c(t)^2 + 0.1c(t)^3 + 0.05c(t)^4$, appear. The distances between the clusters of the pairs of a specific set are also plotted in the figure. Clearly, these distances have not the same length for all the clusters pairs of the set.

Moreover, in the linear case, all the labels of the clusters pairs are ordered (property of symmetric labels). This is not true in all the cases of nonlinear channels.

In an attempt to investigate the property of labels symmetry, to determine the pairs of clusters of a set and to extract information about the values of channel taps we focus our attention in two specifique cases a) even valued nonlinearities and b) odd valued nonlinearities.

1) Even valued nonlinearities:

Theorem 3: Consider a channel with even order nonlinearity of the form: $f(t) = c(t) + bc(t)^{2\nu}$. In the nonlinear -even ordered- channel case, the property of labels symmetry is valid if

$$|b| < (c_{k1} - c_{j1})/(c_{k1}^{2\nu} - c_{j1}^{2\nu}) \tag{6}$$

where, the clusters with representatives $\mathbf{f}_k = \mathbf{c}_k + b\mathbf{c}_k^{2\nu}$ and $\mathbf{f}_j = \mathbf{c}_j + b\mathbf{c}_j^{2\nu}$ form a pair of clusters and $c_{k1} > c_{j1}$.

Proof: Let us consider that the transmitted data pass through a linear filter. Then, the clusters representatives are denoted by $\mathbf{c}_k$, $k = 1, ..., Q$, and they are labeled according to Theorem 2. For example, consider the clusters $\mathbf{c}_k$ and $\mathbf{c}_j$ forming a pair of clusters where

$$-c_{k1} < -c_{j1} < 0. \tag{7}$$

Then, the label of $-\mathbf{c}_k$ is $I(t-i) = -1$ and the label of $-\mathbf{c}_j$ is $I(t-i) = +1$.

In the sequel, we assume that, the output of the linear channel passes through a memoryless even ordered nonlinearity, $f(t) = c(t) + bc(t)^{2\nu}$. Obviously, the clusters with representatives $-\mathbf{f}_k = -\mathbf{c}_k + b\mathbf{c}_k^{2\nu}$ and $-\mathbf{f}_j = -\mathbf{c}_j + b\mathbf{c}_j^{2\nu}$ correspond to transmitted data $I(t-i) = -1$ and $I(t-i) = +1$, respectivelly. However, it can be observed that for some values of b (obviously, $b > 0$) it may hold:

$$-c_{k1} + bc_{k1}^{2\nu} > -c_{j1} + bc_{j1}^{2\nu}. \tag{8}$$

Or equivalently:

$$-\mathbf{f}_k > -\mathbf{f}_j.$$

That is, due to the effect of the nonlinear function, the cluster $-\mathbf{f}_k$ has been moved to the right side of the cluster $-\mathbf{f}_j$. In this case, the cluster $-\mathbf{f}_k$ will be assigned the label -1 although lying in the right side and $-\mathbf{f}_j$ is labeled as +1 although lying in the left side of the pair. Equation (8) can also be written as:

$$b(c_{k1}^{2\nu} - c_{j1}^{2\nu}) > c_{k1} - c_{j1}.$$

Since, $c_{k1} - c_{j1} > 0$ (see (7)) then the value of b for which the labeling condition of Theorem 2 is violated is:

$$b > (c_{k1} - c_{j1})/(c_{k1}^{2\nu} - c_{j1}^{2\nu}).$$

By proceeding in the same way for $c_{k1} > c_{j1} > 0$ and $b < 0$ we conclude that Theorem 2 is not valid for b:

$$b < (c_{j1} - c_{k1})/(c_{k1}^{2\nu} - c_{j1}^{2\nu}).$$

Consequently, in general, the property of labels symmetry is not valid for b:

$$|b| > (c_{k1} - c_{j1})/(c_{k1}^{2\nu} - c_{j1}^{2\nu}). \quad (9)$$

■

For example, this condition for $x = 2$ implies that:

$$|b| > 1/(\sum_{n \neq i} 2h(n)).$$

When the symmetry of labels is valid we can proceed for the estimation of $h(i)$'s as described in the followings.

Theorem 4: The coefficients, $h(i), i = 0, ..., L$, are related to the horizontal distances, $l_{k1}(i)$, of appropriately chosen clusters pairs by the following relation:

$$2^L h(i) = \sum_{k=1}^{2^{L+1}} l_{k1}(i), \quad i = 0, ..., L. \quad (10)$$

Proof: For the ease of writing, consider a quadratic nonlinearity of the form

$$f(t) = c(t) + bc(t)^2.$$

In this case, the clusters representatives can be expressed as: $\mathbf{f}_k = \mathbf{c}_k + b\mathbf{c}_k^2$, $(k = 1, ..., Q)$ where:

$$f_{k1} = \sum_{i=0}^{L} I_{ki}h(i) + b\sum_{i=0}^{L} I_{ki}^2 h(i)^2 + b\sum_{i,j=0 \ i \neq j}^{L} I_{ki}h(i)I_{kj}h(j)$$

and

$$f_{k2} = \sum_{i=1}^{L+1} I_{ki}h(i-1) + b\sum_{i=1}^{L+1} I_{ki}^2 h(i-1)^2 +$$

$$b\sum_{i,j=1 \ i \neq j}^{L+1} I_{ki}h(i-1)I_{kj}h(j-1).$$

The two dimensional data constellation is characterized by 2^L 'rectangulars' of four clusters each. For example, in Fig. 6 we observe four rectangulars formed by the nonlinear channel $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$ and $b = 0.1$. The clusters with respective representatives: $\mathbf{f}_1, \mathbf{f}_2, \mathbf{f}_3, \mathbf{f}_4$ form a rectangular since: $|f_{11} - f_{21}| = |f_{31} - f_{41}|, |f_{12} - f_{32}| = |f_{22} - f_{42}|$. The (2^{L+1}) horizontal lines of the rectangulars have endpoints clusters which share the same transmitted data except I_{k0}.

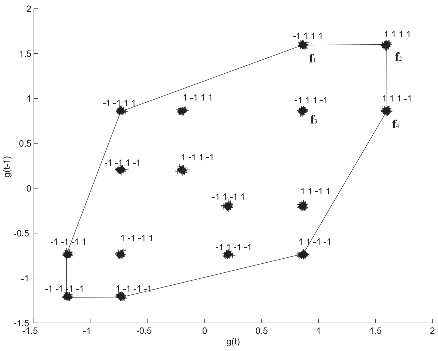

Fig. 6. Clusters formed by the channel $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$ and the nonlinear function $f(t) = c(t) + 0.1c(t)^2$.

The length, $l_{i1}(0)$, $i = 1, ..., 2^{L+1}$, of a horizontal edge of a rectangular is equal to

$$l_{i1}(0) = 2h(0) + 4bh(0)\sum_{j=1}^{L} I_{ij}h(j). \quad (11)$$

The 2^L possible values of $l_{i1}(0)$ correspond to the 2^L different combinations of I_{ij} $(j = 1, ..., L)$. By adding the lengths of all the horizontal lines of the rectangulars we get

$$\sum_{i=1}^{2^{L+1}} l_{i1}(0) = 2^L h(0). \quad (12)$$

This is true since in the sum appear all the possible combinations of I_{ij}'s which, due to the bipolar nature of data, cancel each other.

Consider now the clusters pairs having vertical distance equal to the lengths of the horizontal lines, i.e., $f_{i2} - f_{j2} = l_{k1}(0)$. These pairs will obviously share the same data except I_{i1}. Thus, by adding the corresponding horizontal distances, $l_{i1}(1)$, we get:

$$\sum_{i=1}^{2^{L+1}} l_{i1}(1) = 2^L h(1). \quad (13)$$

By proceeding in the same way for all channel taps we get:

$$2^L h(i) = \sum_{k=1}^{2^{l+1}} l_{k1}(i), \quad i = 0, ..., L. \quad (14)$$

It can be easily shown that (14) is also true in general, for all even valued nonlinearities. ■

2) Odd valued nonlinearities: In the case that odd order nonlinearities are considered it can be easily shown that the property of Theorem 4 is no more true. For example, in the case of third order nonlinearites and a channel with $L+1$ taps the clusters representatives can be expressed as:

$$f_{k1} = \sum_{i=0}^{L} I_{ki}h(i) + b\sum_{i=0}^{L} I_{ki}^3 h(i)^3 + b\sum_{i,j=0 \ i \neq j}^{L} I_{ki}^2 h(i)^2 I_{kj}h(j) +$$

$$b\sum_{i,j,n=0 \ i \neq j \neq n}^{L} I_{ki}h(i)I_{kj}h(j)I_{kn}h(n)$$

and

$$f_{k2} = \sum_{i=1}^{L+1} I_{ki} h(i-1) + b \sum_{i=1}^{L+1} I_{ki}^3 h(i-1)^3 +$$

$$b \sum_{i,j=1 \ i \neq j}^{L+1} I_{ki}^2 h(i-1)^2 I_{kj} h(j-1) +$$

$$b \sum_{i,j,n=1 \ i \neq j \neq n}^{L+1} I_{ki} h(i-1) I_{kj} h(j-1) I_{kn} h(n-1).$$

The length of an horizontal edge of a rectangular is equal to:

$$f_{k1} - f_{j1} = l_{i1}(0) = 2h(0) +$$

$$6bh(0) \sum_{n,j=1}^{L} I_{ij} h(j) I_{in} h(n) + 2bh(0)^3. \quad (15)$$

When all horizontal lines are added the following result arise:

$$\sum_{i=1}^{2^{L+1}} l_{i1}(0) = 2^L h(0) + 2^L 3bh(0) \sum_{j=0}^{L} h(j)^2 + 2^L bh(0)^3. \quad (16)$$

That is, if we follow the previously described procedure for the calculation of $h(i)$, the values of the estimated $\hat{h}(i)$ are not exact, i.e.,

$$\hat{h}(i) = h(i) + bh(i)^3 + 3bh(i) \sum_{j=0, j \neq i}^{L} h(j)^2.$$

Consequently, in this case, the channel taps are not precisely estimated by the proposed algorithm.

B. Symbol by Symbol Blind Clustering Equalizer for Nonlinear channels

In the case of nonlinear -even valued- channel and for the cases that the property of symmetric labels holds, a variation of the proposed algorithm is adopted. First the 2^L rectangulars of clusters characterizing the two dimensional data constellation are located. Then, all horizontal edges are added and the $h(0)$ tap is determined (Theorem 4). Then, the pairs that have vertical distance equal to a horizontal line segment are located. Subsequently, the horizontal distances of these pairs are added and the coefficient $h(1)$ is determined. We proceed sequentially until $h(L)$ is found.

Once the values of $h(i)$ $(i = 0, ..., L)$ are determined, the maximum estimated tap, $|h(d)|$, is chosen. Then, the pairs that are separated by the max horizontal distance $(2|h(d)|)$ are determined. Labeling is performed by assigning the -1 to the left cluster of a pair and the +1 to the right cluster of the same pair. Once labeling has been completed the data detection step follows in a similar manner as in the linear case described before. The Symbol by Symbol Clustering Based Blind Equalizer for nonlinear channels is tabulated in Table II.

TABLE II
SYMBOL BY SYMBOL CLUSTERING BASED ALGORITHM - NON LINEAR CASE

Symbol by Symbol Blind Equalizer for Nonlinear Channels
A. Estimation of the two dimensional clusters
A1. Estimation of the clusters representatives, $\mathbf{f}_k$, $k = 1, ..., Q$
Unsupervised learning
A2. Labeling of clusters
a) Find the 2^L rectangulars b) Add all the horizontal edges of the rectangulars, determine $\|h(0)\|$ c) Determine the pairs with vertical distance equal to a horizontal edge. Add all the horizontal distances of these pairs and determine $\|h(1)\|$. Proceed sequentially until all $\|h(n)\|$ are determined. d) Determine the maximum tap $\|h(d)\|$. e) Label the pairs of clusters (found in c) separated by the maximum horizontal distance left cluster: $I(t-d) = -1$, right cluster: $I(t-d) = +1$
B. Data detection
Nearest neighbor rule.

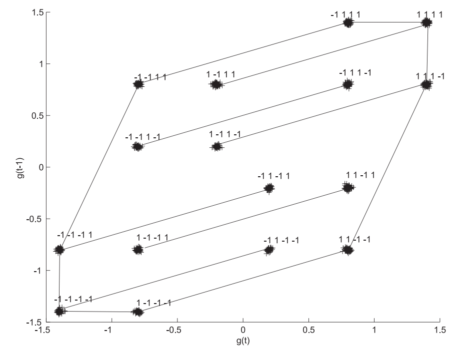

Fig. 7. Clusters formed by the linear channel $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$ and binary data.

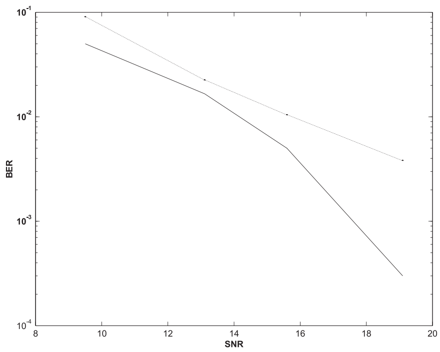

Fig. 8. Performance Comparison. Linear Channel: $H(z) = 0.3 + 0.8z^{-1} + 0.3z^{-2}$, '-': proposed equalizer, '.': blind CMA equalizer.

V. Simulation results

A. Linear channel

In the first experiment data are assumed bipolar and a non minimum phase channel is used where $H(z) = 0.3+0.8z^{-1}+0.3z^{-2}$. The Signal to Noise Ratio is defined as

$$SNR = 10log(S_g/S_w)$$

where, S_g and S_w are the signal and the noise power respectively. In this experiment the performance of the proposed equalizer is compared to the performance of a blind equalizer based on the Godard algorithm (Constant Modulus Algorithm, CMA) [11].

In Fig. 7 appear the clusters formed in the two dimensional space by this specific channel. The convex hull as well as the values of the sequence of transmitted bits: $I(t)I(t-1)I(t-2)I(t-3)$ are also plotted. The horizontal and the vertical lengths of the convex hull edges are: [0.6 0], [1.6 0.6], [0.6 1.6] [0 0.6] (each length couple corresponds to two edges). The line segments which are parallel to $E_i = [1.6\ 0.6]$ are also plotted. The maximum channel tap is $|h(d)| = 1.6/2 = 0.8$. Consequently, the proposed algorithm choose the pairs with horizontal and vertical distance equal to [1.6 0.6]. Obviously, by choosing this set of pairs the biggest horizontal separation is imposed between the clusters pairs. All the respective pairs of clusters are labeled as: $I(t-1) = 1$ the right cluster and $I(t-1) = -1$ the left cluster.

The performance of the two equalizers appears in Fig. 8. Clearly, the performance of the proposed equalizer outperforms the performance of the CMA equalizer.

B. Nonlinear channel

In the second experiment data are assumed bipolar and a minimum phase filter is adopted with $H(z) = 1 + 0.5z^{-1}$. The output of the linear filter is passed through a memoryless quadratic nonlinearity: $f(t) = c(t) + 0.2c(t)^2$. The proposed equalizer exhibits better performance compared to the performance of the CMA equalizer (Fig. 9). For comparison reasons, the performance of a blind Clustering Based Sequence Equalizer (CBSE) which performs the labeling procedure by using Hidden Markov Model [4] is included. The performance of the latter is better compared to the performance of the proposed equalizer at the expense of a much more increased complexity. Moreover, the performance of the proposed equalizer is also compared to the performance of a nonlinear equalizer of RLS type which models the channel nonlinearities as Volterra series. From the figure it is seen that the performance of the two equalizers is comparable, however, the proposed equalizer has a smaller complexity as discussed later.

Next, the performance of the proposed equalizer is investigated in case of even and odd valued nonlinearities. The channel is a cascade of the linear filter $H(z) = 1+0.5z^{-1}$ and the nonlinearity $f(t) = c(t)+0.15c(t)^2+0.1c(t)^3+0.05c(t)^4$ [12]. Although the channel taps are not precisely estimated by the algorithm, the proposed equalizer can be applied at least for the case of mild odd valued nonlinearities as it is illustrated by this simple example. The performance of the four equalizers is illustrated in Fig. 10.

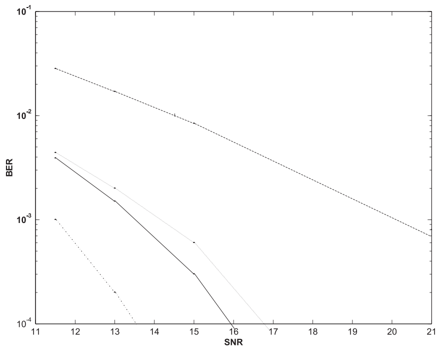

Fig. 9. Performance Comparison. Nonlinear Channel: $H(z) = 1+0.5z^{-1}$, $f(t) = c(t)+0.2c(t)^2$, '-': proposed equalizer, '.': blind Volterra, '- -': blind CMA, '.-': blind CBSE equalizer.

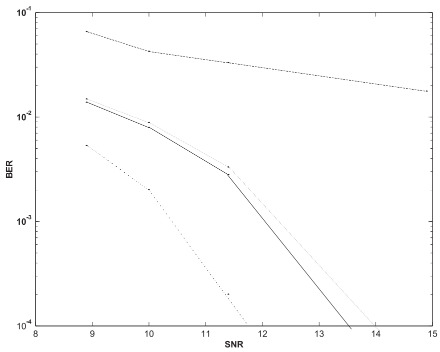

Fig. 10. Performance Comparison. Nonlinear Channel: $H(z) = 1+0.5z^{-1}$, $f(t) = c(t)+0.15c(t)^2+0.1c(t)^3+0.05c(t)^4$, '-': proposed equalizer, '.': blind Volterra, '- -': blind CMA, '.-': blind CBSE equalizer.

The above results have been verified on a variety of channels and channel nonlinearities and indicate the effectiveness of the algorithm.

C. Complex Data

The proposed equalizer can also be applied to the case of complex transmitted data, i.e., $I(t) = I_R(t) + jI_I(t)$. Then, the detection of the transmitted data is decomposed into two subproblems: detection of real part of data and detection of imaginary part of data [6]. Then the algorithm is applied first to the real part of the received data, $g_R(t)$ and then to the imaginary part of the received data, $g_I(t)$. The output of the algorithm corresponds to the real and imaginary transmitted data respectively, i.e, $I_R(t)$, $I_I(t)$. Consider, for example, the case of 4-QAM data and channel with $H(z) = 1 + 0.5z^{-1}$. Fig. 11 illustrates the two dimensional clusters formed by the real part of the transmitted data. Clearly, this constellation is identical with the constellation formed in the case of binary data (Fig. 2) and contains all the information related to the channel, as in the real data case.

D. Computational complexity

The complexity of the proposed equalizer depends on the complexity of a) the unsupervised clustering algorithm b) the

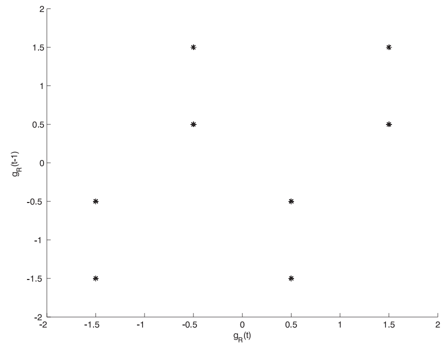

Fig. 11. The real part of clusters formed by the linear channel $H(z) = 1 + 0.5z^{-1}$ and 4-QAM data.

labeling procedure and c) the decision task. The clustering and labeling algorithms are batch procedures performed once using a finite number of data. The total complexity is substantially reduced compared to the complexity of the other cluster based equalizers which a) adopt a complicated labeling algorithm (e.g., an HMM procedure [4]) and b) operate on a sequence decision mode. It should also be emphasized the fact that the complexity of the decision step is very small. More precisely, the decision step consists of a simple comparison of the incoming data with all clusters representatives (2^{L+2}). However, only a small number of comparisons need to be performed since data are first compared with the middle cluster (clusters are assumed to be ordered in the output of the representatives identification algorithm). Next comparison is limited to the half of clusters and the incoming data is compared with the middle cluster of this section and so on. The procedure is repeated until the nearest cluster is found. This procedure gives rise to a very small amount of comparisons.

E. Non identifiable channels

As most clustering algorithms, the proposed equalizer is appropriate for channels that their clusters are not overlapped in the 2- dimensional space. In the linear case this can arise due to the channel nature, or due to very high noise level. In the nonlinear case, the type of nonlinearity can be an extra reason of clusters overlapping. Moreover, as it is already noted, the algorithm is not appropriate for high levels of even nonlinearities which make clusters to exchange positions and thus alter the labels ordering.

VI. Conclusion

A new blind cluster based symbol by symbol equalizer has been proposed. The equalizer consists of 3 steps: a) clusters identification through an unsupervised learning algorithm, b) labeling by unraveling the symmetric properties of labels in the two dimensional clusters constellation and c) symbol by symbol data detection. The algorithm is applicable to linear channels, where an exact solution is given. Moreover, it can be applied to nonlinear channels where a mild polynomial nonlinearity is assumed. Extensive simulation provide evidence that the performance of the equalizer is superior to the performance of a CMA-based blind equalizer. An additional advantage of the proposed method is the low complexity associated to the decision step.

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