Modelling the Occurrence of Defects and Change Requests during User Acceptance Testing

Kevin McDaid, and Simon P. Wilson

Abstract—Software developed for a specific customer under contract typically undergoes a period of testing by the customer before acceptance. This is known as user acceptance testing and the process can reveal both defects in the system and requests for changes to the product. This paper uses nonhomogeneous Poisson processes to model a real user acceptance data set from a recently developed system. In particular a split Poisson process is shown to provide an excellent fit to the data. The paper explains how this model can be used to aid the allocation of resources through the accurate prediction of occurrences both during the acceptance testing phase and before this activity begins.

Keywords—User acceptance testing. Software reliability growth modelling. Split Poisson process. Bayesian methods.

I. INTRODUCTION

The testing of modern software systems to remove remaining defects is a difficult and costly business. Under the traditional Waterfall development methodology, it begins with unit testing and progresses to integration testing to ensure the separate components interface correctly. It is usual to finish with a prolonged amount of system testing where a combination of testing methods can be used. These seek to remove remaining defects and to establish that the system complies with the requirements for the software.

In the case of a system developed for a single customer, known as bespoke software, it is also standard to include a final testing phase immediately before release, conducted collaboratively by the customer and the developing organization, to ensure that the product complies with the requirements from the customer’s perspective. This is known as User acceptance Testing. A detailed description of this activity can be found in [1] and [2]. This phase can reveal defects or errors in the software. Unlike traditional system testing, it can also reveal issues that are effectively additional to or changes to the requirements. These are known as change requests and the developing organization would normally charge the customer the cost of implementing these changes to the system.

The best practice of Software Reliability Engineering, as detailed in [3], has developed a range of probabilistic models to predict the reliability of a system during operation using the occurrence time of defects during testing. These are known as Software Reliability Growth models (SRGM’s) and they have focussed almost exclusively on defect data form the system testing phase. This paper examines the application of a particular class of models, known as nonhomogeneous Poisson processes, to defect and change request data occurring during user acceptance testing of a recently developed commercial bespoke distributed database system. Building on these models the paper proposes a split Poisson process model to represent the data.

In addition to predicting the achieved reliability of a software system, test and project managers can use SRGM’s to predict the number of defects that will occur over a future time period. This can allow for more accurate resource allocation and possibly, depending on the exact release criteria used by the firm, support the decision when to terminate the system testing phase of the lifecycle ([4], [5]).

Of course, accurate predictions early in the testing process are of most value to project and test managers. Unfortunately, during the initial stages of system testing the amount of defect data available is limited and standard approaches to fitting SRGM’s, such as maximum likelihood methods, can be problematic ([6], [7], [8]). This paper addresses this problem in the context of user acceptance testing using Bayesian methods that combine the limited existing defect data with the expert opinion of key personnel. In this way improved estimates for the parameters of the SRGM’s can be obtained, which in turn can lead to better defect and change request occurrence predictions.

The traditional approach to software development relies heavily on the system testing phase to ensure the released product is highly reliable. Experience has shown that without substantial testing at this stage it is likely that the product will most likely contain a large number of faults with the potential to lead to regular failure during operation. This in turn will result in high post-release costs and a significant loss of consumer confidence. Thus, it is important to allocate a significant amount of time to the activity of system testing. However, it is also important to avoid over testing the product. This can result in a piece of software which, although very reliable, may be overpriced and possibly obsolete. On the whole it is crucial for the test manager to achieve a balance between under testing and over testing. This is also an issue for the customer organization who must decide how long to allow between under testing and over testing. This is also an issue for the customer organization who must decide how long to allow for the user acceptance testing phase. In fact the decision is more complicated in their case as they must also allow for the difference between the cost of implementing change requests before and after acceptance of delivery of the product.

This paper contributes to the existing knowledge base in a number of ways. It develops and assesses, based on real data, an original model for an under-researched problem, namely the occurrence of defects and change requests during user acceptance testing. It demonstrates its application to prediction before and during the testing process through the combination

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of defect data with expert knowledge.

The paper is summarized as follows. Section II introduces the area of Software Reliability Engineering and explains briefly nonhomogeneous Poisson process models and their application to defect data. The rationale for their use with defect and change request data arising from user acceptance testing is also explained. A split Poisson process models is developed and compared with independent Poisson process models for defects and change request. The comparison is made on the basis of real data which is also detailed in this section. Section III explains how to apply the models to the prediction of occurrences. A Bayesian model is developed to ensure that stable predictions can be made throughout the user acceptance testing phase. The quality of these predictions are assessed. The method used to elicit expert opinion is also explained. IV concludes the work with a short summary.

II. A Probability Model for Occurrences During User Acceptance Testing

This section introduces the data under examination and introduces a number of nonhomogeneous Poisson processes to model the occurrence of change request and defects.

A. User Acceptance Testing Data

The system under investigation was developed using a V-model approach by a large software company for a government department. The on-line system, supported by significant batch and database facilities, included a range of functionality from basic detail capture to complex rule-based calculations. Development required approximately 3,500 person days of effort over a period of eighteen months involving 29 different personnel. A more detailed description is provided in [9] and [10].

The available data for the software system is taken from the pre-release system testing and user acceptance testing phases and the post-release operational phase. In this paper we examine the occurrences over the user acceptance testing period only. Before considering the application of software reliability growth models we must establish a suitable unit of time. A difficulty arises here due to the lack of information on the time spent by personnel testing the system. The analysis in this paper uses calendar days as the unit of time, with days where UAT testing did not take place excluded from the analysis. While weekend and Christmas periods were clear cases of inactivity, every effort, including examination of time sheets and discussions with senior staff, has been made to exclude days when testing activity did not take place.

The data gathered during the user acceptance testing phase distinguishes between defects and change requests, where change requests are user-suggested changes to the system. Defects and change requests when taken together are termed issues. Figure II-A shows a plot of the occurrence of defects and change requests over time, where time is the standardized time as described earlier. Note that in the first 50% of the time (45 days) 72% of the issues were found (237 out of 329). Also note that the data fails to show any continuity between system and user-acceptance testing. The rate of defect occurrence at the end of the system testing phase is more than twice that at the start of the user acceptance testing phase reflecting the different testing strategies. In system testing, effort is focused on the revelation of the highest possible number of failures whereas in user acceptance testing the emphasis is on the verification of the system through the application of real inputs by potential users.

To finish this subsection we note that the user acceptance testing was conducted by the customer with the help of the developers. This raises the question as to whether the testing can be considered as statistical or based on a more structured coverage approach. While there is little information available on the exact inputs applied, the fact that this testing was driven by the customer using real data would indicate that the method may be closer to statistical rather than a structured coverage-based approach. That said, other research, [11], supports the use of the class of models we consider in Section II-B for defect data arising through testing strategies that involve the selection of inputs to maximize some measure of achieved coverage.

B. Nonhomogeneous Poisson Process Model

Numerous probabilistic models are available in SRE for the purpose of reliability and defect occurrence prediction. These models are usually classified as data-domain or time-domain models, ([12] and [13]), with each category containing several sub-categories. A widely accepted group of these models are the class of non-homogeneous Poisson processes (NHPP) where the total number of defects expected is finite. These were adopted from hardware reliability models between the early 1970s and the mid 1990s. Of these models four have achieved prominence, namely the Goel and Okumoto [14] model and Yamada S-Shaped model [15], each based on two parameters, and the three parameter Weibull and Log-logistic models [11]. The mean value functions for each of these is shown in Table I.

A relatively simple explanation of the Goel-Okumoto model is possible through the mean value function presented, which gives the cumulative number of defects that would be expected to occur before each time point, $T$. For the Goel-Okumoto model, the cumulative number of defects $D(t)$ at time $t$ is given by:

$$D(t) = \int_0^t \lambda(s) \, ds$$

where $\lambda(s)$ is the rate function. The mean value function $M(t)$ is the expected value of $D(t)$:

$$M(t) = \int_0^t \lambda(s) \, ds$$

Fig. 1. Cumulative occurrence of software defects and change requests.
model the mean value function has two parameters \((a, b)\). Typically, these are estimated from the occurrence times of defects during testing. For this form the average number of defects discovered by time \(T\), \(m(T)\), is of course increasing over time but the rate of increase, representing the rate at which new defects come to light, slows with the highest rate value at the beginning of testing. The other models differ from the Goel-Okumoto in shape as the rate of occurrence initially increases before then decreasing. For the Goel-Okumoto model the \(a\) parameter represents the number of defects that would be found were testing to continue at infinitum. The \(b\) parameter represents the rate at which defects come to light. More specifically, it gives (approximately) the proportion of the remaining defects that would be discovered in a single unit of testing.

The NHPP software reliability models contain many assumptions, some of which are unrealistic, including the instant and perfect repair of defects. However, in practice, \([16]\) is a good example, the models has been found to provide a good mechanism for modelling software failure data. Estimation of the parameters of these models is normally performed by maximizing the likelihood or density function which, in the case of ungrouped data, can be written as

\[
f(t_1, t_2, \ldots, t_n) = e^{-m(T)} \prod_{i=1}^{n} \Lambda(t_i),
\]

where \(\Lambda(t)\) is the rate function found by differentiating the mean value function and \(T\) is the time at which testing is censored with \(n\) occurrences at \(t_1, t_2, \ldots, t_n\) before this point.

In the case of the Goel-Okumoto model with parameters \(a\) and \(b\) the specific form is

\[
(ab)^n e^{-a(1-e^{-bT})} e^{-b \sum_{i=1}^{n} t_i}.
\]

The maximum likelihood value for the \(a\) parameter is given by

\[
a = \frac{n}{1 - e^{-bT}},
\]

where the value for \(b\) is found by numerically solving the following:

\[
\frac{n}{b} = \sum_{i=1}^{n} t_i + \frac{nT e^{-bT}}{1 - e^{-bT}}.
\]

However, as other authors have documented \([6]\) and \([17]\), there can be significant issues with this approach. Specifically, there are combinations of defect occurrence values for which the maximum likelihood estimates for the Goel-Okumoto model do not exist. This situation occurs most often during the early stages of testing. However, a further problem arises where, although the maximum likelihood estimates for the parameters can be derived, the resulting values are unrealistic. This again predominantly occurs early in testing and often yields very high values for the \(a\) parameter in the case of the Goel-Okumoto and the Yamada S-shaped model. We highlight this problem in Section III through the real industry example. Note that this drawback also applies to the Weibull and Log-logistic models.

Importantly, the mean value functions for the NHPP models discussed above can all be written in the form \(a(C(t))\) where \(a\) is the number of defects that may be discovered were testing to continue at infinitum and \(C(t)\) can be thought of as the percentage coverage achieved over time. It is this structure that is used in \([11]\) to argue that these models are suitable for application in the case where the defect data arises through testing that is coverage-based rather than operational profile based. We now turn our attention to the fitting of these models.

### C. Independent Poisson Process Model

The simplest approach to modelling the occurrence of defect and change request data would assume that the processes were independent and fit nonhomogeneous Poisson process models to the defect and change request data separately. We have fitted the four models to each of the series. The Goel-Okumoto, Weibull and Log-logistic models result in very similar maximum likelihood values with the S-shape model performing relatively poorly. Of these, the Goel-Okumoto model requires one fewer parameters and should thus be considered as the best choice to model each of the defect and change request data sets. A formal calculation of the Akaike Information Criterion (AIC) supports this conclusion. It is not surprising that the Goel-Okumoto model is chosen given the shape of the curves. Figure II-C shows the data and the fitted Goel-Okumoto models. It is clear that the model represents the actual data reasonably well with some problems with the fit to the data during the period from day 20 to 60.

Based on the maximum likelihood methods the model values were found to be \(a=266\) and \(b=0.017\) for the defects and \(a=143\) and \(b=0.022\) for the change requests. The relative values for the rate parameters indicate that the user acceptance testing phase may be more successful at revealing change requests than revealing failures. The question we next consider is whether a simpler model using three rather than four parameters can be found to fit the data.

### D. Split Poisson Process Model

Previously we fitted independent Poisson process models to the defect and change request data. A closer examination of the user acceptance testing process may reveal clues as to a potential alternative model. Within the process it seems that when an issue arose it was later classified as a defect or a change request. This would indicate an underlying process where the occurrences are separated into two types. We
assume a probability, \( \theta \), that the occurrences are of type I (defects) and \( 1 - \theta \) that the occurrences are of type II (change requests). If this value of \( \theta \) does not change over time then this is the case of a split Poisson process. Suppose defects occur at times \( t_1, t_2, \ldots, t_n \) and change requests at times \( s_1, \ldots, s_m \) before censoring time \( T \) then the density function, \( f(t_1, t_2, \ldots, t_n, s_1, \ldots, s_m) \), is given by

\[
(ab)^{n+m}e^{-(1-e^{-\theta T})}e^{-b(\sum t_i + \sum s_i)} \left[ \theta^n(1 - \theta)^m \right],
\]

which can also be written in the following form

\[
((1 - \theta)ab)^{n+m}e^{-(1-\theta T)}e^{-b(\sum t_i)} \times (1-\theta)^{n+m}e^{-b(\sum s_i)}. \tag{6}
\]

This shows that the occurrence of defects and change requests can be treated as independent nonhomogeneous Poisson processes with mean value functions given by \( \theta a (1 - e^{-\theta T}) \) and \( (1 - \theta)a (1 - e^{-b T}) \) respectively. This is a well-known result from Poisson process theory and implies in this study that the occurrence of defects follows a Goel-Okumoto model with parameters \( a \) and \( b \), and the occurrence of change requests follows an independent (given model parameters) Goel-Okumoto model with parameters \( a(1 - \theta) \) and \( b \).

The structure of the density function in Equation 5 implies that the maximum likelihood estimates for the \( a \) and \( b \) parameters can be found using the solution presented in Equations 3 and 4. The maximum likelihood estimates for \( \theta \) is the proportion of defects in the data, namely \( \frac{n}{n+m} \). We fit the split Poisson process model to the data and display the result in Figure 3. The common rate parameter is estimated as 0.0185 with the overall \( a \) parameters given by 406 with \( \theta = 0.63 \) yielding \( a \) values for the defects and change requests given by 255 and 150 respectively. Crucially, the AIC value for this model is 173.75 which is less than the value of 174.76 for the four parameter Goel-Okumoto models presented earlier. This provides some indication that the benefit of a reduced parameter model (three as opposed to four) outweighs the reduction in fit of the model. Having established the model we will show shortly how it can be use for prediction purposes.

As the mean value function for the four models presented in Table I can be written as \( a[C(t)] \) it is relatively straightforward to develop a split process in the case of each of these models.

III. PREDICTION OF DEFECT AND CHANGE REQUEST OCCURRENCE

The power of a model is in its potential to provide accurate predictions for future occurrences. In this section we examine how this can be achieved both before and after the user acceptance testing phase.

A. Issues with Early Prediction

As mentioned earlier, estimation of the parameters of these models is normally performed using maximum likelihood methods. It is well known ([16], [17]) that there can be significant issues with this approach as there are combinations of defect occurrence values for which the maximum likelihood estimates for the Goel-Okumoto model do not exist. A further problem arises where, although the maximum likelihood estimates for the parameters can be derived, the resulting values are unrealistic. These difficulties are most likely to occur early in testing and often result in very high values for the \( a \) parameter. We illustrate this process in Figure 4 where the split Poisson process model is fitted based at the 20 and 30 percent time points using the data available up to that point. The figure shows the poor performance of the predicted cumulative number of defects and change requests following that point. Note that it is not possible to fit the data using occurrences up to the 10 % point.

A further criticism of the maximum likelihood approach is the fact that it ignores the expert knowledge of key project personnel accumulated over possibly years of involvement with the industry and organization in question. This knowledge should influence the selection of the parameters and should in particular protect against the selection of extreme values for the parameters. The approach also suffers from the lack of a mechanism to predict the occurrences prior to the commencement of testing, when predictions are of most value to the firm. To counter both these drawbacks we next present a Bayesian version of the split Poisson process model and apply it to the prediction problem.
B. Bayesian Split Poisson Process Model

Assuming the split Poisson process model in the previous section, we adopt a Bayesian model by placing prior distributions on the model parameters, $a$, $b$ and $\theta$. Independent prior gamma distributions are placed on the $a$ and $b$ parameters with a prior beta distribution placed on the $\theta$ parameter. These assumptions are flexible and allow us to incorporate a wide variety of prior knowledge.

Specifically, we assume that the $a$ is represented by a prior gamma distribution with parameters $\tau$ and $\lambda$, $b$ by a prior gamma distribution with parameters $\alpha$ and $\mu$ and $\theta$ by a prior beta distribution with parameters $\omega$ and $\rho$. The choice of a prior gamma distribution is consistent with [18] and [2]. Thus the prior structure is as follows:

$$f(a, b, \theta | \tau, \lambda, \alpha, \mu, \omega, \rho) = \frac{a^{\tau-1}e^{-\lambda a}b^{\mu-1}e^{-\mu b} \Gamma(\alpha) \Gamma(\mu) \theta^{\omega-1}(1-\theta)^{\rho-1} \Gamma(\omega+\rho)}{\Gamma(\alpha) \Gamma(\mu+\omega+\rho)} (7)$$

As the prior distribution for the proportion, $\theta$ and the Goel-Okumoto model parameters $a$ and $b$ can be separately we can use available results to generate prediction methods for the number of defects and change requests based on the posterior distribution.

We next illustrate how to evaluate the number of defects that would be expected to occur after $T_1$ units of acceptance testing, assuming $T(T \leq T_1)$ units of testing has already taken place resulting in $n$ defects at times $t_1, t_2, \ldots, t_n$ and $m$ change requests at $s_1, s_2, \ldots, s_m$. This is developed as

$$E(N_D(T_1)|\bar{N}_D, \hat{\bar{N}}_D, \bar{N}_CR, T_1) = \frac{\Gamma(n + m + \tau + i + 1)}{i!(1+\lambda)^i \sum_{j=0}^{\infty} \frac{(1+\lambda)^j \Gamma(n + m + j + 1)}{j!(1+\lambda)^j}} \frac{(1+\lambda)^{n+m+\tau+i+1}}{\sum_{j=0}^{\infty} \frac{(1+\lambda)^j \Gamma(n + m + j + 1)}{j!(1+\lambda)^j}} \frac{Y^\alpha}{\lambda^{\tau+1} \mu^{\rho+1}}$$

where

$$Y = \frac{\Gamma(n + m + \tau + j)}{\sum_{j=0}^{\infty} \frac{(1+\lambda)^j \Gamma(n + m + j + 1)}{j!(1+\lambda)^j}} \frac{Y^\alpha}{\lambda^{\tau+1} \mu^{\rho+1}}$$

and $\bar{N}_D(T_1)$ represents the combined number of defects and change requests occurring after time $T_1$. The prediction for the number of defects before testing begins at which point only prior information is available can be shown to be

$$E(N_D(T_1)|\tau, \lambda, \alpha, \omega, \rho) = \frac{\omega}{\omega+\rho} \frac{\tau}{\lambda} \left( \frac{\mu}{\mu+T_1} \right)^\alpha (9)$$

The prediction for the number of change requests, corresponding to Equation 8, has an identical second term with the first term given by $\omega+n+\rho+m$. Note that the expansion of the $E_{a,b}(N_{D,CR}(T_1))$ term in Equation 8 follows from work in [18]. The fact that the posterior distribution for the proportion, $\theta$, follows a beta distribution with parameters $\omega+n$ and $\rho+m$ is used to expand the $E_{a,b}(\theta)$ term.

C. Assessment of Predictions

We next apply the Bayesian split Poisson process model to the data presented previously. We assume two separate sets of prior values for the model parameters, one which represents the situation where the expert opinion is very close to the true values and the second where the prior opinion yields estimates for $a$ and $b$ which are higher than the ideal values and the prior values for the beta distribution represent a uniform distribution. The second set illustrates the situation where the expert provide poor, yet not completely unrealistic, prior information. Table II shows the two selected parameter sets. The mean and standard deviation of the prior distributions are also presented as, for comparison purposes, are the maximum likelihood estimates based on all the available defect and change request data.

The predictions for the remaining number of defects and change requests based on the 20 and 30 percent points are presented in Figure 5. These predictions, for both sets of parameters, are a significant improvement on those presented in Figure 4.

IV. CONCLUSION

This work develops a relatively simple model for the occurrence of change requests and defects during the user acceptance testing phase for a bespoke piece of software. This can be of benefit to development firms in their efforts to forecast the required resources to support the repair of defects and the implementation of change requests. We illustrate the issues with prediction using the model and develop a Bayesian model to overcome these problems and at the same time allow for the incorporation of expert opinion. Similarly, the split nonhomogeneous Poisson process model can also be of benefit to the customer organization who can use it to predict the total cost and to decide how long to conduct user acceptance testing before accepting the product for operation.
TABLE II
PRIV PARAMETERS SELECTIONS COMPARED WITH MAXIMUM LIKELIHOOD ESTIMATES

<table>
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<tr>
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<th>Set 1</th>
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<th>Set 2</th>
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<td>Mean a</td>
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<td>100</td>
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<tr>
<td>Mean b</td>
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<tr>
<td>MLE θ</td>
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Fig. 5. Predictions at 20 and 30% points using Bayesian split Poisson process.

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