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Using stresses obtained from a low detailed FE model and located at a reference point to quickly calculate the free-edge stress intensity factors of bonded joints

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***Abstract*—**The present study focuses on methods allowing a convenient and quick calculation of the SIFs in order to predict the static adhesive strength of bonded joints. A new SIF calculation method is proposed, based on the stresses obtained from a FE model at a reference point located in the adhesive layer at equal distance of the free-edge and of the two interfaces. It is shown that, even limiting ourselves to the two main modes, i.e. the opening and the shearing modes, and using the values of the stresses resulting from a low detailed FE model, an efficient calculation of the peeling stress at adhesive-substrate corners can be obtained by this way. The proposed method is interesting in that it can be the basis of a prediction tool that will allow the designer to quickly evaluate the SIFs characterizing a particular application without developing a detailed analysis.

***Keywords*—**Adhesive layer, Bounded joints, Free-edge corner, Stress intensity factor.

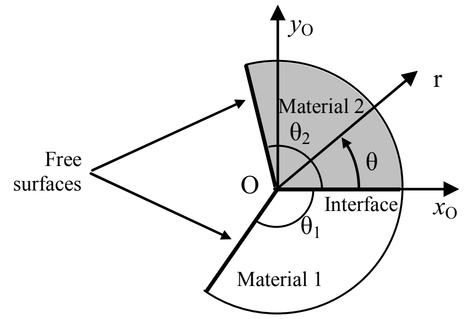
I. Introduction

MANY modern engineering structures contain adhesively bonded joints because they offer certain advantages over mechanical connectors. It becomes important to investigate the mechanical response near the wedge corner under various loading conditions as there are a lot of applications where the risk of adhesive rupture cannot be eliminated. Adhesive failure occurs when the fracture initiates at one of the joint free-edge interface corners, and spreads along the interface. The designer who wishes to predict the static adhesive strength inherent to a new design can refer to the concept of stress intensity factor (SIF) in order to formulate a criterion.

As it is well-known, a stress singularity develops at wedge corners [1]-[2]. The asymptotic free-edge stress fields near an interface corner may be obtained using the Airy's stress function approach. It can be shown that the stresses subjected to a remote mechanical load, and having the local edge geometry shown in Fig. 1, are of the form:

$$\sigma_{ij}^{m} = \sum_{l=1}^{n} K_l \; r^{\lambda_l - 1} \; f_{ijl}^{m}(\theta)$$

where $(i,j) \equiv (r,\theta)$ are plane polar coordinates centred at the interface corner; m is the material number (m = 1,2); λ_l are the eigenvalue of the problem (l = 1,n); f_{ijl} are non-dimensional constant functions of the material elastic properties, the eigenvalue λ_l, the local edge geometry characterized by angles θ_1 and θ_2, and of the polar coordinate θ; and K_l is the wedge corner stress intensity factor associated with the eigenvalue λ_l.



Corner O
Material 1 in region $-\theta_1 \le \theta \le 0$
Material 2 in region $0 \le \theta \le \theta_2$

Fig. 1 General configuration at the interface corner between two dissimilar materials [3]

The K_l are defined such that f_{ijl} = 1 along the interface (θ = 0). Then the peeling stress is obtained directly by:

$$\sigma_{\theta\theta}^{m}(r,0) = \sum_{l=1}^{n} K_l \; r^{\lambda_l - 1}$$

The eigenvalue λ_l are the root of the following equation [4]:

$$G(\theta_1,\theta_2,\alpha,\beta,\lambda) = A\,\beta^2 + 2\,B\,\alpha\,\beta + C\,\alpha^2 + 2\,D\,\beta + 2\,E\,\alpha + F = 0$$

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where :

$$A = 4\,H(\lambda,\theta_1)\,H(\lambda,\theta_2)$$
$$B = 2\,(1+\lambda)^2\,[\sin^2\theta_2\,H(\lambda,\theta_1) + \sin^2\theta_1\,H(\lambda,\theta_2)]$$
$$C = 4\,(1+\lambda)^2\,[(1+\lambda)^2 - 1]\sin^2\theta_2\sin^2\theta_1 + H(\lambda,\theta_2-\theta_1)$$
$$D = 2\,(1+\lambda)^2\,[\sin^2\theta_2\,H(\lambda,\theta_1) - \sin^2\theta_1\,H(\lambda,\theta_2)]$$
$$E = -D + H(\lambda,\theta_1) - H(\lambda,\theta_2)$$
$$F = H(\lambda,\theta_2+\theta_1)$$
$$H(\lambda,\theta) = \sin^2[\theta\,(1+\lambda)] - (1+\lambda)^2\sin^2\theta$$

α and β are Dundurs composite parameters [5] which are defined as :

$$\alpha = \frac{k\,L_2 - L_1}{k\,L_2 + L_1} \qquad \beta = \frac{k\,(L_2-2)-(L_1-2)}{k\,L_2 + L_1}$$

where $k = \mu_1/\mu_2$, $\mu_m = E_m/2\left(1+\nu_m\right)$, E_m and ν_m denote shear modulus, Young's modulus and Poisson's ratio for material *m* = (1,2), respectively, and $L_m = 4(1-\nu_m)$ for plane strain. The value of λ_l can be real or complex in general.

In this paper, as in most of the available studies, only the stress term associated with the smallest positive real eigenvalue λ_1 ($0<\lambda_1<1$) will be considered, the contribution from the higher order terms being neglected. The peeling stress at the interface takes the following simple form:

$$\sigma_{\theta\theta}(r,0) \approx K\,r^{\delta} \quad (\delta < 0) \tag{1}$$

where *r* is the distance from the corner, *K* the SIF and δ the order of the singularity ($\delta = \lambda_1 - 1 < 0$).

Being particular bi-material wedges, the different interface corners of any bonded joint can be analyzed using this approach. The adhesive layer must be considered as such in the model because the objective is to exhibit the stress fields that take place between this adhesive and the substrates in the region where the adhesive fracture initiates. Fig. 2 shows one of the possible geometrical configurations which characterize usually a single lap joint with square edges at the end of the layer. It makes appear four different corners, noted A, B, C and D. Some parameters describing the corners A and C in accordance with the previous method are given in the figure. Other configurations could have been considered as well, for example those related to double lap joint, single strap joint…, or with a spew fillet at the end of the adhesive layer.

Each corner leads to particular values of δ and *K* as it refers to a specific couple of angles and materials. From the materials proposed in Table I, from the interface corner parameters specified in Tables II, the Table III gives the singularity orders which characterize the different corners of the configuration defined in Fig. 2. They vary from $\delta_A = -0.3386$ to $\delta_C = -0.2680$.

TABLE I
PARAMETERS DEFINING THE MATERIALS

	Adherend n°1	Adherend n°2	Adhesive layer
Material	Steel	Aluminium	Epoxy
Young's Modulus	E_1 = 207 GPa	E_2 = 69 GPa	E_a = 3.5 GPa
Poisson's ratio	ν_1 = 0.3	ν_2 = 0.33	ν_a = 0.35

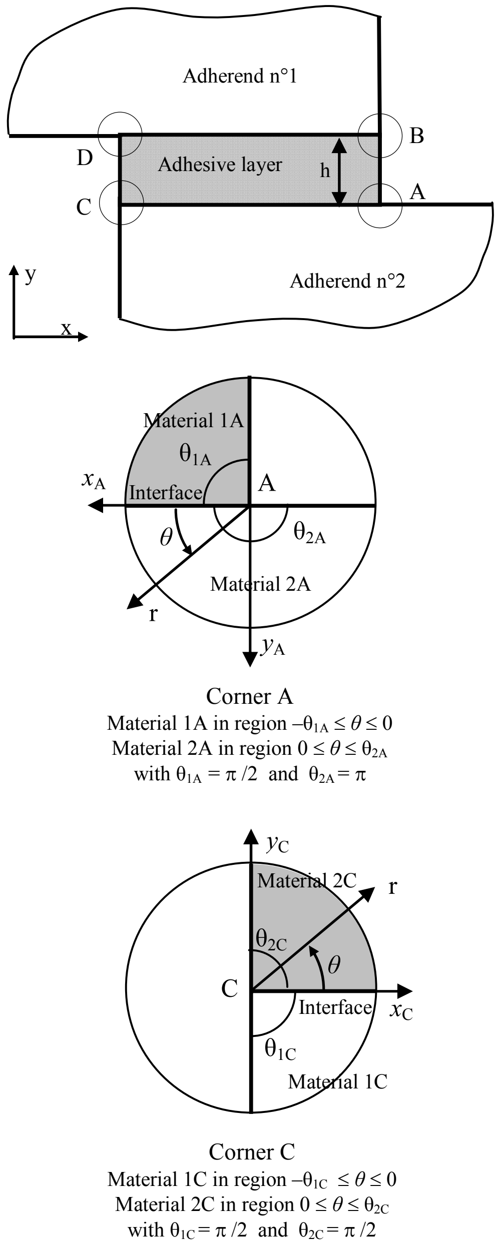

Fig. 2 A single lap joint configuration with square edges at the end of the layer

TABLE II
PARAMETERS DEFINING THE INTERFACE CORNERS

	θ_1	Material 1	θ_2	Material 2
Corner A	$\pi/2$	Adhesive	π	Aluminium
Corner B	π / 2	Steel	π / 2	Adhesive
Corner C	π / 2	Aluminium	π / 2	Adhesive
Corner D	π / 2	Adhesive	π	Steel

TABLE III
SINGULARITY ORDERS

	α	β	$\delta=\lambda_1-1$
Corner A	0.9020	0.2070	-0.3386
Corner B	0.9655	0.2218	-0.3016
Corner C	0.9020	0.2070	-0.2680
Corner D	0.9655	0.2218	-0.3272

Regarding the SIF K, the literature proposes the use of a critical value, noted K_{crit}, as a fracture initiation criterion [6]-[7]. The value of K_{crit}, which characterizes the strength of the interface, must be evaluated from tests for the different configurations that may be encountered (material combinations, corner geometries, bonding processes). During a pre-sizing phase, when it is necessary to check the capability of a new application, the designer must calculate the value of K at the different singularities of the joint, and verify that these values remain below than the related K_{crit}. The evaluation of the magnitude of K has led to recent interest for various joint geometries and loading [6]-[7]. The solution that is generally proposed in the literature to calculate the SIFs is based on the development of FE analyses using very fine mesh near the corner points. The Fig. 3 illustrates one of the first models that have been proposed by Reedy [8]-[10]. In this model, the configuration is idealized as the adherends are considered as rigid. Plane strain finite elements are used, and the applied shear loading is obtained by displacing the layer's lower edge relative to the fixed upper edge. Then, the SIF is determined from FE stresses at the interface by curve fitting using for example a least square algorithm. As illustrated in the Fig. 4, [11]-[12] shown that the extent of the region to be considered is about $10^{-5} < r/h < 10^{-2}$ where h is the adhesive layer thickness. The need in meshing refinement near the corner points is directly dependent on these values.

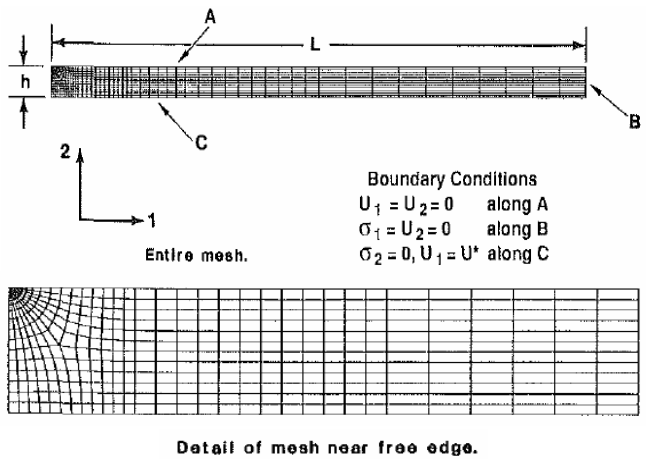

Fig. 3 Typical finite element mesh used in FE analysis [8]

Methods allowing a more direct and quick calculation of K are expected here. The approach of Wang & Rose [12] is one of the more interesting among those given in the literature. They have proposed the following general form, which considers the superposition of two principal loading modes, like in fracture mechanics:

$$K = K_I + K_{II}$$

with :

$$K_I = \sigma^* \; h^{-\delta} \; A(\nu_a) \quad \text{and} \quad K_{II} = \tau^* \; h^{-\delta} \; B(\nu_a) \qquad (2)$$

where:

1) K_I , K_{II} are the SIFs related to mode I (opening mode) and mode II (shearing mode) respectively,
2) σ^* and τ^* are stresses representative of the corner external loading,
3) h is the adhesive layer thickness and δ the singularity order. The factor $h^{-\delta}$, which is based on dimensional considerations, has been proposed first by Reedy [10],
4) A and B are non-dimensional stress intensity functions. They are solely dependent on adhesive Poisson's ratio, because, for simplicity, they are determined once and for all using full finite-element computations in the hypothesis of rigid adherends.

But someone who wants to practice this method is helpless when it seeks to implement it: the main difficulty is linked to the absence of a precise definition of the stresses that have to be extracted from models and used as reference. The use of σ_{Max} and τ_{Max}, the maximal peeling and shearing stresses in the adhesive layer in the vicinity of the corner, has been suggested [12]. But the evaluation of these maximal stresses is not so easy: their localization is unknown a priori, they are dependent on the mesh refinement when FE analyses are used, and they suffer of large approximations when they are calculated by an analytical method.

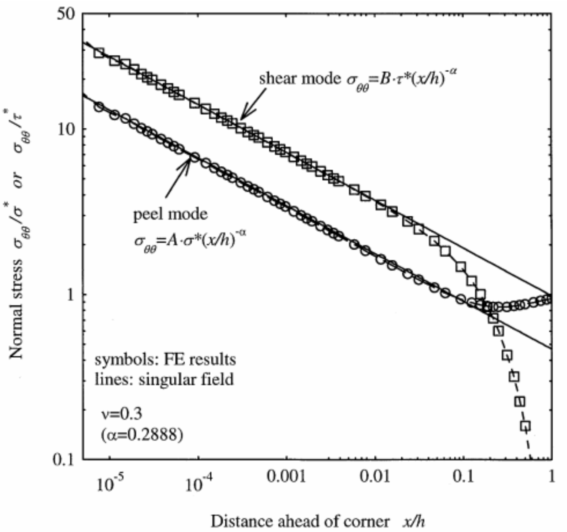

Fig. 4 Distribution of stress near the wedge tip calculated using FE analysis, Wang & al. [12]

II. Defining a New Method to Calculate the Stress Intensity Factor

This paper seeks to define a calculation method of the SIFs which would be:

- Clearly defined: the user should be able to apply this method automatically and systematically, without worrying about the possible influence of arbitrary parameters

- Universal: the method should handle the many cases that may arise in industrial applications. Also, analytical methods are to be avoided as they are able to describe the behaviour of joints only in particular cases. By cons, the use of the finite element method seems to be quite suitable, since it allows the calculation of many parameters whatever the geometric shape of the adherends

- Fast: the method must be based on data available at low cost. Having chosen to rely on the finite element method to assess certain parameters, it is preferable to avoid, during the daily use of the method, the need to develop specific and refined models, whose preparation would be time-consuming, and to use instead a meshing as simple as possible, and easy to be obtained.

- Reliable: the method must be able to give an estimate of the SIFs with good accuracy. In preliminary design phase, a fast estimate that leads, at the most critical areas, to an error of less than 10% is often considered more interesting than a very expensive estimate, even if this last one provides more accurate results.

A method that provides a satisfactory answer to all these objectives is proposed below. This method is based on the stresses obtained from a low detailed FE model. It requires only that the end of the bonded joint is meshed as shown in Fig. 5. Five plane strain quadratic elements (1 to 5) have to be located at the interface extremity:

- transversally, the adhesive layer is divided into two elements of equal thickness h/2,
- longitudinally, the last column of elements at the end of the adhesive layer also has a width equal to h/2,
- the two adherends are meshed so that square elements of height h/2 and width h/2 are located in continuity of those mentioned above.

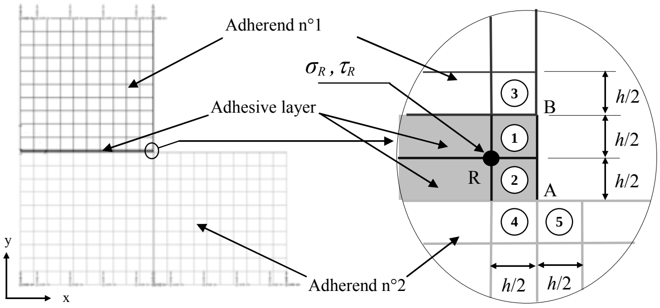

Fig. 5 Typical finite element mesh used in analyses Detail of the mesh at interface corner

The point R located in the adhesive layer at equal distance of the free-edge and of the two interfaces is used as the reference point where to extract the peeling stress σ_R and the shearing stress τ_R that will be used in the calculation of the SIFs. The general expression of a stress intensity factor becomes:

$$K = K_I + K_{II}$$

with :

$$K_I = C_{R,I}\ \sigma_R\ h^{-\delta} \quad \text{and} \quad K_{II} = C_{R,II}\ \tau_R\ h^{-\delta} \qquad (3)$$

where $C_{R,I}$ and $C_{R,II}$ are two parameters dependent on the couple of angles and materials which characterises the corner under study. In the configuration of the Fig. 5, the two corners A and B are studied from the same FE analyses, but each corner will lead to K_I and K_{II} specific values.

The procedure allowing the identification of the parameters $C_{R,I}$ and $C_{R,II}$ which appear in (3) and which are related to the corner A is described below. The procedure is based on results derived from a pattern of study. The geometrical parameters of the pattern which has been chosen are presented in the Fig. 6. The adhesive thickness *h* may be fixed arbitrary as it is directly taken into consideration in (3), then $C_{R,I}$ and $C_{R,II}$ do not depend on its value. The dimensional parameter *e* driving the whole geometry must be chosen large related to *h*. For instance, *e* = 100 *h* can be used.

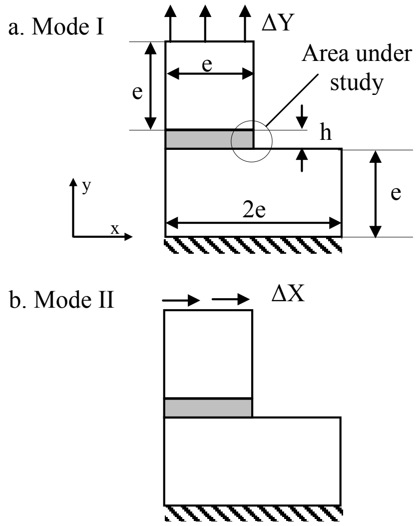

Fig. 6 Geometric parameters and boundary conditions of the pattern of study

The identification of $C_{R,I}$ and $C_{R,II}$ will be done in the three following phases:

Phase 1. The geometry of the pattern is meshed using quadratic plane strain finite elements, the meshing being relatively sparse, but respecting the constraints proposed in Fig. 5: it must make appear the five square elements whose side dimensions are h/2. Two analyses are then realised:

- one related to the mode I. The boundary conditions to be used are those given in Fig. 6a: the lower edge is fixed and the upper edge is displaced transversally by ΔY (the value is arbitrary). Then the stresses $\sigma_{R,I}$ and $\tau_{R,I}$ are evaluated.
- one related to the mode II. As illustrated in Fig. 6b, the upper edge is now displaced longitudinally by ΔX. The stresses $\sigma_{R,II}$ and $\tau_{R,II}$ are then calculated.

Phase 2. The geometry of the pattern of study is now meshed using a highly refined meshing around the corners at the end of the adhesive layer. The Fig. 7 illustrates the kind of detailed FE model which can be used. At the corner, the smallest side element length is about 3.E-5mm. The same analyses than in phase 1 are then operated. The object of these two calculations is to determine the two peeling stress fields along the interface, $\sigma^{ref}_{\theta\theta\ I}(r,0)$ and $\sigma^{ref}_{\theta\theta\ II}(r,0)$. These fields will be used as reference during the following phase.

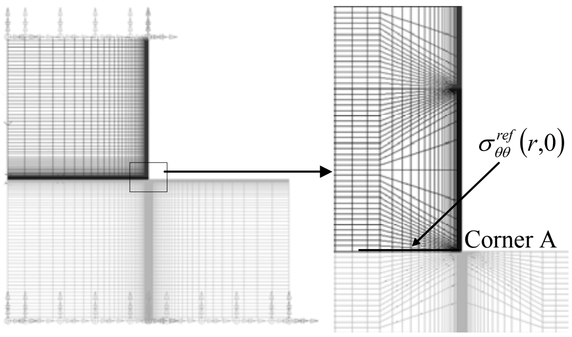

Fig. 7 Typical refined meshing

Phase 3. The object of this phase is to determine once and for all the two parameters $C_{R,I}$ and $C_{R,II}$ which will allow the calculation of the SIF directly from a sparse FE model. (1) and (3) may be rewritten in:

$$\begin{aligned}\sigma_{\theta\theta}(r,0) &= K\, r^{\delta} = (K_I + K_{II})\, r^{\delta} \\ &= (C_{R,I}\;\; \sigma_R + C_{R,II}\;\; \tau_R)\;\; h^{-\delta}\;\; r^{\delta}\end{aligned} \quad (4)$$

Then, the identification of $C_{R,I}$ and $C_{R,II}$ can be done by minimizing the distance between the fields calculated using (4) and the reference fields, i.e.:

- Mode I: the field calculated using (4) with $\sigma_{R,I}$, $\tau_{R,I}$ and the reference field $\sigma_{\theta\theta\, I}^{ref}(r,0)$
- Mode II: the field calculated using (4) with $\sigma_{R,II}$, $\tau_{R,II}$ and the reference field $\sigma_{\theta\theta\, II}^{ref}(r,0)$

Table IV gives the values that could be obtained when considering the single lap configuration of Fig. 2 and the parameters proposed in Table II and Table III.

TABLE IV
$C_{R,I}$ AND $C_{R,II}$ RELATED TO THE CONFIGURATION DEFINED IN FIG.2

	$\delta = \lambda_1 - 1$	$C_{R,I}$	$C_{R,II}$
Corner A	-0.3386	0.439	-0.830
Corner B	-0.3016	0.496	0.798
Corner C	-0.2680	0.499	0.730
Corner D	-0.3272	0.430	-0.936

III. EVALUATING THE PROPOSED METHOD

The accuracy of the method proposed in the previous section will now be evaluated. The influence of three main parameters will be analyzed:

- the load direction, particularly the combination of the two principal modes,
- the thickness of the adhesive layer,
- the combination of adhesive and adherend materials.

A. *Mixed-mode loading*

The structure illustrated in Fig. 8 is used as support for this study. Its forms refer to those of an Arcan device, allowing the application of the load at various incidence angles. Five angles are considered:

- load F_1 : pure opening-mode loading (mode I),
- loads F_2, F_3 and F_4 : the incidence angles are respectively equal to π/8 , π/4 and 3π/8 (mixed-modes),
- load F_5 : pure shearing-mode loading (mode II).

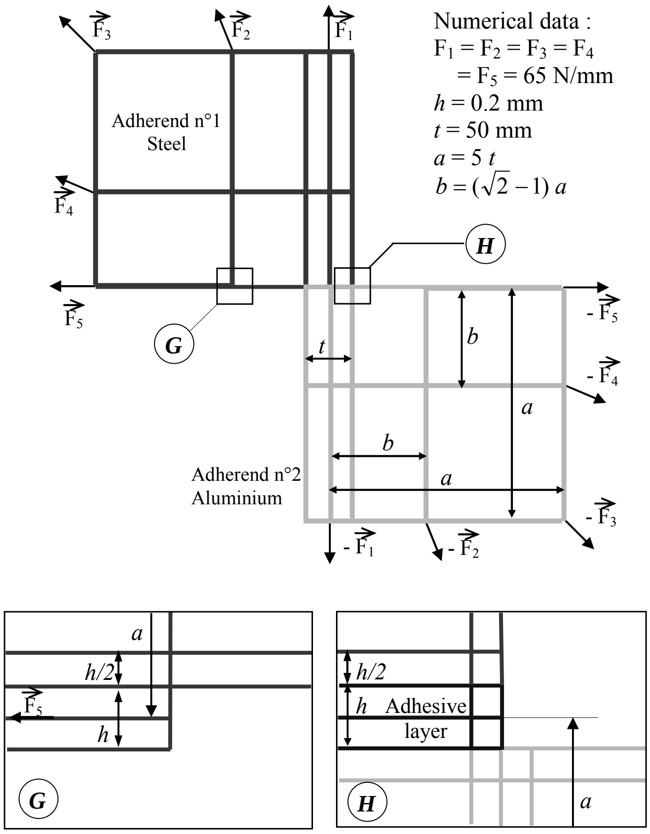

Fig. 8 Geometry and loadings of an Arcan shaped structure

The Fig. 8 makes appear the geometrical partitions that have been made in order to obtain the appropriate loading angles ($b = a\,(\sqrt{2} - 1)$ leads to the orientations required by F_2 and F_4). The shape of the adhesive layer is the same than in Fig. 1: single lap configuration with square edges at the two ends. Materials are those defined in Table I.

In a first time, the method described in Section II is implemented. The structure is meshed using plane strain quadratic elements. The meshing is relatively sparse: the model counts 5265 nodes and 1680 elements considering the whole structure. It satisfies the constraint related to the presence of five elements at each end of the adhesive layer (Fig. 5). Then a FE analysis is done for each different loading. The SIFs, noted K_{method}, can be calculated for each corner using (3) with:

- the stresses σ_R and τ_R obtained from the FE analyses,
- $C_{R,I}$ and $C_{R,II}$ obtained in the previous section.

In a second time, the structure is analyzed using a highly refined mesh in the region of the interface corners (87661 nodes and 28900 elements to model the whole structure).

From the peeling stress distribution obtained at the interface, a SIF reference value, noted K_{ref}, is calculated by optimization at each corner.

The performance of the proposed method can be evaluated by comparing K_{method} to K_{ref}. Table V gives K_{method} and the gap relative to the reference:

$$\text{Gap/ref} = 100\,\frac{K_{\text{method}} - K_{\text{ref}}}{K_{\text{ref}}}$$

TABLE V
VALUE OF THE SIFS (MPA/MM^δ) AND THE GAPS/REFERENCE (%) FOR DIFFERENT LOADINGS

	Corner A		Corner B	
	K_{method}	Gap/ref	K_{method}	Gap/ref
Load F_1	1105	0.19	1452	0.59
Load F_2	2525	0.25	612	1.75
Load F_3	3552	0.63	-345	2.88
Load F_4	4032	1.63	-1255	0.72
Load F_5	3889	1.40	-1984	0.54

	Corner C		Corner D	
	K_{method}	Gap/ref	K_{method}	Gap/ref
Load F_1	1005	0.13	1357	0.06
Load F_2	877	0.33	2303	0.26
Load F_3	603	0.87	2888	0.64
Load F_4	233	4.37	3029	1.08
Load F_5	-179	9.15	2704	1.55

Table V allows identifying the highest values of the SIFs. K is maximal at corner B for load F_1, then at corner A for the other loads. The load intensity remaining constant ($F = 65$N/mm), the peeling stress related to the mode II is more important than the one due to the mode I. The gaps are small wherever the values of K are positive and significant (less than 2%). This shows the interest of the proposed method.

Gaps become more important at the corner C when the weight of the mode II increases. But it can be noticed that the value of K is close to zero, and then it is quite normal that the gap increases. Moreover, when K is negative, the peeling stress $\sigma_{\theta\theta}(r,0)$ at the interface is compressive, and the probability to generate an adhesive failure is very low.

B. *Thickness of the adhesive layer*

Using the same structure than in section III.*A*, and considering only the pure shearing-mode (mode II), which is the most disadvantageous relative to the performance of the method, the adhesive layer thickness h is now considered as a variable. Table VI gives the SIFs obtained by the proposed method (K_{method}) and the gaps relative to the reference for four different thicknesses: 0.2, 0.3, 0.4 and 0.5 mm.

As shown in Table VI, the proposed method leads to accurate results at the corners submitted to important peeling stresses (corners A and D). The differences increase with the thickness of the adhesive layer, but still within acceptable limits. These results seem logical since the reference point moves away from the corner area when the joint thickness increases.

C. *Combination of materials*

Another set of materials is considered below. The structure remains the same than in section III.*A*, the loading being kept the most critical ones (mode II).

The adherend n°1 is chosen more rigid than the steel (Silicon Carbide, noted SiC; E_1 = 441 GPa), and the adhesive more compliant (E_a = 1.9 GPa instead of 3.5 GPa). Data related to the new set of materials are given in Table VII. The adherend n°2 remains unchanged (aluminium).

As shown in Table VIII, the SIF of the corner A will lead to highest peeling stress. The precision of the proposed method seems to be satisfactory (0.78% where the attention must be focused).

It can be noticed that the value of K at the corner A is largely lower with this set of materials than the one obtained in section III.*A*: 2421 MPa/mm^(-0.3905) instead of 3889 MPa/mm^(-0.3386). The adhesive stiffness decrease (E_a has been divided by 1.8) seems to be highly most influent on the value of K than the adherend stiffness increase (E_1 has been multiplied by 2.1). But it would be necessary to introduce the two associated values of K_{crit} in the reasoning to be able to conclude on the relative strength of the two types of adhesive joint.

TABLE VI
VALUE OF THE SIFS (MPA/MM^ δ) AND THE GAPS/REFERENCE (%) FOR DIFFERENT THICKNESSES

Thickness	Corner A		Corner B	
(mm)	K_{method}	Gap/ref	K_{method}	Gap/ref
0.2	3889	1.40	-1984	0.54
0.3	3883	3.02	-2141	0.87
0.4	3913	4.23	-2291	0.93
0.5	3960	5.23	-2431	0.76

Thickness	Corner C		Corner D	
(mm)	K_{method}	Gap/ref	K_{method}	Gap/ref
0.2	-179	9.15	2704	1.55
0.3	-418	15.46	2878	4.56
0.4	-640	17.43	3035	6.78
0.5	-843	17.26	3178	8.53

TABLE VII
PARAMETERS DEFINING THE NEW SET OF MATERIALS

	Adherend n°1	Adherend n°2	Adhesive layer
Material	Silicon Carbide	Aluminium	Other adhesive
Young's Modulus	E_1 = 441 GPa	E_2 = 69 GPa	E_a = 1.9GPa
Poisson's ratio	ν_1 = 0.16	ν_2 = 0.33	ν_a = 0.46

TABLE VIII
VALUE OF THE SIFS (MPA/MM^ δ) AND THE GAPS/REFERENCE (%) FOR ANOTHER COMBINATION OF MATERIALS

Corner A		Corner B	
K_{method}	Gap/ref	K_{method}	Gap/ref
2421	0.78	-1433	1.83

Corner C		Corner D	
K_{method}	Gap/ref	K_{method}	Gap/ref
-227	4.43	1726	0.39

IV. APPLICATION

As example, the analysis of a conventional single lap joint is proposed, the two adherends being now two plates whose thicknesses are relatively thick compared to those used above. Fig. 9 gives the different dimensions of the assembly under study. Materials are those introduced in Table I (steel, aluminum and epoxy). The two plates experience a traction load. It is well known that, in this configuration, the flexion of the plates induces, at the extremity of the bonded joint, twice shearing and peeling effects. The structure has been meshed first using the prescriptions of the proposed method, and then using a highly refined meshing in order to evaluate the accuracy of the results.

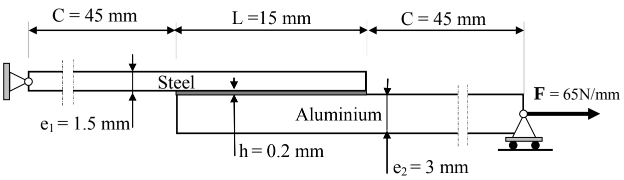

Fig. 9 Geometric profile of the assembly

Table IX shows values of the SIFs obtained by the proposed method, and also provides the differences observed in relation to reference calculations. The accuracy is good at the two corners D and A where the peeling stresses are highest.

TABLE IX
VALUE OF THE SIFS (MPA/MM^ δ) AND THE GAPS/REFERENCE (%)

Corner A		Corner B	
K_{method}	Gap/ref	K_{method}	Gap/ref
10320	1.52	-3183	1.89

Corner C		Corner D	
K_{method}	Gap/ref	K_{method}	Gap/ref
2135	26.5	15830	1.50

V. CONCLUSION

The Stress Intensity Factor K is one of the parameters commonly used to express the static adhesive strength criterions of bonded joints. But, to determine the value of K that characterizes a new application, it is necessary, if a direct approach is not available, to develop a very refined FE modeling around the singularities where stress concentrations happen.

The method defined in this paper aims to avoid the designer has to spend time in cumbersome modeling work, especially during preliminary design phases. The following expression is proposed to obtain a more direct estimate of the SIF:

$$K = \left(C_{R,I}\ \sigma_R + C_{R,II}\ \tau_R\right) h^{-\delta}$$

It refers to the two conventional fracture modes: the opening-mode (mode I) and the shearing-mode (mode II). It requires identifying first, once and for all, for each corner, the two constant parameters $C_{R,I}$ and $C_{R,II}$ characterizing the influence of each mode. A well-definite calculation process is given to lead to the determination of these parameters. Then, during the daily use of the method, the designer is asked to implement a low detailed FE model of the structure under study, but following precise prescribed meshing rules at the extremity of the bounded joint, and to extract from this model the two stresses σ_R and τ_R present at the point R located in the adhesive layer at equal distance of the free-edge and of the two interfaces.

An evaluation of the method accuracy is proposed by measuring the gaps between its results and reference values obtained from highly refined analyses. A structure which looks like an Arcan device is considered in order to make vary the direction of the loading, and then the relative weight of the modes. The same structure allows evaluating also the influence of the thickness of the adhesive layer, and the combination of the materials. It is shown that the gaps remain low at the corners where the values of K are significant.

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