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Terminal Velocity of a Bubble Rise in a Liquid Column

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***Abstract*—**As it is known, buoyancy and drag forces rule bubble's rise velocity in a liquid column. These forces are strongly dependent on fluid properties, gravity as well as equivalent's diameter. This study reports a set of bubble rising velocity experiments in a liquid column using water or glycerol. Several records of terminal velocity were obtained. The results show that bubble's rise terminal velocity is strongly dependent on dynamic viscosity effect. The data set allowed to have some terminal velocities data interval of 8.0 – 32.9 cm/s with Reynolds number interval 1.3 – 7490. The bubble's movement was recorded with a video camera. The main goal is to present an original set data and results that will be discussed based on two-phase flow's theory. It will also discussed, the prediction of terminal velocity of a single bubble in liquid, as well as the range of its applicability. In conclusion, this study presents general expressions for the determination of the terminal velocity of isolated gas bubbles of a Reynolds number range, when the fluid proprieties are known.

***Keywords*—**Bubbles, terminal velocity, two phase-flow, vertical column.

I. Introduction

WHEN rising through an infinite stagnant liquid, the single bubble's terminal velocity is of fundamental importance in gas liquid two phase flow's theory.

As is known, the single isolated gas bubble's rising velocity in a liquid large column depends on buoyancy and drag forces. Interactions between forces happen due to surface tension, viscosity, inertia and buoyancy produce a various effects which are quite often proved by different bubble shapes and trajectories.

Many industrial processes include bubble columns for promoting mass transfer, high pressure evaporators and so on.

Air bubble's velocity dependence has been determined experimentally by numerous investigators [1]-[12], among others].

For single isolated smallest bubbles, which are approximately perfect spheres due to surface tension dominant effect of on their shape, Stokes solution [13] provides a reasonably accurate description.

$$u_\infty = \frac{1}{18}\frac{g d_e^2\left(\rho_l - \rho_g\right)}{\mu_l} \tag{1}$$

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where g is the acceleration due gravity, d_e the equivalent bubble diameter (diameter of a sphere with same volume as the bubble), μ_l the dynamic viscosity of liquid, ρ_l the density of liquid and ρ_g the density of gas.

When isolated bubbles are very large, surface tension effects and viscosity are despicable and rise's velocity is given by Davies and Taylor's [14]

$$u_\infty = 0.707\sqrt{g d_e} \tag{2}$$

For intermediate size bubbles, both effects of liquid inertia, surface tension, viscosity and cleanliness are important, as well as whether bubbles rise in straight lines, oscillate, or describe a spiral path. Many correlations are presented in the speciality literature.

In the present study, a single isolated bubble's terminal velocity was investigated theoretically and experimentally. Two liquids with different viscosities were considered (water or glycerol). The influence of the wall column using the results of Collins [15] was considered.

II. Theory

In this study we applied dimensional analysis to determine dimensionless groups that influence single isolated gas bubble's velocity in a stagnant liquid, rising in a large container filled with different viscosity liquids. Physically, the velocity depends on seven parameters

$$u_\infty = u_\infty\left(g, d_e, \Delta\rho, \rho_l, \mu_l, \sigma_{l/g}\right) \tag{3}$$

where $\sigma_{l/g}$ is the surface tension of liquid and $\Delta\rho = \left(\rho_l - \rho_g\right)$ the apparent density.

The equation's (3) dimensional analysis can be obtained through traditional techniques [16] where the chosen independent variables were g, d_e e $\Delta\rho$.

Thus, the four dimensionless groups are

$$\Pi_1 = \frac{u_\infty}{g^{1/2} d_e^{1/2}} \tag{4}$$

$$\Pi_2 = \frac{\rho_l}{\Delta\rho} \tag{5}$$

The group Π_2 it is always very close to unity.

$$\Pi_3 = \frac{\mu_l}{g^{1/2}\Delta\rho d_e^{3/2}} \tag{6}$$

$$\Pi_4 = \frac{\sigma_{l/g}}{g\Delta\rho d_e^2} \tag{7}$$

Also

$$\Pi_5\left(=\frac{\Pi_1\Pi_2}{\Pi_3}\right) = \frac{\rho_l u_\infty d_e}{\mu_l} = \text{Re} \tag{8}$$

and,

$$\Pi_6\left(=\Pi_1{}^2\Pi_2\right) = \frac{u_\infty^2\rho_l}{gd_e\Delta\rho} \tag{9}$$

For values of $\Pi_1 = \phi\left(\Pi_3{}^{-1}\right)$ may be written

$$u_\infty = k\frac{gd_e^2\Delta\rho}{\mu_l} \tag{10}$$

where *k* is a experimental constant e $\Delta\rho = \left(\rho_l - \rho_g\right)$.

If $k = \frac{1}{18}$, (10) is equivalent to (1).

In this study isolated bubbles were large enough and surface tension effects may be negligible. According to Harmathy [17] when surface tension dominates, the dimensionless group Π_4^{-1} is important therefore it was called by Eotvos number.

III. Experimental Method

The experimental technique adopted in this study is easier to understand by Figure's 1 analysis.

The horizontal tank wall cross section was a quadrangle "20cm×20cm" with four vertical walls of transparent acrylic filled with water or glycerol to a depth of 150 cm.

The bubbles were generated just above the centre of the board by hemispherical cup (only for atmospheric pressure), which was supported so that it could be rotated about a horizontal axis between two walls as shown Fig. 1.

It was introduced air and it was trapped inside this inverted cup so that, when rotated, a spherical cap bubble was produced near to the base of the tank and on the axis of the cylinders.

By adjusting rotation's rate it was possible to minimize secondary bubbles production.

Each bubble was collected in a graduated cylinder at the top of the tank in order to determine its volume as it is shown in Fig. 1.

In order to minimize the error of the air bubble diameter, and for the same bubble size, the gathered bubbles inside a cylinder graduated were counted.

The total volume was divided later by the number of air bubbles gathered.

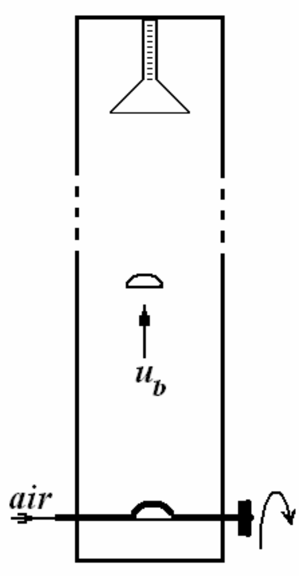

Fig. 1 Schematic of the experimental apparatus

To determine smaller bubble's volume it was used a graduate syringe.

The bubbles were photographed and followed with a video camera.

All the experiments at were made at room temperature of about 20 ºC.

In our experiments, each single isolated gas bubble on the rise through water (1×10^{-3} Pa.s) or glycerol (1.4 Pa.s) was timed as it passed between two marks, at the tank wall.

All measurements were made from the top (nose) of the bubble.

Using a time interval, between two marks which define the referential distance, the experimental velocity (instantaneous velocity) can be calculated.

Photographs were obtained for single isolated gas bubble in water or glycerol.

Two examples, air – water system and air – glycerol system, are shown in Fig. 2 and Fig. 3.

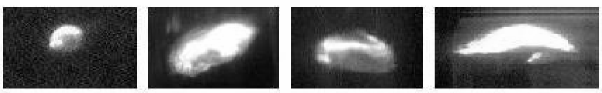

Fig. 2 Air – water system: typical pictures for high values of Re

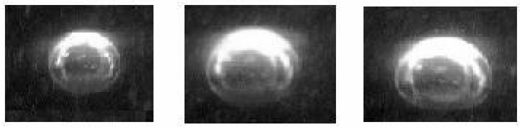

Fig. 3 Air – glycerol system: typical pictures for low values of Re

IV. Results and Discussion

The Fig. 4 show for the air – water and air – glycerol system the instantaneous velocity and the Fig. 5 the relationship between the drag coefficient and the Re (Reynolds number).

The Fig. 6 and Fig. 7 shows values of the terminal velocity of air bubbles for the two systems studied.

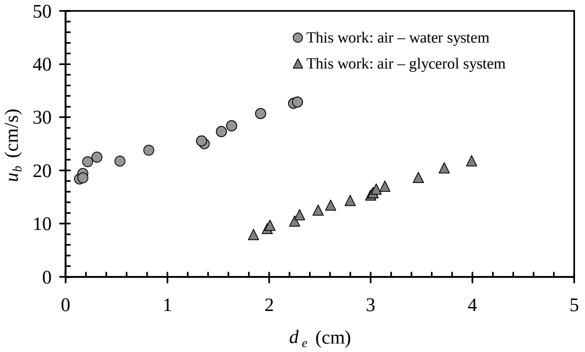

Fig. 4 Instantaneous velocity

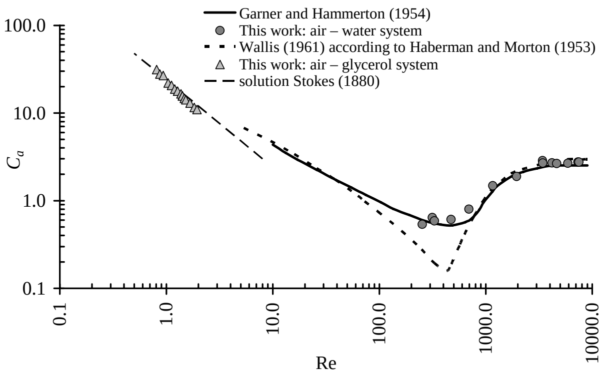

Fig. 5 C_d versus Re

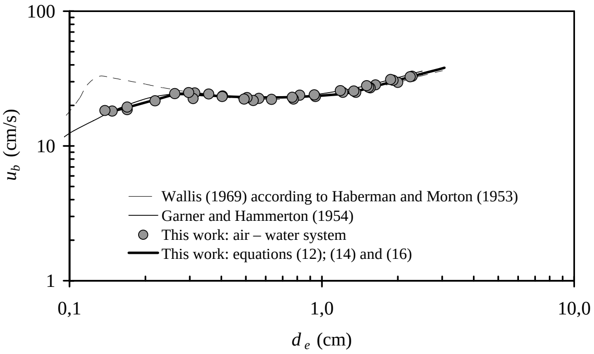

Fig. 6 Terminal velocity of air bubbles in air – water system

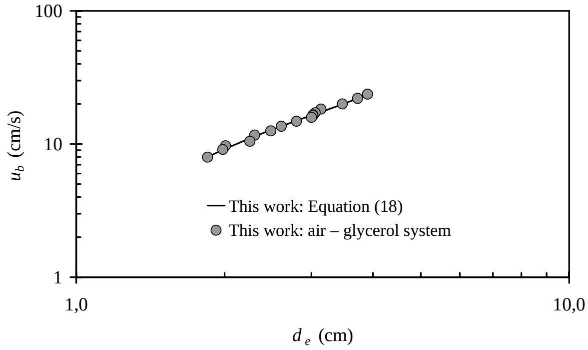

Fig. 7 Terminal velocity of air bubbles in air – glycerol system

For the air – water system, according the Fig. 6, the experimental value shows an agreement with the value found by Wallis [11] according Haberman and Morton [5] and Garner and Hammerton [3] for equivalent bubble diameter above 0.3 cm.

As it can be comprehended from Fig. 5, the Wallis line's [11] deviation according to Haberman and Morton [5] is due gas bubble rising in filtered or distilled water with equivalent diameter below 0.3 cm. This deviation does not happen for value of $d_e \geq 0.3$ cm.

When inertia is dominant, it is possible to develop a global expression to predict a single bubble's velocity when its diameter and liquid and gas physical properties are known.

A good approximation description of the experimental points is defined by the function ($3425 \leq \mathrm{Re} \leq 7490$, $25.5 \leq u_b \leq 32.9$ cm/s; $1.34 \leq d_e \leq 2.28$ cm and $2.68 \leq C_d \leq 2.76$, where C_d correspond to the drag coefficient) is given by

$$\Pi_6^{1/2} = (0.694 \pm 0.021) \tag{11}$$

Equation (11) with algebraic manipulation leads to

$$u_b = (0.694 \pm 0.021)\left(\frac{g d_e \Delta\rho}{\rho_l}\right)^{1/2} \tag{12}$$

Equation (12) has an excellent agreement with Davies Taylor's [14] given by (2) when the bubbles are very large and surface tension effects and viscosity are negligible and also container walls influence.

In the course, for $695 \leq \mathrm{Re} \leq 3425$, $22.5 \leq u_b \leq 25.5$ cm/s and $0.31 \leq d_e \leq 1.34$ cm, is obtained

$$\Pi_6^{1/2} = (877.193\Pi_3 + 0.289)^{1/2} \tag{13}$$

when the constants of polynomial are determined.

Equation (13) with algebraic manipulation leads to

$$u_b = \left(0.289 \frac{g d_e \Delta\rho}{\rho_l} + 877.193 \frac{\mu_l g^{1/2}}{\rho_l d_e^{1/2}} \right)^{1/2} \quad (14)$$

Also, for $255 \le \text{Re} \le 695$, $18.3 \le u_b \le 22.5$ cm/s and $0.14 \le d_e \le 0.31$ cm, is obtained

$$\Pi_6^{1/2} = (1.500 \pm 0.045) \quad (15)$$

Equation (15) with algebraic manipulation leads to

$$u_b = (1.500 \pm 0.045) \left(\frac{g d_e \Delta\rho}{\rho_l} \right)^{1/2} \quad (16)$$

For the air – glycerol system and when the dynamic viscosity is dominant ($1.3 \le \text{Re} \le 8.3$ with $8.0 \le u_b \le 24.0$ cm/s; $3.90 \le d_e \le 1.85$ cm and $9.1 \le C_d \le 38.1$) it was possible to develop a global expression to predict a single bubble's velocity when equivalent bubble diameter and liquid and gas physical proprieties are known.

Fig. 7 shows rise velocity's dependence on bubble volume for air bubbles in glycerol.

A nearly good description of experimental points is given by $\Pi_3 = \varphi(\Pi_6)$ or

$$\Pi_6^{1/2} = -\left(0.529\Pi_3 - 2.386 \times 10^{-2}\right)^{1/2} + 0.415 \quad (17)$$

when the constants of polynomial are determined.

Equation (17) with algebraic manipulation leads to

$$u_b = 0.415 \frac{g d_e \Delta\rho}{\rho_l} - \left(0.529 \frac{g^{1/2} \mu_l}{\rho_l d_e^{1/2}} - 2.386 \times 10^{-2} \frac{g d_e \Delta\rho}{\rho_l} \right)^{1/2} \quad (18)$$

V. Research in Course

When rising through an infinite stagnant liquid, a single bubble's terminal velocity study is very important in a system gas – liquid in the two phase flow field.

The experiment research still goes on in order to obtain a large range of terminal velocity data for a single bubble rising in water or glycerol or mixtures water – glycerol.

In a near future it will be possible to develop expressions to predict the velocity of a single bubble when the equivalent bubble diameter (or volume of the bubble) and the physical proprieties of the liquid and gas are known.

Also, the surface tension – dominate regime will researched, well as effect of external (shape oscillation of the bubble and accumulation of contaminants).

VI. Conclusion

During the research, all experiments were carried out at constant temperature and surface tension was not considered.

Experimental data show that it is possible to work out expressions to accurately predict terminal velocity of isolated gas bubbles rising in water or glycerol.

Inertia and viscosity dominant expressions that were presented seem to be much easier to use. In practice, these expressions are easily used as long as the equivalent bubble diameter and liquid and gas physical properties are known.

The predictions of the correlations are shown to be in good agreement with experimental data and the range of Reynolds number is well defined.

Nomenclature

Roman Letters

k	constant	(10)
C_d	drag coefficient	[-]
d_e	equivalent bubble diameter	[m]
g	gravitational acceleration	[m/s^2]
Re	Reynolds number	[-]
u_∞	terminal velocity (without effect wall)	[m/s]
u_b	terminal velocity (without effect wall)	[m/s]

Greek Letters

μ_l	dynamic viscosity of the liquid	[Ns/m^2]
ρ_g	density of gas	[kg/m^3]
ρ_l	density of liquid	[kg/m^3]
$\sigma_{l/g}$	surface tension of liquid	[N/m]
$\Delta\rho$	apparent density $= (\rho_l - \rho_g)$	[kg/m^3]
Π_i	dimensionless parameter	[-]

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