Design of Variable Fractional-Delay FIR Differentiators

Jong-Jy Shyu, Soo-Chang Pei and Min-Han Chang

***Abstract*—**In this paper, the least-squares design of variable fractional-delay (VFD) finite impulse response (FIR) digital differentiators is proposed. The used transfer function is formulated so that Farrow structure can be applied to realize the designed system. Also, the symmetric characteristics of filter coefficients are derived, which leads to the complexity reduction by saving almost a half of the number of coefficients. Moreover, all the elements of related vectors or matrices for the optimal process can be represented in closed forms, which make the design easier. Design example is also presented to illustrate the effectiveness of the proposed method.

***Keywords*—**Differentiator, variable fractional-delay filter, FIR filter, least-squares method, Farrow structure.

I. Introduction

DIGITAL differentiators are generally used to calculate the instantaneous rate of change of digital signals, and have wide applications in instrument and measurement, biomedical engineering and signal processing [1]–[4] etc. For the past decade, several techniques have been proposed to design digital differentiators, including eigenfilter approach, quadratic optimization, maxflat technique, Simpson integration rule, Newton-Cotes formula, bilinear transformation and semi-infinite programming approach [5]–[15] etc. Also, there is a branch of trend concerning the design of variable fractional-delay (VFD) digital filters which are applied to where the delay characteristic need to be adjustable online without designing a new filters [16]–[24].

In this paper, the design of VFD FIR differentiators is proposed. First, the used transfer function is formulated such that the differentiators can be implemented by Farrow structure [16]. Then the symmetry on filter coefficients is analyzed, and it is found that each of the subfilters in Farrow structure can be designed by the conventional linear-phase FIR filters with symmetric/antisymmetric coefficients. In this paper, a least-squares method is used to the optimal design of VFD FIR differentiators, and all of the elements of related vectors or matrices can be represented in closed forms. Finally, a design example is presented to demonstrate the efficiency of the proposed method. Also, it is found that although the existing iterative weighted-least-squares method such as the technique in [25] can be applied to minimize the peak absolute error of variable frequency response, but it is not recommended because the delay responses are very sensitive to the variation of magnitude responses around d.c. frequency.

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II. Problem Formulation and Design Example

Conventionally, for designing a differentiator the desired frequency response is generally given by

$$H_c(\omega) = j\omega e^{-jI\omega}, \qquad |\omega| \le \omega_p \tag{1}$$

where I is the prescribed delay and ω_p is the cutoff frequency. In this paper, it is extended to design a VFD FIR differentiator, and the desired variable frequency response becomes

$$H_d(\omega, p) = j\omega e^{-j(I+p)\omega}, \quad |\omega| \le \omega_p, \quad -0.5 \le p \le 0.5 \tag{2}$$

where p is a adjustable parameter which is used to control the delay of a differentiator online.

In this paper, the used transfer function can be characterized by

$$H(z,p) = \sum_{n=0}^{N} h_n(p) z^{-n} \tag{3}$$

where the coefficients $h_n(p)$ are expressed as the polynomials of p

$$h_n(p) = \sum_{m=0}^{M} h(n,m) p^m, \tag{4}$$

hence (3) can be formulated into

$$\begin{aligned} H(z,p) &= \sum_{n=0}^{N}\sum_{m=0}^{M} h(n,m) p^m z^{-n} \\ &= \sum_{m=0}^{M} G_m(z) p^m \end{aligned} \tag{5}$$

where the subfilters $G_m(z)$ are given by

$$G_m(z) = \sum_{n=0}^{N} h(n,m) z^{-n}, \qquad 0 \le m \le M \tag{6}$$

and the transfer function can be implemented by Farrow structure [16].

Notice that (2) can be further represented by

$$H_d(\omega,p)=e^{-jI\omega}\left(\omega\sin(p\omega)+j\omega\cos(p\omega)\right)$$
$$=e^{-jI\omega}j\omega\sum_{m=0}^{\infty}\frac{(-jp\omega)^m}{m!}$$
$$\cong\sum_{m=0}^{M}(-1)^m\frac{(j\omega)^{m+1}}{m!}e^{-jI\omega}p^m \qquad (7)$$

for sufficient large M. Comparing (5) and (7), it can be found that the frequency response of the subfilter $G_m(z)$ is inherently used to approximate $(-1)^m(j\omega)^{m+1}e^{-jI\omega}/m!$, for $0\le m\le M$. So it is reasonable to choose the impulse response of $G_m(z)$ to be antisymmetric for even m and symmetric for odd m, and the frequency response of (5) for even N can be formulated into

$$H\left(e^{j\omega},p\right)$$
$$=\sum_{m=0}^{M_e}\left[\sum_{n=0}^{N}h(n,2m)e^{-jn\omega}\right]p^{2m}+\sum_{m=0}^{M_o}\left[\sum_{n=0}^{N}h(n,2m+1)e^{-jn\omega}\right]p^{2m+1}$$
$$=e^{-j\frac{N}{2}\omega}\left[j\sum_{m=0}^{M_e}\sum_{n=1}^{N/2}a(n,m)p^{2m}\sin(n\omega)+\sum_{m=0}^{M_o}\sum_{n=0}^{N/2}b(n,m)p^{2m+1}\cos(n\omega)\right] \qquad (8)$$

where

$$\begin{cases}M_e=M_o+1=\frac{M}{2}, & \text{for even } M,\\ M_e=M_o=\frac{M-1}{2}, & \text{for odd } M\end{cases} \qquad (9)$$

and

$$a(n,m)=2h\left(\tfrac{N}{2}-n,2m\right)=-2h\left(\tfrac{N}{2}+n,2m\right),$$
$$1\le n\le\tfrac{N}{2},\qquad 0\le m\le M_e \qquad (10a)$$

$$b(n,m)=\begin{cases}h\left(\frac{N}{2},2m+1\right), & n=0,\quad 0\le m\le M_o,\\ 2h\left(\frac{N}{2}-n,2m+1\right) & \\ =2h\left(\frac{N}{2}+n,2m+1\right), & 1\le n\le\frac{N}{2},\quad 0\le m\le M_o.\end{cases} \qquad (10b)$$

It is noted that $h\left(\frac{N}{2},2m\right)=0$ for $0\le m\le M_e$, and obviously $I=N/2$. Also, the case for odd N can be extended in the same manner.

Define

$$\boldsymbol{a}=\left[a(1,0),\cdots,a\left(\tfrac{N}{2},0\right),\cdots,a(1,M_e),\cdots,a\left(\tfrac{N}{2},M_e\right)\right]^T, \qquad (11a)$$

$$\boldsymbol{b}=\left[b(0,0),\cdots,b\left(\tfrac{N}{2},0\right),\cdots,b(0,M_o),\cdots,b\left(\tfrac{N}{2},M_o\right)\right]^T, \qquad (11b)$$

$$\boldsymbol{s}(\omega,p)=\left[\sin(\omega),\cdots,\sin\left(\tfrac{N}{2}\omega\right),\cdots,p^{2M_e}\sin(\omega),\cdots,p^{2M_e}\sin\left(\tfrac{N}{2}\omega\right)\right]^T \qquad (11c)$$

and

$$\boldsymbol{c}(\omega,p)=\left[p,\cdots,p\cos\left(\tfrac{N}{2}\omega\right),\cdots,p^{2M_o+1},\cdots,p^{2M_o+1}\cos\left(\tfrac{N}{2}\omega\right)\right]^T \qquad (11d)$$

where the superscript ^{T} denotes the transpose operator, Eq.(8) can be further represented by

$$H\left(e^{j\omega},p\right)=e^{-j\frac{N}{2}\omega}\left(j\boldsymbol{a}^T\boldsymbol{s}(\omega,p)+\boldsymbol{b}^T\boldsymbol{c}(\omega,p)\right). \qquad (12)$$

In this paper, the objective error function is given by

$$e(\boldsymbol{a},\boldsymbol{b})=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|H_d(\omega,p)-H\left(e^{j\omega},p\right)\right|^2d\omega\,dp$$
$$=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|\omega\sin(p\omega)+j\omega\cos(p\omega)-j\boldsymbol{a}^T\boldsymbol{s}(\omega,p)-\boldsymbol{b}^T\boldsymbol{c}(\omega,p)\right|^2d\omega\,dp$$
$$=e(\boldsymbol{a})+e(\boldsymbol{b}) \qquad (13)$$

where

$$e(\boldsymbol{a})=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|\omega\cos(p\omega)-\boldsymbol{a}^T\boldsymbol{s}(\omega,p)\right|^2d\omega\,dp \qquad (14a)$$

and

$$e(\boldsymbol{b})=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\left|\omega\sin(p\omega)-\boldsymbol{b}^T\boldsymbol{c}(\omega,p)\right|^2d\omega\,dp. \qquad (14b)$$

Eq.(14a) can be further represented by

$$e(\boldsymbol{a})=s_a+\boldsymbol{r}_a^T\boldsymbol{a}+\boldsymbol{a}^T\boldsymbol{Q}_a\boldsymbol{a} \qquad (15)$$

where

$$s_a=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\omega^2\cos^2(p\omega)\,d\omega\,dp, \qquad (16a)$$

$$\boldsymbol{r}_a=-2\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\omega\cos(p\omega)\boldsymbol{s}(\omega,p)\,d\omega\,dp \qquad (16b)$$

and

$$\boldsymbol{Q}_a=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\boldsymbol{s}(\omega,p)\boldsymbol{s}^T(\omega,p)\,d\omega\,dp. \qquad (16c)$$

By the Taylor series expansion of cosine function,

$$s_a=\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\omega^2\left[\tfrac{1}{2}+\tfrac{1}{2}\cos(2p\omega)\right]d\omega\,dp$$
$$=\frac{\omega_p^{\,3}}{6}+\frac{1}{2}\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\omega^2\left[\sum_{k=0}^{\infty}(-1)^k\frac{(2p\omega)^{2k}}{(2k)!}\right]d\omega\,dp$$
$$\cong\frac{\omega_p^{\,3}}{6}+\sum_{k=0}^{K}\frac{(-4)^k}{(2k)!}\frac{0.5^{2k+1}}{2k+1}\frac{\omega_p^{\,2k+3}}{2k+3} \qquad (17a)$$

and the elements of $\boldsymbol{r}_a$ and $\boldsymbol{Q}_a$ can be derived as follows:

$$\boldsymbol{r}_a(i)=-2\int_{-0.5}^{0.5}\int_{0}^{\omega_p}\omega\cos(p\omega)p^{2m}\sin(n\omega)\,d\omega\,dp$$
$$=-2\int_{-0.5}^{0.5}\int_{0}^{\omega_p}p^{2m}\omega\sin(n\omega)\left[\sum_{k=0}^{\infty}(-1)^k\frac{(p\omega)^{2k}}{(2k)!}\right]d\omega\,dp$$
$$\cong-4\sum_{k=0}^{K}\frac{(-1)^k}{(2k)!}\frac{0.5^{2m+2k+1}}{2m+2k+1}\int_{0}^{\omega_p}\omega^{2k+1}\sin(n\omega)\,d\omega,$$
$$0\le i\le\tfrac{N}{2}(M_e+1)-1, \qquad (17b)$$

$$\boldsymbol{Q}_a(i,l) = \int_{-0.5}^{0.5}\int_0^{\omega_p} p^{2m}\sin(n\omega)p^{2\hat{m}}\sin(\hat{n}\omega)\,d\omega\,dp$$
$$= \frac{0.5^{2m+2\hat{m}+1}}{2m+2\hat{m}+1}\left[\frac{\sin((n-\hat{n})\omega_p)}{n-\hat{n}} - \frac{\sin((n+\hat{n})\omega_p)}{n+\hat{n}}\right],$$
$$0 \le i,l \le \tfrac{N}{2}(M_e+1)-1. \tag{17c}$$

In(17), $n = \text{mod}\left(i,\frac{N}{2}\right)+1$, $m = \left\lfloor\frac{i}{N/2}\right\rfloor$, $\hat{n} = \text{mod}\left(l,\frac{N}{2}\right)+1$ and $\hat{m} = \left\lfloor\frac{l}{N/2}\right\rfloor$ where $\text{mod}(x,y)$ denotes the remainder when the integer x is divided by the integer y, and $\lfloor u \rfloor$ denotes the largest integer less than or equal to the real number u. Also, the K in (17) must be chosen large enough as in [22], and $K = 10$ is used in this paper.

Similarly,

$$e(\boldsymbol{b}) = s_b + \boldsymbol{r}_b^T\boldsymbol{b} + \boldsymbol{b}^T\boldsymbol{Q}_b\boldsymbol{b} \tag{18}$$

where

$$s_b \cong \frac{\omega_p^{\,3}}{6} - \sum_{k=0}^{K}\frac{(-4)^k}{(2k)!}\frac{0.5^{2k+1}}{2k+1}\frac{\omega_p^{\,2k+3}}{2k+3} \tag{19a}$$

and the elements of $\boldsymbol{r}_b$ and $\boldsymbol{Q}_b$ are given by

$$\boldsymbol{r}_b(i) \cong -4\sum_{k=0}^{K}\frac{(-1)^k}{(2k+1)!}\frac{0.5^{2m+2k+3}}{2m+2k+3}\int_0^{\omega_p}\omega^{2k+2}\cos(n\omega)\,d\omega,$$
$$0 \le i \le \left(\tfrac{N}{2}+1\right)(M_o+1)-1, \tag{19b}$$

$$\boldsymbol{Q}_b(i,l) = \frac{0.5^{2m+2\hat{m}+3}}{2m+2\hat{m}+3}\left[\frac{\sin((n-\hat{n})\omega_p)}{n-\hat{n}} + \frac{\sin((n+\hat{n})\omega_p)}{n+\hat{n}}\right],$$
$$0 \le i,l \le \left(\tfrac{N}{2}+1\right)(M_o+1)-1. \tag{19c}$$

In (19), $n = \text{mod}\left(i,\frac{N}{2}+1\right)$, $m = \left\lfloor\frac{i}{N/2+1}\right\rfloor$, $\hat{n} = \text{mod}\left(l,\frac{N}{2}+1\right)$ and $\hat{m} = \left\lfloor\frac{l}{N/2+1}\right\rfloor$.

Once (17) and (19) are found, the coefficient vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ can be determined by differentiating (13) with respect to $\boldsymbol{a}$ and $\boldsymbol{b}$,

$$\frac{\partial e(\boldsymbol{a},\boldsymbol{b})}{\partial \boldsymbol{a}} = \frac{\partial e(\boldsymbol{a})}{\partial \boldsymbol{a}} = \boldsymbol{r}_a + 2\boldsymbol{Q}_a\boldsymbol{a}\,, \tag{20a}$$

$$\frac{\partial e(\boldsymbol{a},\boldsymbol{b})}{\partial \boldsymbol{b}} = \frac{\partial e(\boldsymbol{b})}{\partial \boldsymbol{b}} = \boldsymbol{r}_b + 2\boldsymbol{Q}_b\boldsymbol{b} \tag{20b}$$

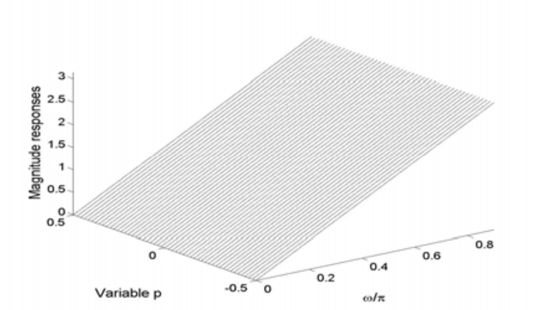
and then setting the above results to zeros, which yields

$$\boldsymbol{a} = -\frac{1}{2}\boldsymbol{Q}_a^{-1}\boldsymbol{r}_a \tag{21a}$$

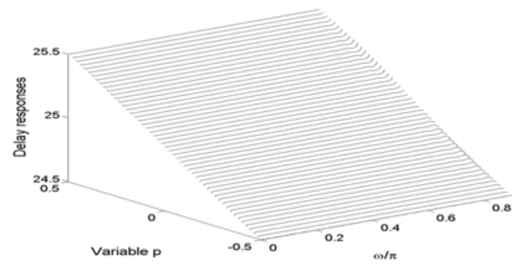
and

$$\boldsymbol{b} = -\frac{1}{2}\boldsymbol{Q}_b^{-1}\boldsymbol{r}_b. \tag{21b}$$

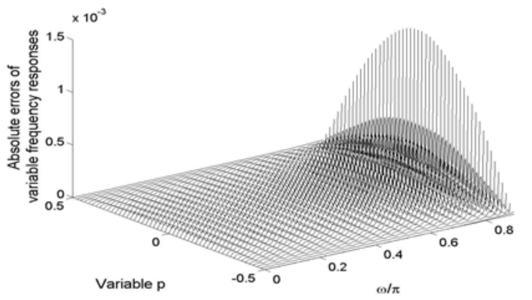
To evaluate the performance of the designed method, the



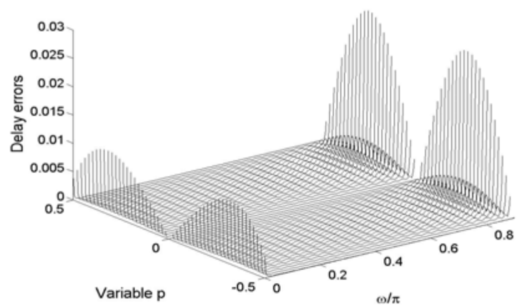
(a)



(b)



(c)



(d)

Fig. 1 Design of an $N = 50,\ M = 7,\ \omega_p = 0.9\pi$ VFD FIR differentiator. (a) Magnitude responses. (b) Delay responses. (c) Absolute errors of variable frequency responses. (d) Absolute delay errors.

normalized root-mean-squares error of the variable frequency response, the maximum absolute error of variable frequency response and the maximum absolute delay error are defined by

$$\varepsilon_2 = \left[\frac{\int_{-0.5}^{0.5}\int_0^{\omega_p}\left|H_d(\omega,p) - H(e^{j\omega},p)\right|^2 d\omega\,dp}{\int_{-0.5}^{0.5}\int_0^{\omega_p}\left|H_d(\omega,p)\right|^2 d\omega\,dp}\right]^{1/2} \times 100\%, \tag{22a}$$

$$\varepsilon_m = \max\left\{\left|H_d(\omega,p)-H(e^{j\omega},p)\right|, 0\le\omega\le\omega_p, -0.5\le p\le 0.5\right\} \quad (22b)$$

and

$$\varepsilon_\tau = \max\left\{\left|\tau_d(\omega,p)-\tau(\omega,p)\right|,\ 0<\omega\le\omega_p,\ -0.5\le p\le 0.5\right\}, \quad (22c)$$

respectively, where $\tau_d(\omega,p)$ and $\tau(\omega,p)$ are the ideal delay response and the actual delay response of the designed VFD FIR differentiator. To compute (22b) and (22c), the frequency ω and the parameter p are uniformly sampled at the step sizes $\omega_p/400$ and 1/50, respectively. Notice that the delay errors for $\omega=0$, $-0.5\le p\le 0.5$ are not included in (22c) because the magnitude responses over that area are almost zero.

Example: This example deals with the design of an $N=50$, $M=7$, $\omega_p=0.9\pi$ VFD FIR differentiator, and the magnitude responses and delay responses are shown in Fig.1(a) and (b), respectively, while Fig.1(c) and (d) present the absolute errors of variable frequency responses and the absolute delay errors, respectively. The related errors are list as below:

$$\varepsilon_2 = 0.00503772\%$$
$$\varepsilon_m = 0.0014095$$
$$\varepsilon_\tau = 0.02612531.$$

For the VFD FIR differentiator design, it is not recommended to incorporate a weighting function in (13) such that the peak absolute error of variable frequency responses can be minimized, because the delay responses near $\omega=0$ are very sensitive to the variation of magnitude responses. For example, if the iterative weighted-least-squares method in [25] is applied, about five iterations are needed to minimize the maximum absolute error of variable frequency responses, and $\varepsilon_m = 0.0003173$ which is much smaller than that shown above, but the maximum absolute delay error becomes $\varepsilon_\tau = 0.05014075$ which is much larger than that shown above.

III. Conclusions

In this paper, the proposed least-squares method has been successfully applied to the design of VFD FIR differentiators. It can be found that the dominant difference between general VFD FIR digital filters [22] and VFD FIR differentiators is that only the middle coefficient of the first subfilter is required for the former but not for the latter. Although the proposed system can be replaced by cascading a differentiator with a variable fractional-delay filter, but however, which will leads to a larger delay. Obviously, the technique proposed in this paper can also be applied to the design of VFD higher-order or fractional-order FIR differentiators.

References

[1] S. Usui and I. Amidror, "Digital low-pass differentiation for biological signal processing," *IEEE Trans. Biomed. Eng.*, vol.BME-29, no.10, pp.686-693, Oct. 1982.

[2] R. C. Kavanagh, "FIR differentiators for quantized signals," *IEEE Trans. Signal Process.,* vol.49, no.11, pp.2713-2720, Nov. 2001.

[3] J. Luo, J. Bai, P. He and K. Ying, "Axial strain calculation using a low-pass digital differentiator in ultrasound elastography," *IEEE Trans. Ultrason., Ferroelect., Freq. Contr.*, vol.51, no.9, pp.1119-1127, Sep. 2004.

[4] S. L. Lin and S. Mourad, "On-chip rise-time measurement," *IEEE Trans. Instrum. Meas.,* vol.53, no.6, pp.1510-1516, Dec. 2004.

[5] S.-C. Pei and J.-J. Shyu, "Eigenfilter design of high-order digital differentiators," *IEEE Trans. Acoust., Speech, Signal Process.*, vol.37, no.4, pp.505-511, Apr. 1989.

[6] S. Sunder, Y. Su, A. Antoniou and W.-S. Lu , "Design of digital differentiators satisfying prescribed specifications using optimization techniques," in *Proc. ISCAS* 1989, May 1989, pp.1652-1655.

[7] B. Kumar, S. C. D. Roy and H. Shah, "On the design of FIR digital differentiators which are maximally linear at the frequency π/p, $p\in\{\text{positive integers}\}$," *IEEE Trans. Signal Process.,* vol.40, no.9, pp.2334-2338, Sep. 1992.

[8] E. Hermanowicz and M. Rojewski, "Design of FIR first order digital differentiators of variable fractional sample delay using maximally flat error criterion," *Electron. Lett.*, vol.30, no.1, pp.17-18, Jan. 1994.

[9] M. A. Al-Alaoui, "Novel IIR differentiator from the Simpon integration rule," *IEEE Trans. Circuits Syst. I, Fundament. Theory Appl.*, vol.41, no.2, pp.186-187, Feb. 1994.

[10] X. Zhang and T. Yoshikawa, "Design of full band IIR digital differentiators," in *Proc. IEEE Int. Conf. Neural Net. Signal Process.*, Dec. 2003, pp.652-655.

[11] C.-C. Tseng, "Digital differentiator design using fractional delay filter and limit computation," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol.52, no.10, pp.2248-2259, Oct. 2005.

[12] N. Q. Ngo, "A new approach for the design of wideband digital integrator and differentiator," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol.53, no.9, pp.936-940, Sep. 2006.

[13] M. A. Al-Alaoui, "Linear phase low-pass IIR digital differentiators," *IEEE Trans. Signal Process.,* vol.55, no.2, pp.697-706, Feb. 2007.

[14] C.-C. Tseng and S.-L. Lee, "Design of digital differentiator using difference formula and Richardson extrapolation," vol.2, no.2, pp.177-188, Jun. 2008.

[15] C. Y.-F. Ho, B. W.-K. Ling, Y.-Q. Liu, P. K.-S. Tam and K.-L. Teo, "Optimum design of discrete-time differentiators via semi-infinite programming approach," *IEEE Trans. Instrum. Meas.,* vol.57, no.10, pp.2226-2230, Oct. 2008.

[16] C. W. Farrow, "A continuously variable digital delay element," in *Proc. ISCAS* 1988, May 1988, pp.2641-2645.

[17] T. I. Laakso, V. Välimäki, M. Karjalainen and U. K. Laine, "Splitting the unit delay: Tools for fractional delay filter design," *IEEE Signal Process. Mag.*, vol.13, pp.30-60, Jan. 1996.

[18] W.-S. Lu and T.-B. Deng, "An improved weighted least-squares design for variable fractional delay FIR filters," *IEEE Trans. Circuits Syst. I, Analog Digit. Signal Process.*, vol.46, no.8, pp.1035-1040, Aug. 1999.

[19] H. Johansson and P. Löwenborg, "On the design of adjustable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II, Analog Digit. Signal Process.*, vol.50, no.4, pp.164-169, Apr. 2003.

[20] C.-C. Tseng, "Design of variable fractional delay FIR filter using symmetry," in *Proc. ISCAS* 2004, May 2004, pp.III/477-III/480.

[21] H. Zhao and J. Yu, "A simple and efficient design of variable fractional delay FIR filters," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol.53, no.2, pp.157-160, Feb. 2006.

[22] T.-B. Deng, and Y. Lian, "Weighted-least-squares design of variable fractional-delay FIR filters using coefficient symmetry," *IEEE Trans. Signal Process.*, vol.54, no.8, pp.3023-3038, Aug. 2006.

[23] E. Hermanowice and H. Johansson, "A complex variable fractional-delay FIR filter structure," *IEEE Trans. Circuits Syst. II, Exp. Briefs*, vol.54, no.9, pp.785-789, Sep. 2007.

[24] T.-B. Deng, "Symmetric structures for odd-order maximally flat and weighted-least- squares variable fractional-delay filters," *IEEE Trans. Circuits Syst. I, Reg. Papers*, vol.54, no.12, pp.2718-2732, Dec. 2007

[25] C. Y. Chi and Y. T. Kou, "A new self-initiated optimum WLS approximation method for the design of linear phase FIR digital filters," in *Proc. ISCAS* 1991, June 1991, pp.168-171.

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