Delay-Dependent Stability Criteria for Linear Time-Delay System of Neutral Type

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Abstract—This paper proposes improved delay-dependent stability conditions of the linear time-delay systems of neutral type. The proposed methods employ a suitable Lyapunov-Krasovskii's functional and a new form of the augmented system. New delay-dependent stability criteria for the systems are established in terms of Linear matrix inequalities (LMIs) which can be easily solved by various effective optimization algorithms. Numerical examples showed that the proposed method is effective and can provide less conservative results.

Keywords—Neutral systems; Time-delay; Stability; Lyapunov method; LMI.

I. INTRODUCTION

Time-delay occurs in various physical, industrial and engineering systems such as aircrafts, biological systems, neural networks, networked control systems, and so on. It has been shown that the delay is a source of oscillation, poor performance or instability of control systems. Therefore, the study on stability analysis for time-delay systems has been widely investigated. For more details, see [1]-[22].

In general, stability analysis for time-delay systems can be classified into two types. One is the delay-dependent stability analysis which includes the information on the size of delay, and another is the delay-independent stability analysis which do not. Generally speaking, the former is less conservative then the latter particularly when the time-delay is small. In delay-dependent stability analysis, an important issue is to enlarge the feasibility region of stability criteria or to provide an upper bound of time delays for guaranteeing asymptotic stability of time-delay systems. Therefore, a great number of results on time-delay systems have been reported in the literature [5]-[12]. Above all, Ariba and Gouaisbaut [12] proposed some new stability criteria by an augmented model of time-varying delay systems and presented one by a form of the Lyapunov-Krasovskii's functional that includes a triple-integral term.

On the other hand, some practical systems can be modeled by using the model of time-delay systems of the neutral type, which have delays in both its state and the derivatives of the state. This system refers to the time-delay systems which are the amount of the past state variables affects the current state variables. The examples of time-delay systems of

neutral type include lossless transmission lines (LC circuit), partial element equivalent circuit (PEEC) [23], the control of constrained manipulators with delay measurements [24], the system which need the information of the past state variables, and so on. The various approach to the delay-dependent stability analysis for time-delay system of neutral type have been investigated in the literature [13]-[22], on account of theoretical and practical importance for time-delay systems of neutral type.

In this paper, we propose improved delay-dependent stability criteria for linear time-delay systems of neutral type. By constructing a suitable Lyapunov-Krasovskii's functional and a new augmented system, new delay-dependent criteria are derived in terms of LMIs which can be solved efficiently by using the interior-point algorithms [25]. Numerical examples are included to show the effectiveness of the proposed method. The organization of the paper is as follows. In Section 2, we formulate the solved problem, and review the general lemmas which are needed to derive new stability criteria. In Section 3, we deal with new stability criteria for time-delay systems. In Section 4, based on the results in Section 3, three numerical examples and PEEC model which is a practical example are given for a comparison of the previous results. Finally, in Section 5, we summarize the results in this paper.

Notation: $\mathbf{R}^n$ is the n-dimensional Euclidean space, $\mathbf{R}^{m \times n}$ denotes the set of $m \times n$ real matrix. $\mathbf{C}_{n,h} = \mathbf{C}([-h, 0], \mathbf{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into $\mathbf{R}^n$, with the topology of uniform convergence. $X > 0$ (respectively, $X \geq 0$) means that the matrix X is a real symmetric positive definite matrix (respectively, positive semi-definite). I denotes the identity matrix with appropriate dimensions. $\| \cdot \|$ refers to the Euclidean vector norm and the induced matrix norm. $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix.

II. PROBLEM STATEMENTS

Consider the following linear systems with time-delay

$$\begin{aligned}
\dot{x}(t) - C\dot{x}(t-h) &= Ax(t) + A_d x(t-h), \quad \forall t > 0, \\
x(s) &= \phi(s), \quad \forall s \in [-h, 0], \quad h > 0, \quad (1)
\end{aligned}$$

where $x(t) \in \mathbf{R}^n$ is the state vector, A, A_d, $C \in \mathbf{R}^{n \times n}$ are known constant matrices with appropriate dimension, $\phi(s) \in \mathbf{C}_{n,h}$ is a given continuous vector valued initial function, h is a constant time-delay.

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International Journal of Computer and Information Engineering
Vol:4, No:10, 2010

In this paper, we assume $\|C\| < 1$ which implies that the system (1) satisfies Lipschitz condition in $\dot{x}(t-h)$ with a constant less than 1 (see [1, pp.29-30] for details).

To derive stability analysis of the system (1), we introduce two equivalent systems. One is to transform the original system (1) to the following equivalent system by integrating both terms of Eq.(1)

$$\int_{t-h}^{t} \dot{x}(s)ds - C\int_{t-h}^{t} \dot{x}(s-h)ds$$
$$= A\int_{t-h}^{t} x(s)ds + A_d\int_{t-h}^{t} x(s-h)ds, \qquad (2)$$

and another is the differentiating system (1)

$$\ddot{x}(t) - C\ddot{x}(t-h) = A\dot{x}(t) + A_d\dot{x}(t-h). \qquad (3)$$

Eq.(1)-(3) can be re-arranged to give the following new augmented system:

$$\mathcal{E}\dot{z}(t) - \mathcal{C}\dot{z}(t-h) = \mathcal{A}z(t) + \mathcal{A}_dz(t-h), \qquad (4)$$

where

$$\mathcal{E} = \begin{bmatrix} I & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \\ I & 0 & 0 \\ 0 & I & 0 \end{bmatrix}, \quad \mathcal{C} = \begin{bmatrix} C & 0 & 0 \\ 0 & C & 0 \\ 0 & 0 & C \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$\mathcal{A} = \begin{bmatrix} A & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & A \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix}, \quad \mathcal{A}_d = \begin{bmatrix} A_d & 0 & 0 \\ 0 & A_d & 0 \\ 0 & 0 & A_d \\ 0 & -I & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$z(t) = \begin{bmatrix} \int_{t-h}^{t} x(s)ds \\ x(t) \\ \dot{x}(t) \end{bmatrix}. \qquad (5)$$

Remark 1: The augmented system (4) is a new one and not considered in other literature. By differentiating and integrating system (1), three equations (1), (2), and (3) can be represented as one dynamic system which have more information than system (1). Also, by constructing a suitable Lyapunov-Krasovskii's functional for augmented state $z(t)$, a less conservative stability condition may be derived, which provide an improved feasible region for system (1).

The goal of this paper is to investigate the delay-dependent stability analysis of system (1) with time-invariant delay.

Before deriving our main results, we give the following fact and lemmas.

Fact 1: (Schur Complement) Given constant matrices Σ_1, Σ_2, Σ_3 with $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0. \qquad (6)$$

Lemma 1: [3] Let $\zeta \in \mathbf{R}^n$, $\Phi = \Phi^T \in \mathbf{R}^{n \times n}$, and $B \in \mathbf{R}^{m \times n}$ such that $rank(B) < n$. The following statement are equivalent:
(i) $\zeta^T\Phi\zeta < 0$, $\forall B\zeta = 0$, $\zeta \neq 0$,
(ii) $B^{\perp^T}\Phi B^{\perp} < 0$ where $B^{\perp}$ is a right orthogonal complement of B.

Lemma 2: [4] For any constant matrix $M \in \mathbf{R}^{n \times n}$, $M = M^T > 0$, a scalar $\gamma > 0$ and a vector function $x : [0, \gamma] \to \mathbf{R}^n$ such that the integrations concerned are well defined, then

$$\gamma\int_0^{\gamma} x^T(s)Mx(s)ds$$
$$\geq \left(\int_0^{\gamma} x(s)ds\right)^T M\left(\int_0^{\gamma} x(s)ds\right). \qquad (7)$$

To derive a less conservative stability criterion, we use the following lemma, to be utilized in deriving an upper bound of double-integral terms.

Lemma 3: For any scalar $h > 0$ and any constant matrix $M = M^T > 0$, the following inequality holds:

$$\frac{h^2}{2}\int_{t-h}^{t}\int_s^t x^T(u)Mx(u)duds$$
$$\geq \left(\int_{t-h}^{t}\int_s^t x(u)duds\right)^T M\left(\int_{t-h}^{t}\int_s^t x(u)duds\right). \qquad (8)$$

Proof: From Lemma 2, the following inequality holds

$$(t-s)\int_s^t x^T(u)Mx(u)du$$
$$\geq \left(\int_s^t x(u)du\right)^T M\left(\int_s^t x(u)du\right), \qquad (9)$$

where $t - h \leq s \leq t$.
By using Fact 1, Eq.(9) is equivalent to the following

$$\begin{bmatrix} \int_s^t x^T(u)Mx(u)du & \int_s^t x^T(u)du \\ \int_s^t x(u)du & (t-s)M^{-1} \end{bmatrix} \geq 0. \qquad (10)$$

Integration of (10) from $t-h$ to t yields

$$\begin{bmatrix} \int_{t-h}^{t}\varphi(t,s)ds & \int_{t-h}^{t}\int_s^t x^T(u)duds \\ \int_{t-h}^{t}\int_s^t x(u)duds & \int_{t-h}^{t}(t-s)M^{-1}ds \end{bmatrix} \geq 0, \qquad (11)$$

where $\varphi(t,s) = \int_s^t x^T(u)Mx(u)du$.
Eq.(11) is equivalent to inequality (8) according to Fact 1. ∎

III. MAIN RESULTS

In this section, we propose new stability criteria for time-delay system (1). Before introducing our main results, the notation of several matrices are defined for simplicity:

$$B = [\mathcal{A} \quad \mathcal{A}_d \quad -\mathcal{E} \quad \mathcal{C}],$$
$$\zeta^T(t) = [z^T(t) \quad z^T(t-h) \quad \dot{z}^T(t) \quad \dot{z}^T(t-h)]. \qquad (12)$$

Now, we have the following theorem.

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Theorem 1: For a given positive scalar h, the system (1) with time-invariant delay is asymptotically stable if $\|C\| < 1$ and there exist positive definite matrices $\mathcal{P} = [P_{ij}]_{3\times3}$, $\mathcal{Q}_1 = [Q_{1,ij}]_{3\times3}$, $\mathcal{Q}_3 = [Q_{3,ij}]_{3\times3}$, $\mathcal{R} = [R_{ij}]_{3\times3}$, and any matrix $\mathcal{Q}_2 = [Q_{2,ii}]_{3\times3}$ satisfying the follow LMIs:

$$B^{\perp^T}\Phi B^{\perp} < 0. \tag{13}$$

where

$$\Phi = \begin{bmatrix} \mathcal{Q}_1 - \mathcal{R} & \mathcal{R} & \mathcal{P} + \mathcal{Q}_2 & 0 \\ \star & -\mathcal{Q}_1 - \mathcal{R} & 0 & -\mathcal{Q}_2 \\ \star & \star & \mathcal{Q}_3 + h^2\mathcal{R} & 0 \\ \star & \star & \star & -\mathcal{Q}_3 \end{bmatrix}. \tag{14}$$

Proof: Let us choose the well-known Lyapunov-Krasovskii's functional candidate as

$$V = V_1 + V_2 + V_3, \tag{15}$$

where

$$\begin{aligned} V_1 &= z^T(t)\mathcal{P}z(t), \\ V_2 &= \int_{t-h}^{t} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix} ds, \\ V_3 &= h\int_{t-h}^{t}\int_{s}^{t} \dot{z}^T(u)\mathcal{R}\dot{z}(u)duds. \end{aligned} \tag{16}$$

First, the time-derivative of V_1 can be calculated as

$$\dot{V}_1 = 2z^T(t)\mathcal{P}\dot{z}(t). \tag{17}$$

Second, the time-derivative of V_2 can be obtained as

$$\begin{aligned} \dot{V}_2 &= \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} \\ &\quad - \begin{bmatrix} z(t-h) \\ \dot{z}(t-h) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(t-h) \\ \dot{z}(t-h) \end{bmatrix}. \end{aligned} \tag{18}$$

Finally, calculating the time-derivative of V_3 lead to

$$\dot{V}_3 = h^2\dot{z}^T(t)\mathcal{R}\dot{z}(t) - h\int_{t-h}^{t}\dot{z}^T(s)\mathcal{R}\dot{z}(s)ds. \tag{19}$$

By using Lemma 2, an upper bound of integral term of $\dot{V}_3$ can be obtained as

$$\begin{aligned} &-h\int_{t-h}^{t}\dot{z}^T(s)\mathcal{R}\dot{z}(s)ds \\ &\leq -\left(\int_{t-h}^{t}\dot{z}(s)ds\right)^T \mathcal{R}\left(\int_{t-h}^{t}\dot{z}(s)ds\right) \\ &= \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}^T \begin{bmatrix} -\mathcal{R} & \mathcal{R} \\ \star & -\mathcal{R} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}. \end{aligned} \tag{20}$$

From (16)-(20), the time-derivative of V has a new upper bound as

$$\dot{V} \leq \zeta^T(t)\Phi\zeta(t), \tag{21}$$

where $\zeta(t)$ and Φ are defined in (12) and (14), respectively.

In addition, the system (1) with time-invariant delay can be rewritten as $B\zeta(t) = 0$ where B is defined in (12). By

Lemma 1, the inequality $\zeta(t)\Phi\zeta(t) < 0$ is equivalent to the inequality $B^{\perp^T}\Phi B^{\perp} < 0$. Therefore, if for all $\zeta(t)$ such that $B\zeta(t) = 0$, the LMI (13) are satisfied, then the system (1) with time-invariant delay is guaranteed to be asymptotically stable. This completes our proof. ∎

Theorem 1 is derived by utilizing a well-known double integral form of Lyapunov-Krasovskii's functional. If a triple-integral form of Lyapunov-Krasovskii's functional are included, an improved delay-dependent stability criterion which will be introduced in Theorem 2 can be derived.

Theorem 2: For a given positive scalar h, the system (1) with time-invariant delay is asymptotic stable if $\|C\| < 1$ and there exist positive definite matrices $\mathcal{P} = [P_{ij}]_{3\times3}$, $\mathcal{Q}_1 = [Q_{1,ij}]_{3\times3}$, $\mathcal{Q}_3 = [Q_{3,ij}]_{3\times3}$, $\mathcal{R} = [R_{ij}]_{3\times3}$, S, and any matrix $\mathcal{Q}_2 = [Q_{2,ii}]_{3\times3}$ satisfying the follow LMIs:

$$B^{\perp^T}\hat{\Phi}B^{\perp} < 0. \tag{22}$$

where

$$\begin{aligned} \hat{\Phi} &= \begin{bmatrix} \mathcal{Q}_1 - \mathcal{R} & \mathcal{R} & \mathcal{P} + \mathcal{Q}_2 & 0 \\ \star & -\mathcal{Q}_1 - \mathcal{R} & 0 & -\mathcal{Q}_2 \\ \star & \star & \mathcal{T} & 0 \\ \star & \star & \star & -\mathcal{Q}_3 \end{bmatrix}, \\ \mathcal{T} &= \mathcal{Q}_3 + h^2\mathcal{R} + \mathcal{S}, \\ \mathcal{S} &= \begin{bmatrix} -S & hS & 0 \\ \star & -h^2S & 0 \\ \star & \star & (h^2/2)^2S \end{bmatrix}. \end{aligned} \tag{23}$$

Proof: Let us choose the Lyapunov-Krasovskii's functional candidate that contains a triple-integral term as

$$V = V_1 + V_2 + V_3 + V_4, \tag{24}$$

where

$$\begin{aligned} V_1 &= z^T(t)\mathcal{P}z(t), \\ V_2 &= \int_{t-h}^{t} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(s) \\ \dot{z}(s) \end{bmatrix} ds, \\ V_3 &= h\int_{t-h}^{t}\int_{s}^{t} \dot{z}^T(u)\mathcal{R}\dot{z}(u)duds, \\ V_4 &= \frac{h^2}{2}\int_{t-h}^{t}\int_{s}^{t}\int_{u}^{t} \dot{z}^T(v)\Pi^T S\Pi\dot{z}(v)dvduds, \end{aligned} \tag{25}$$

and Π in V_4 is defined as $\Pi = [0 \ \ 0 \ \ I]$.

First, with the similar method of the proof of Theorem 1, the time-derivative of V_1, V_2 and V_3 can be calculated as

$$\begin{aligned} \dot{V}_1 &= 2z^T(t)\mathcal{P}\dot{z}(t), \\ \dot{V}_2 &= \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(t) \\ \dot{z}(t) \end{bmatrix} \\ &\quad - \begin{bmatrix} z(t-h) \\ \dot{z}(t-h) \end{bmatrix}^T \begin{bmatrix} \mathcal{Q}_1 & \mathcal{Q}_2 \\ \star & \mathcal{Q}_3 \end{bmatrix} \begin{bmatrix} z(t-h) \\ \dot{z}(t-h) \end{bmatrix}, \\ \dot{V}_3 &\leq h^2\dot{z}^T(t)\mathcal{R}\dot{z}(t) \\ &\quad + \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}^T \begin{bmatrix} -\mathcal{R} & \mathcal{R} \\ \star & -\mathcal{R} \end{bmatrix} \begin{bmatrix} z(t) \\ z(t-h) \end{bmatrix}. \end{aligned} \tag{26}$$

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Calculating the time-derivative of V_4 lead to

$$
\begin{aligned}
\dot{V_4} &= (h^2/2)^2 \dot{z}^T(t) \Pi^T S \Pi \dot{z}(t) \\
&\quad -(h^2/2) \int_{t-h}^t \int_s^t \dot{z}^T(u) \Pi^T S \Pi \dot{z}(u) duds \\
&= (h^2/2)^2 \ddot{x}^T(t) S \ddot{x}(t) \\
&\quad -(h^2/2) \int_{t-h}^t \int_s^t \ddot{x}^T(u) S \ddot{x}(u) duds.
\end{aligned}
\tag{27}
$$

and Π is defined in (25).

By using Lemma 3, an upper bound of double-integral term of $\dot{V_4}$ can be obtained as

$$
\begin{aligned}
&-(h^2/2) \int_{t-h}^t \int_s^t \ddot{x}^T(u) S \ddot{x}(u) duds \\
&\leq -\left(\int_{t-h}^t \int_s^t \ddot{x}(u) duds \right)^T S \left(\int_{t-h}^t \int_s^t \ddot{x}(u) duds \right) \\
&= \begin{bmatrix} \dot{x}(t) \\ \int_{t-h}^t \dot{x}(t)ds \end{bmatrix}^T \begin{bmatrix} -h^2 S & hS \\ \star & -S \end{bmatrix} \begin{bmatrix} \dot{x}(t) \\ \int_{t-h}^t \dot{x}(t)ds \end{bmatrix}.
\end{aligned}
\tag{28}
$$

Using the similar method shown in the proof of Theorem 1, the LMIs (22) can be easily obtained. ∎

IV. NUMERICAL EXAMPLES

In this section, we provide four examples to show the less conservativeness of the proposed new stability criterion in this paper.

Example 1: Consider the neutral system (1) with the following parameters

$$
A = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix}, \; A_d = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},
$$
$$
C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix}, \; 0 \leq c < 1.
\tag{29}
$$

Table I shows the results of the upper bound of time-delay with different c. It can be seen that Theorem 2 in this paper provides larger delay bound than the previous results given in Table I.

The example shows that Theorem 1 and 2 obtain the less conservative results step by step. Based on the well-known Lyapunov-Krasovskii functional (16) with the current state of new augmented system (4), Theorem 1 is proposed, which is proved to less conservative than the results in [7], [16] and [14]. In Theorem 2, for further improved result, we add the triple-integral term of $\ddot{x}(t)$ on (16) since we consider the current state $\int_{t-h(t)}^t x(s)ds$ of the integrating system (2). These results in Theorems indicate that the presented stability conditions relieve the constraint of the stability caused by time-delay.

Example 2: Consider the neutral system in the form of (1) with

$$
A = \begin{bmatrix} -0.9 & 0.2 \\ 0.1 & -0.9 \end{bmatrix}, \; A_d = \begin{bmatrix} -1.1 & -0.2 \\ -0.1 & -1.1 \end{bmatrix},
$$
$$
C = \begin{bmatrix} -0.2 & 0 \\ 0.2 & -0.1 \end{bmatrix}.
\tag{30}
$$

TABLE I
UPPER BOUNDS OF TIME-DELAY WITH DIFFERENT c (EXAMPLE 1).

c	0	0.1	0.5	0.9
Fridman et al. [7]	4.47	3.49	1.14	0.13
Xu et al. [16]	4.47	3.58	1.46	0.32
Wu et al. [14]	4.47	4.35	3.67	1.41
Ours (Theorem 1)	4.47	4.42	3.69	1.41
Ours (Theorem 2)	5.30	5.21	4.20	1.49

TABLE II
UPPER BOUNDS OF TIME-DELAY (EXAMPLE 2).

Mathods	Upper bounds
Zhao et al. [18]	1.7856
Kwon et al. [19]	1.8266
Nian et al. [22]	1.9132
Ours (Theorem 2)	2.0054

TABLE III
UPPER BOUNDS OF TIME-DELAY WITH DIFFERENT β (EXAMPLE 4).

β	-2.105	-2.103	-2.1
Yue et al. [15]	1.1413	0.3892	0.2553
Kwon et al. [20]	1.1410	0.3892	0.2553
Ours (Theorem 2)	1.4483	0.4917	0.3214

In Table II, the results of the upper bound of time-delay for guaranteeing stability are compared with the previous results. It also can be shown that the proposed stability criterion for this system improves the stability region.

Example 3: Consider the system (1) with

$$
A = \begin{bmatrix} -1.7073 & 0.6856 \\ 0.2279 & -0.6368 \end{bmatrix},
$$
$$
A_d = \begin{bmatrix} -2.5026 & -1.0540 \\ -0.1856 & -1.5715 \end{bmatrix},
$$
$$
C = \begin{bmatrix} 0.0558 & 0.0360 \\ 0.2747 & -0.1084 \end{bmatrix}.
\tag{31}
$$

In [16], for the above system, the obtained the upper bound of time-delay was 0.5735. By Theorem 1 in [21], it was shown that the upper bound of time-delay with the same condition was 0.6189. By applying Theorem 2, it can be obtained that the upper bound of time-delay is 0.6612, which is larger delay bound than one in [16] and [21].

When $C = 0$, the upper bound of time-delay obtained 0.6903 in [16] and 0.7918 in [21]. However, by using Theorem 2, one can obtain the upper bound of time-delay is 0.8431. This result of our criterion gives a larger delay bound than one in [16] and [21].

Example 4: Consider the following PEEC model:

$$
A = 100 \times \begin{bmatrix} \beta & 1 & 2 \\ 3 & -9 & 0 \\ 1 & 2 & -6 \end{bmatrix},
$$
$$
A_d = 100 \times \begin{bmatrix} 1 & 0 & -3 \\ -0.5 & -0.5 & -1 \\ -0.5 & -1.5 & 0 \end{bmatrix},
$$
$$
C = \frac{1}{72} \times \begin{bmatrix} -1 & 5 & 2 \\ 4 & 0 & 3 \\ -2 & 4 & 1 \end{bmatrix}.
\tag{32}
$$

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In Table III, the results for different condition of β are compared with the results in [15] and [20]. From Table III, it can be shown that our result for this example gives larger upper bound of time-delay than the ones in [15] and [20].

V. CONCLUSION

In this paper, new delay-dependent stability criteria for linear time-delay systems of neutral type is proposed. To obtain a less conservative result, an augmented Lyapunov-Krasovskii's functional that includes a triple-integral term is used to improve the feasible region of stability criterion. Numerical examples have been given to show the superiority of the presented criteria and its improvement over the existing results.

ACKNOWLEDGMENT

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