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Packing Theory for Natural and Crushed Aggregate to Obtain the Best Mix of Aggregate: Research and Development

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***Abstract*—**Concrete performance is strongly affected by the particle packing degree since it determines the distribution of the cementitious component and the interaction of mineral particles. By using packing theory designers will be able to select optimal aggregate materials for preparing concrete with low cement content, which is beneficial from the point of cost. Optimum particle packing implies minimizing porosity and thereby reducing the amount of cement paste needed to fill the voids between the aggregate particles, taking also the rheology of the concrete into consideration. For reaching good fluidity superplasticizers are required. The results from pilot tests at Luleå University of Technology (LTU) show various forms of the proposed theoretical models, and the empirical approach taken in the study seems to provide a safer basis for developing new, improved packing models.

***Keywords*—**Aggregate mix, Computer program, Concrete mix design, Models of packing.

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I. INTRODUCTION

CONCRETE behavior is affected by the packing degree of the concrete components, making it necessary for engineers working to consider, in detail, particle packing concepts and their influence on concrete behavior for being able to select suitable fine aggregate material.

The aim of optimizing concrete mixing is to prepare concrete with the being as densely packed as possible. The amount of binder for filling the aggregate voids can be minimized still keeping the freshly mixed concrete (workability) sufficiently fluid.

A minimum amount of binder is beneficial not only from economical points of view but also to reduce shrinkage and creep and thereby obtain a product that is more durable and strong than one with more binder. The w/c ratio is a strength-controlling parameter that is affected by the packing concept. Particle packing models give a basis for mix designs not only for traditional concrete but also for selecting mix proportions for special concrete like high performance, self-compacting and high strength concrete [1].

II. OBJECTIVE AND SCOPE

The aim of the study has been to identify and normalize current packing models for comparison. A second aim has been to test the validity of selected theoretical models for simulating packing of aggregate (natural and crushed) by comparing them with results from new experimental work. New software has been worked out based on theoretical models, showing good agreement between the microstructural constitutions of natural aggregate and crushed aggregate. Comparison of the packing degree for natural aggregate and crushed aggregate has also been in focus. The argument is that use of natural aggregate in concrete should be minimized for environmental preservation.

III. RESEARCH SIGNIFICANCE

Use of validated models for obtaining optimal particle packing of aggregates provides a general basis for alternative aggregate combinations, implying that this can give designers scientific tools for selecting the best of several possible aggregate size spectra and hence to a cheaper concrete with improved quality without comprehensive testing.

Use of validated models will also reduce the necessary number of experiments required for using the recipe in practice. The models are incorporated in user-friendly software as illustrated in the present paper.

IV. DEFINITION OF PACKING

The degree of packing is expressed in terms of the amount of solid aggregate minerals per unit volume. The mathematical expression is simply “unity minus porosity” [2]; the degree of packing is function of the grading curve and the shape of the particles.

V. PACKING MODELS

A. Historical review

Comprehensive studies and derivation of packing concepts were initiated in the 19th century, and one of the first reports on particle packing for concrete production was published by R. Feret in 1892 [3],[4].

Packing techniques in preparation of concrete have been used in Scandinavia as early as 1896 for providing concrete durability in marine environment. Most of the literature on packing was published in the 1930s describing the optimization of packing followed by research made by Furnas in 1929 and by Westmann and Hugill in 1930 [3]. In 1989, Petersen showed the use of packing concepts in relation to the mechanical and the rheological properties.

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Petersen found that the model by Aim and Goff gave the best fit of the theoretical to the experimental packing densities for small particle diameter ratios and that the model defined by Toufar et. al [3] gave the best fit for larger diameter ratios.

Goltermann et al. [3] used three models in their tests, i.e. the Aim model, the Toufar model and the modified Toufar model. A large variety in particle size and size distribution of natural and crushed aggregates was considered in this study and the results showed that the Toufar model, and especially the modified Toufar model, agrees very well with the measured packing degrees. Correction factors would hence not be required. The Aim model did not fit the test results and could not be used for the aggregates.

B. Sedran et al. Models and software

The French concrete experts Sedran and Larrard [5] have developed a method that uses a new method for concrete mixing. Their software, Bétonlab, is consistent with their mathematical models. The first version of this software was available in1992. In addition to assessing the packing degree by use of the models, the authors showed that one can calculate the fresh concrete properties and also the compressive strength.

C. Toufar's model

The model proposed by Golterman et al., [3] based on the Toufar's model have been validated by comparing around 800 test results from multiple sources. The packing degree predicted by this model is expressed as in (1):

$$\varphi = \frac{1}{\left[\frac{y_1}{\varphi_1} + \frac{y_2}{\varphi_2} - y_2\left(\frac{1}{\varphi_2} - 1\right)k_d k_s\right]} \tag{1}$$

Where:

y_1/φ_1	is the bulk volume of the fine particles
y_2/φ_2	is the bulk volume of the coarse particles
$y_2(1/\varphi_2 - 1)$	is the void volume between the coarse particles
k_d	a factor that determines the influence of the diameter ratio
k_s	a statistical factor

Toufar et. al. also showed that as in (2) applies:

$$k_d = \frac{d_2 - d_1}{d_1 + d_2} \tag{2}$$

And that each of the fine particles is located between four of the coarse particles, leading as in (3):

$$k_s = 1 - \frac{1+4x}{(1+x)^4} \tag{3}$$

Where, x is the (bulk volume of the fine particles)/ (void volume between the coarse particles) and that it obtained as in (4).

$$x = \frac{\left(\frac{y_1}{y_2}\right)\left(\frac{\varphi_2}{\varphi_1}\right)}{(1-\varphi_2)} \tag{4}$$

The packing grades from tests have shown that it does not increase when a small amount of fine particles is added. This is because the fine particle is confined in the void between the four coarse particles. This over-idealized approach was corrected by alternating expression k_s as shown in (5) and (6):

$$k_s = k_0\left(\frac{x}{x_0}\right) \text{ For } x < x_0 \tag{5}$$

$X_0 = 0.4753$ and $k_0 = 0.3881$

$$k_s = 1 - \frac{1+4x}{(1+x)^4} \text{ For } x \geq x_0 \tag{6}$$

The Toufar model and the modified Toufar model are based on a number of assumptions that are not realistic, especially concerning shape and size variations [3].

The first two assumptions are overcome by introducing a characteristic diameter for the aggregates and by using the measured "eigenpacking" degrees for the aggregates according to Goltermann et al. [3]. Moreover, the authors stated that the void diameter is a central parameter for the particle distribution and should be the basis for ascribing characteristic diameters of the aggregate particles [5].

D. The 4C-Packing Model and Software

4C-Packing, is a model that can be used for calculating the packing of any combination of solid constituent's in concrete (aggregate, cement, fly ash etc.) [6]. 4C-Packing is based on a linear packing model developed on the basis of principles of packing of binary mixtures, extended to deal also with multi-component mixtures Stoval et al., [7]. Combination of empirical model data and this packing program makes it possible to optimize concrete composition for getting optimal properties, and at the same time to minimize the cement content and consequently the price. This software has been used as a tool for comparing results from other theoretical models and from practical tests of the packing degree. The basic packing formula is as shown in (7) [6]:

$$packing = Minimum_{i=1}^{n}\left(\alpha i + (1-\alpha i)\sum_{j=1}^{i-1} g(i,j)\phi i + \sum_{j=i+1}^{n} f(i,j)\phi j\right) \tag{7}$$

Where:

α : is the mono-disperse packing

Ø: is the volume fraction

f(i,j) : interaction function for the "wall effect" Small particles near to the wall of the container or the large particles cannot be packed as in bulk.

g(i,j) : interaction function for the effect of small particles misfitting in voids between large particles without disturbing the packing of the large ones. This effect is characterized by a

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so called “μ-value” [6], [8].

The previous model has been converted into a computer program which is available at the Concrete center of the Danish Technological Institute [9], the input and output of the program being shown in table I.

TABLE I
INPUT AND OUTPUT DATA BY 4C-PROGRAM

	Input	Output
Materials	Particle density Size distribution (grading curve) Eigen-packing Unit cost	
Calculations	“μ-value” GC subdivision	Packing density Packing diagrams Combined grading curve Concrete mix Grading factor

VI. EXPERIMENTAL PACKING METHODS

No typical technique exists that is appropriate for determination of the packing of aggregates. Based on practice and experience, it has been found to be suitable to compact the aggregates in such a way that the densest packing is achieved. This is not obtained by vibration, but by a shared shaking-tapping process. The procedure is described in detail in the user manual, [10]. The accuracy for determination of packing is around ± 2 %. This means that for a nominal value of 0.70, an interval of 0.69 to 0.71 can be expected.

VII. MATERIALS

Four different aggregate mixes described in table II have been investigated. One of them was a natural aggregate material that has been tested by the first author at a LTU laboratory, and the three other were crushed aggregate tested by [5] at the same laboratory. Different mixtures have been investigated. They are characterized by the size diagrams in Fig. 1, 2, 3 and 4.

TABLE II
AGGREGATE TYPES USED

Aggregate study 1 (natural aggregate)		Aggregate study 2 (crushed aggregate)		Aggregate study 3 (crushed aggregate)		Aggregate study 4 (crushed aggregate)	
Riksten	0-8 mm	Riksten	0.5-1 mm	Enhörna	0-4 mm	Kållered	0-0.5 mm
Riksten	8-16 mm	Riksten	2-4 mm	Enhörna	4-8 mm	Kållered	0.5-1 mm
Riksten	16-27 mm	Riksten	8-11 mm	Enhörna	8-11 mm	Kållered	1-2 mm

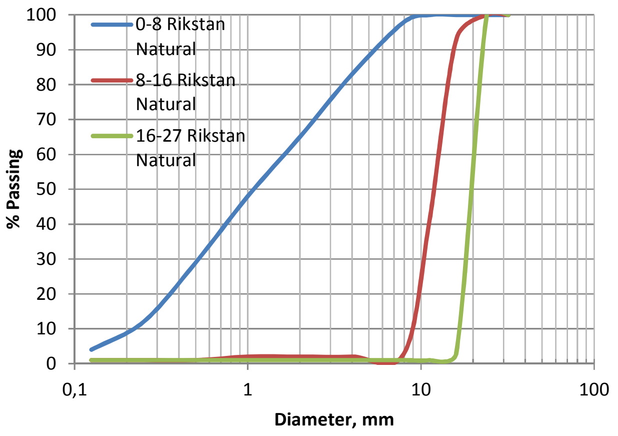

Fig. 1 Sieve curves showing the 0-8 mm, 8-16 mm and 16-27 mm fractions for the natural Riksten material.

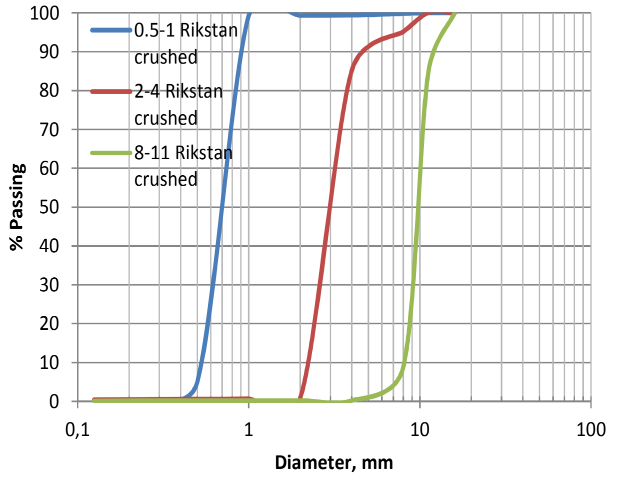

Fig. 2 Sieve curves showing the 0.5-1 mm, 2-4 mm and 8-11 mm fractions for the crushed Riksten material

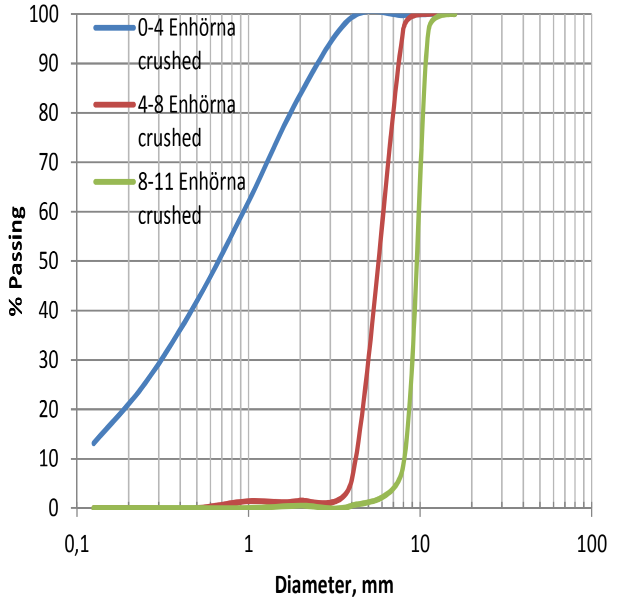

Fig. 3 Sieve curves showing the 0-4 mm, 4-8 mm and 8-11 mm fractions for the crushed Enhörna material finally

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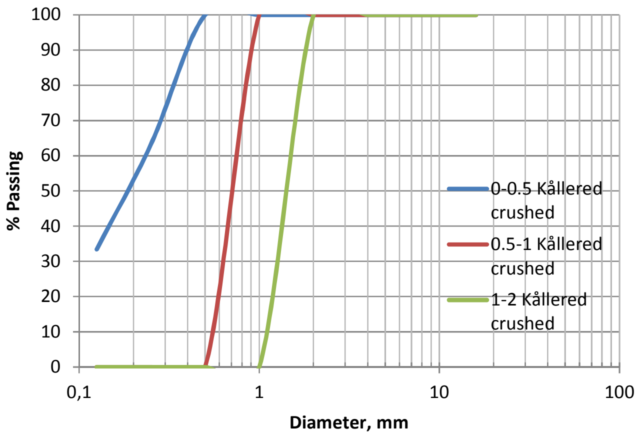

Fig. 4 Sieve curves showing the 0-0.5 mm, 0.5-1 mm and 1-2 mm fractions for the crushed Kållered material

VIII. RESULTS, ANALYSES AND DISCUSSION

A. Comparison of Theoretical Packing and Experimental tests aggregate type 1

Fig. 5 shows a graphical representation of the results from packing degree tests. One can see that 4C-Packing give results that are more in agreement with Toufar's than the average test results for natural aggregate. The theoretical 4C packing gives the highest packing degree for the aggregate composition 40% 0 – 8 mm, 1% 8 – 16 mm and 59% 16 – 27mm see Fig. 6, while the Toufar model gives a maximum packing degree for around 40 – 42% 0 – 8 mm, 18 – 20% 8 – 16 mm and around 40% 16 – 27 mm.

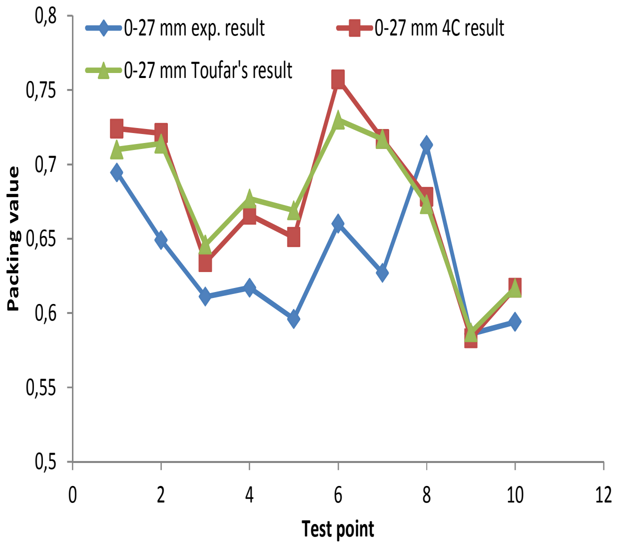

Fig. 5 Representation of the packing results from natural aggregate 1 test with packing from 4C program and Toufar model

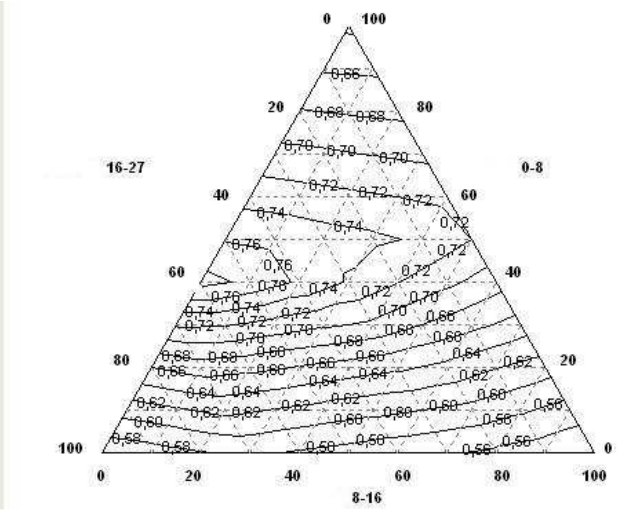

Fig. 6 Ternary diagram showing the Riksten natural material, 0-8, 8-16 and 16-27mm

B. Comparison of Theoretical Packing and Experimental tests aggregate type 2

Fig. 7 shows a graphical representation of the results found for packing degree tests. One can see that 4C-Packing give results that in better agreement with experiments than the Toufar's model. The accuracy can be inferred from the diagram with only 8 test points. The theoretical 4C packing gives a maximum packing degree for the aggregate composition 40% 0.5 – 1 mm, 0% 2 – 4 mm and 60% 8 – 11mm see Fig. 8, while the Toufar model gives a maximum packing degree for 27.3% 0.5 – 1 mm, 26.6% 2 – 4 mm and 46.1% 8 – 11 mm.

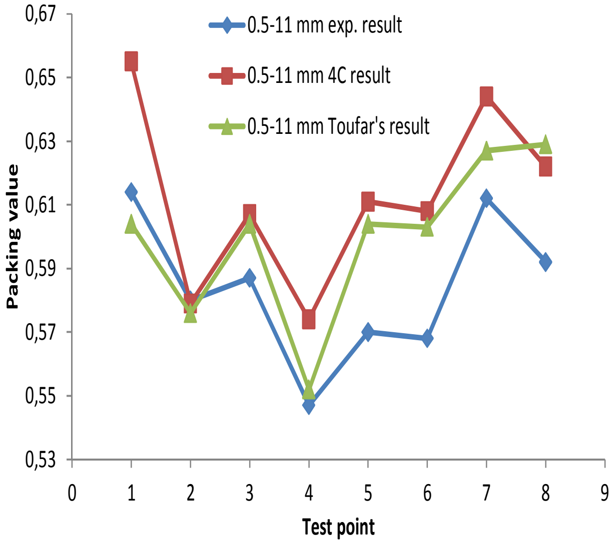

Fig. 7 Representation of the packing results from crushed aggregate 2 tests with packing from 4C program and Toufar model

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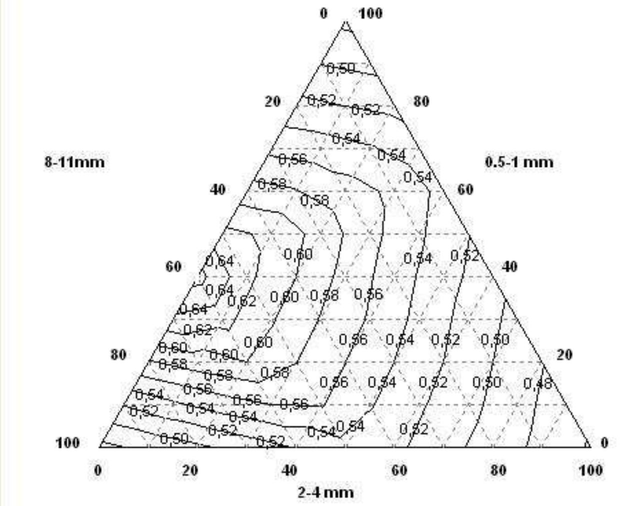

Fig. 8 Ternary diagram showing the Riksten crushed material, 0.5-1, 2-4 and 8-11 mm

C. Comparison of Theoretical Packing with Experimental tests aggregate type 3

Fig. 9 shows clearly that the 4C-Packing software corresponds better than the modified Toufar model also repecting the maximum value of the packing, The theoretical 4C packing gives a highest packing degree for the aggregate composition 47% 0 – 4 mm, 1% 4 – 8 mm and 52% 8 – 11mm see Fig. 10, while the Toufar model gives a highest packing degree for 25.5% 0 – 4 mm, 1% 4 – 8 mm and 73.5% 8 – 11 mm.

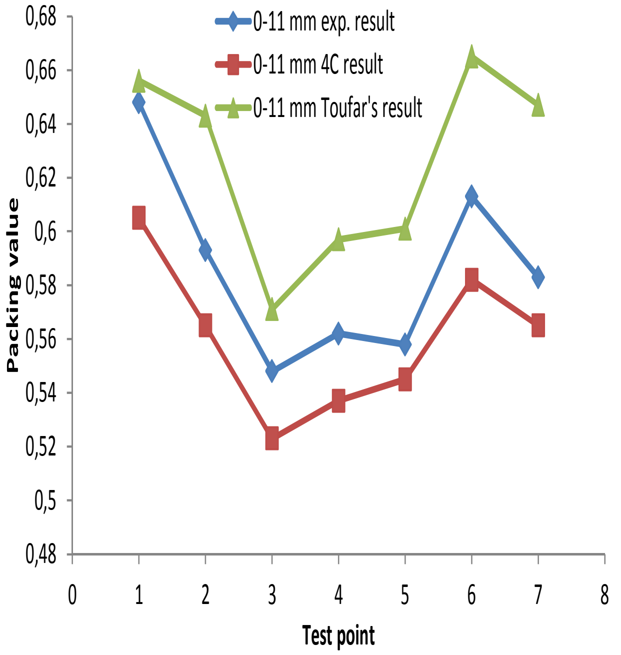

Fig. 9 Representation of the packing results from crushed aggregate 3 tests with packing from 4C program and Toufar model.

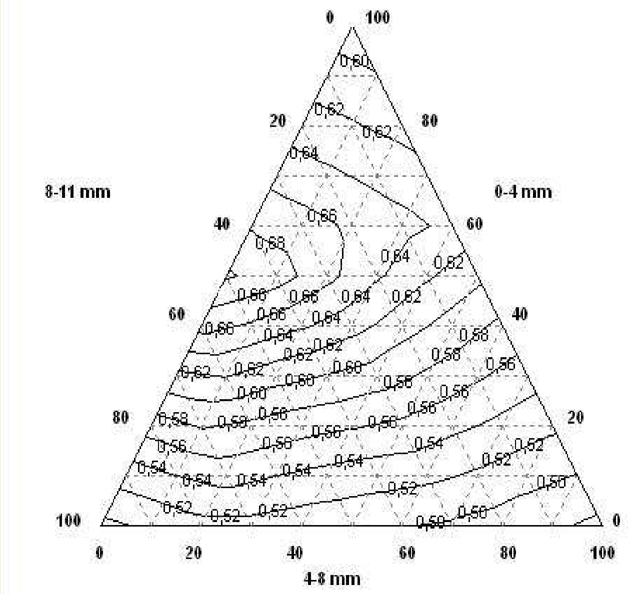

Fig. 10 Ternary diagram showing the Enhörna crushed material, 0-4, 4-8 and 8-11mm

D. Comparison of Theoretical Packing with Experimental tests aggregate type 4

Finally, the variation of the curves from physical tests represented by the lowest curve in Fig. 11 with those derived from the theoretical model test results compared with those from the 4C-Packing model calculations tendencies can be seen in the diagram. The profiles of the sets of curves correspond well with respect to the maximum packing value, The theoretical 4C packing gives a highest packing grade for the aggregate composition 60% 0 – 0.5 mm, 0% 0.5 – 1 mm and 40% 1 – 2mm see Fig. 12, while the Toufar model gives a maximum packing degree for 40% 0 – 0.5 mm, 21% 0.5 – 1 mm and finally 39% 1 – 2 mm.

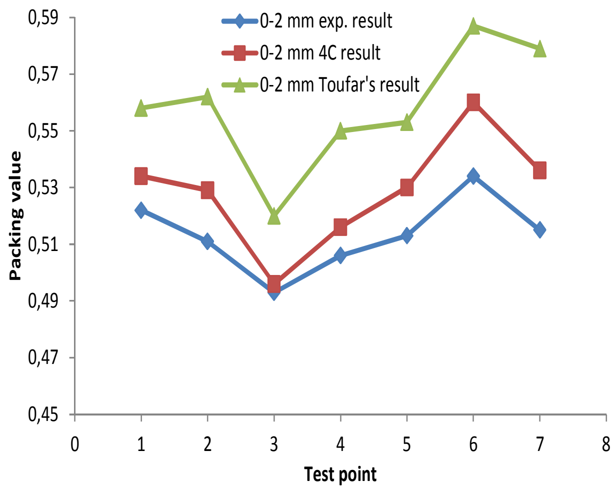

Fig. 11 Representation of the packing results from crushed aggregate 4 tests with packing from 4C program and Toufar model

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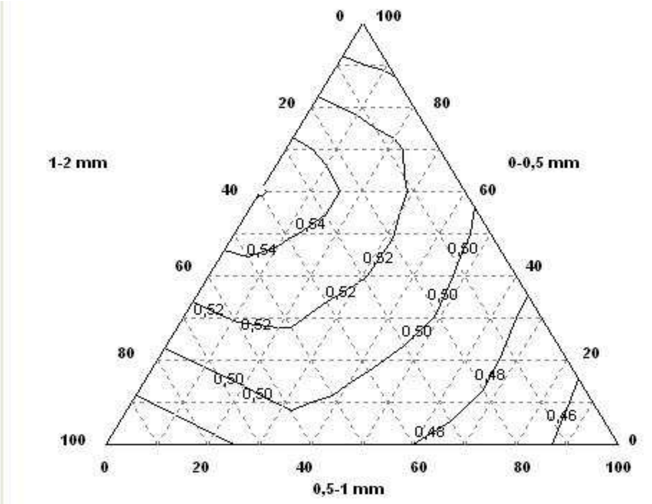
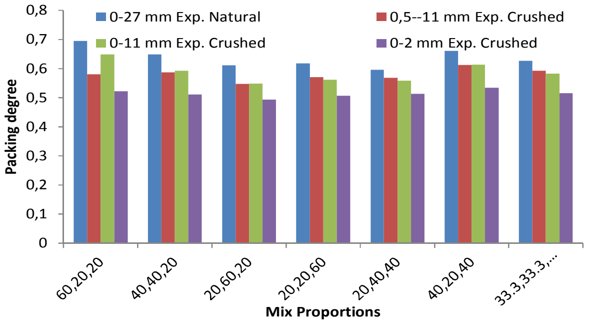


Fig. 12 Ternary diagram showing the Kållered crushed material, 0-0.5, 0.5-1 and 1-2mm

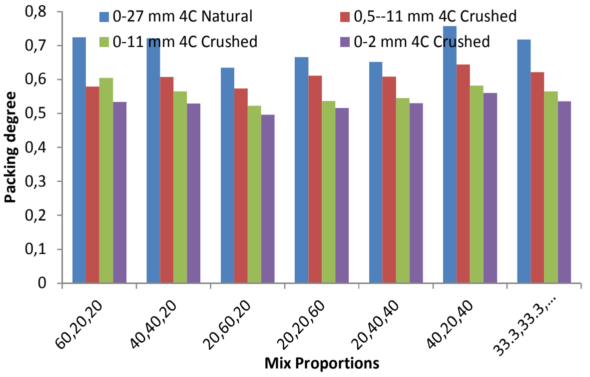
Finally to conclude this part, it has been shown that the 4C-Packing software gives the best fit with experimental data. Moreover, it can be noted that 4C-packing overrates the evolution of packing degree as has also been shown by Powers [1]. In contrast, the Toufar model does not show satisfactory packing degree estimates comparing to the test averages. This particularly obvious when the fine aggregate content ranges between 40% to 60% of the total aggregate [11]. From a practical point of view one finds that optimal composition would require more of the fine aggregate and less of coarse aggregate [2].

E. Comparison of different types of aggregate (Natural and Crushed) Particle Packing

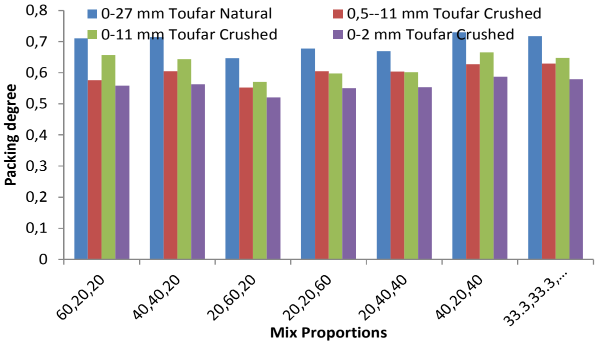
Fig. 13a shows the comparison of particle packing from experimental tests with natural aggregate type 1 with crushed aggregate types 2, 3 and 4 for the same mix proportions. One concludes that the particle packing of natural aggregate is better than of crushed aggregate types. This is obviously caused by the different tails of the materials and by the more irregular shape (lower sphericity and roundness) of the crushed grain populations. The agreement between the actual packing degrees and the theoretically derived ones is illustrated by the b and c diagrams in the Fig. 13. The obvious differences in packing degree suggest that more, systematic studies should be made.



a)



b)



c)

Fig. 13 Comparison between particle packing for natural and crushed aggregate and the same mix proportions, (a) from experimental tests, (b) from 4C program and (c) from Toufar's model

F. Sensitivity of "μ-value" in 4C-Program

One of the input parameters used in 4C program to calculate the packing is "μ-value" which indicates the maximum size ratio between the two particle types where no interaction (loosening) takes place. The small particles will often be unable to fill the voids between the large particles without disturbing their packing; this "loosening-effect" being quantified by this parameter that normally ranges between 0.07 and 0.13. Fig. 14 shows the particle packing calculated by use of the 4C program for the natural aggregate type 1 and for combined mix ratios and the μ -values (0.03, 0.05, 0.07 and 0.09). A sensitivity analysis for the μ-value has shown that differences from 0.05 to 0.09 causes a small increase in packing degree with increasing μ-value.

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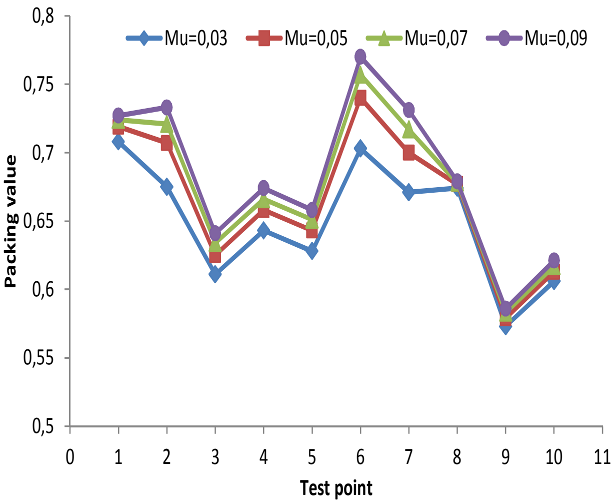

Fig. 14 Comparison particle packing for natural aggregate with different "Mu-value" for the same test point

G.Comparison of Experimental and Theoretical Packing Result

An overview of all the comparisons between experimental and estimated packing degrees for ternary packing is shown in Fig. 15. Test results for aggregate (types 1,2,3 and 4) have been compared to the packing from Toufar's model and 4C program, the figure showing that the theoretical packing correlates very well with experimental results and is compatible with what Golterman et al., got in their comparison [3] Fig. 16 .

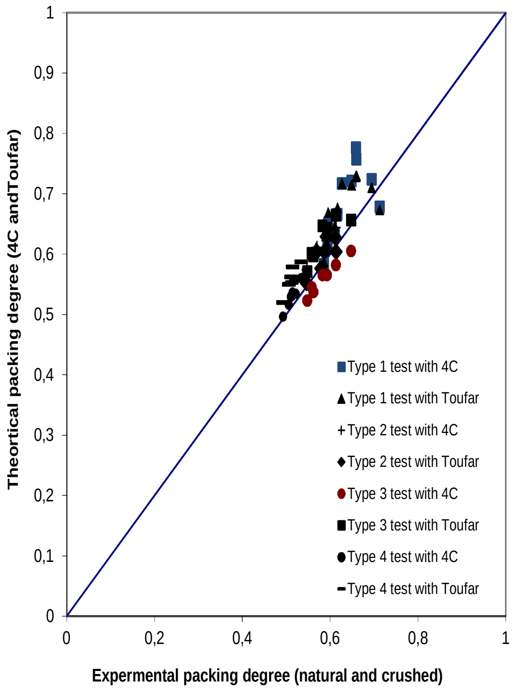

Fig. 15 Total comparison of differences between measured and calculated packing for all data

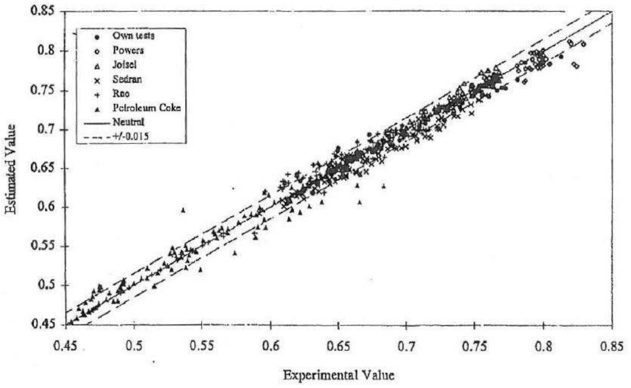

Fig. 16 Experimental packing degree versus estimated packing [3]

IX. CONCLUSION

It can be concluded after having considered theoretical models such as Toufar and 4C-program, as well as the experimental results, that the 4C is valid in principle but that it somewhat overrates the packing degree. It is obvious that the packing is a function of the particle shape and the size distribution. It can be concluded that the natural aggregate ranged (0-27 mm) gives packing values higher than of crushed aggregate for the same aggregate mix proportions for all experimental and theoretical studies. It is also found use of three types of aggregate or more gives optimum packing and good concrete. A suitable content of fine aggregate appears to be 40% to 60% of the total aggregate content. Finally, it can be concluded that the packing concept makes the designer able to reach optimal aggregate selection.

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