

# Analytical Investigation of the Effects of a Standing Ocean Wave in a Wave-Power Device OWC

E.G. Bautista, F. Méndez, O. Bautista, J.C. Arcos

**Abstract**—In this work we study analytically and numerically the performance of the mean heave motion of an OWC coupled with the governing equation of the spreading ocean waves due to the wide variation in an open parabolic channel with constant depth. This paper considers that the ocean wave propagation is under the assumption of a shallow flow condition. In order to verify the effect of the waves in the OWC firstly we establish the analytical model in a non-dimensional form based on the energy equation. The proposed wave-power system has two aims: one is to perturb the ocean waves as a consequence of the channel shape in order to concentrate the maximum ocean wave amplitude in the neighborhood of the OWC and the second is to determine the pressure and volume oscillation of air inside the compression chamber.

**Keywords**—Oscillating water column, Shallow flow, Wave energy.

## I. INTRODUCTION

THE present tendency to overcome the environmental problems caused by thermal energy production is to use all available kinds of renewable energy. This enhances the status of all techniques to convert the power contained in the global movement of the water, the renewable energy widely used for a very long time. Especially the use of wave energy, a huge unexploited global power reservoir becomes a new centre of interest.

The Oscillating Water Column (referred to hereafter as the OWC) wave energy converters device operates mainly on conventional technologies and probably owes its success to its mechanical and structural simplicity. It consists of a partially submerged and hollow structures opened to the sea below the mean sea level. The structure partially encloses a column of water exposed to the incident wave field through the underwater opening and to the atmosphere through a power take-off mechanism. As the waves impinge on the device, pressure oscillation at the column underwater interfaces causes the water column rise and fall within the structure. The air

trapped above the column surface within the structure is alternatively expanded and compressed through the power take-off mechanism, typically a turbine, converting the air static and dynamic pressure into mechanical energy. The principle is suitable for either bottom mounted or floating configurations.

Their conversion efficiency may also be enhanced by beneficial interactions between the structure motion and the column oscillations. For many years, scientists and engineers have been constantly working to develop an effective device for the utilization of ocean wave-energy. An analysis of oscillating water column wave energy-device has been given by [4]. He modeled the device by two closely spaced plates in two dimensions or a narrow tube (of circular cross section) in three dimensions. Under the assumptions of linearized water wave theory and using the method of matched asymptotic expansions he was able to obtain results for the maximum power that such system could absorb. Theoretical analyses for such devices which more realistically allow for spatial variation of the interior free surface were developed by [15] for two dimensional geometries, and by [6] who derived general results for configurations in two and three dimensions. Long arrays of devices are required if wave energy is to provide a significant contribution to large electrical grids. The Hydrodynamic interaction between devices was first studied theoretically for systems of oscillating bodies [2]-[3] and later extended to systems of oscillating pressure distribution by [6]. Reference [14] proposed a theoretical model to simulate the energy conversion, from wave to turbine shaft, of an Oscillating Water Column plant equipped with a Wells air-turbine. The resonance in the OWC has potential for various coastal management purposes such as aquaculture, flushing out of contaminated areas or the recovery of isolated coastal lagoons as breeding grounds, [7]-[9]. Reference [1] proposed a novel OWC geometry based on a vertical duct at the wave-beaten side, an OWC give some much better performances. This is, essentially, due to two reasons. First, an OWC with the additional vertical duct (U-OWC) has an eigenperiod greater than the eigenperiod of a conventional OWC. Second, the amplitude of the pressure fluctuations on the opening of a U-OWC is greater than the amplitude of the pressure fluctuations on the opening of a conventional OWC. Reference [16] conducted a study on the hydroelastic responses of a very-long floating structure (VLFS) in waves connected to a floating oscillating-water-column (OWC) breakwater system by a pin was analyzed by making use of the modal expansion method in two dimensions. The Bernoulli-Euler beam equation for the VLFS is coupled with the

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equations of motions of the breakwater taking account of the geometric and dynamic boundary conditions at the pin.

Three-dimensional numerical model of fixed Oscillating Water Column system was presented by [11] in order to study the regular wave interaction with the OWC. Reference [13] presents a numerical study of waves interacting with a cylindrical point absorber that is directly driving a seabed based linear generator. Other authors have employed computational fluid dynamics in order to analyze the flow characteristics in the OWC, [12], [18].

In this work we present an analytical coupled model whose describe the hydrodynamic behavior motion of the water column oscillation, perturbed by the action of a group waves propagating in a tapered channel with horizontal parabolic shape with constant depth. In order to verify the effect of physical variables in OWC hydrodynamics, the governing equations (Energy and Shallow Water Equations) are expressed in terms of non-dimensionless parameters.

## II. MATHEMATICAL MODEL

A cartesian coordinate system is chosen, in which the  $x$  and  $y$  axis lie on the plane of the still water level and the  $z$  axis is positive upwards. The OWC structure with width  $2H$  is separated from the line coast a distance  $L$  and is submerged a depth  $W$  from the bottom. The barrier between the OWC and the parabolic channels is  $S$  and is small compared with  $H$ . The parabolic channel has an horizontal distribution  $y = (1/4a)x^2 + H$ , where  $a$  is the focus of the parabola. In this study we are considering that channel depth  $h_0$  is constant, Fig. 1 and Fig. 2.

In order to obtain a simple mathematical model that describes the dynamics of the OWC, we apply the well-known energy equation along the streamline which connects the free water surface (1) and the top of the water column (2), Fig. 2.

$$\int_1^2 \frac{\partial V}{\partial t} ds + \frac{1}{\rho}(P_2 - P_1) + \frac{1}{2}(V_2^2 - V_1^2) + (z - \eta) + \frac{K}{2}V_2^2 = 0 \quad (1)$$

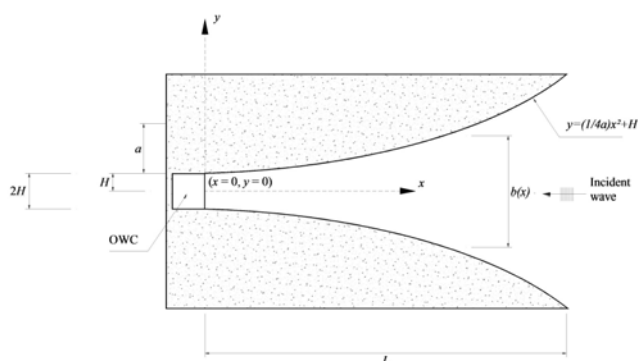


Fig. 1. Schematic horizontal view of a coupled OWC-Channel.

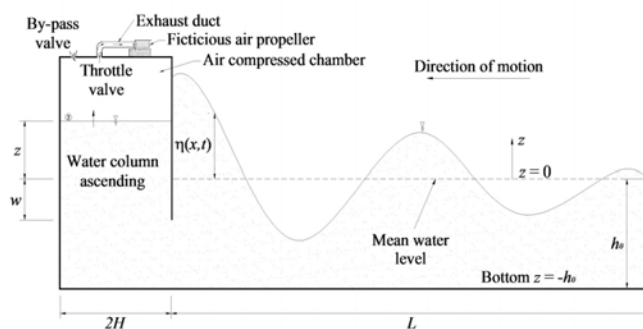


Fig. 2. Schematic cross-section of a coupled OWC-Channel

where  $P_2$  is the pressure inside the air chamber and  $P_1$  is the atmospheric pressure,  $\partial V / \partial t$  is the rate momentum change,  $g$  is gravity,  $K$  is a material friction coefficient of the OWC structure,  $z$  is the amplitude of the mean heave motion of the OWC,  $\eta(x, t)$  is the water wave surface elevation and  $\rho$  is the water density.

Integrating the first term of the left-hand side the next relationship is obtained

$$\int_1^2 \frac{\partial V}{\partial t} ds + \frac{\partial^2 z}{\partial t^2}(z + W) - \frac{\partial^2 \eta}{\partial t^2}(\eta + W) \quad (2)$$

taking into account that  $V_1 = \partial \eta / \partial t$  and  $V_2 = \partial z / \partial t$ .

During actual expansion and faster compression processes of air in a quasi equilibrium process, pressure and volume can be related as a polytrophic process. It will be assumed that the thermodynamic process taking place in the chamber is adiabatic, since the amount of heat exchanged in the relatively small period of time of a wave cycle is likely to be only a small fraction of what would be required to keep the air at constant temperature, on this condition, the pressure inside the air chamber can be expressed as

$$P_2 = P_1 \left[ \frac{V_0}{V_c(z, t)} \right]^k \quad (3)$$

where  $V_0$  and  $V_c(z, t)$  are the volume of the air compression chamber at the initial state and for different states respectively,  $k$  is the specific heat ratio and is defined as  $k = C_p / C_v$ ,  $C_p$  is the change in enthalpy with temperature at constant pressure and  $C_v$  is the change in internal energy with temperature at constant volume, in this work we consider  $k = 1.4$ . Substituting (2) and (3) in (1), result

$$(z + W) \frac{\partial^2 z}{\partial t^2} - (\eta + W) \frac{\partial^2 \eta}{\partial t^2} + \frac{1}{\rho} \left[ P_1 \left( \frac{V_0}{V_c} \right)^k - P_1 \right] + \frac{1}{2} \left[ \left( \frac{\partial z}{\partial t} \right)^2 - \left( \frac{\partial \eta}{\partial t} \right)^2 \right] + g(z - \eta) + \frac{K}{2} \left( \frac{\partial z}{\partial t} \right)^2 = 0 \quad (4)$$

Equation (4) is function of several physical variables that govern the phenomenon, its solution depend of every term, which implies a enormous possible combinations, to avoid this problem the next non-dimensional variable are introduced.

$$\tau = \omega t, \quad Z = \frac{z}{\lambda}, \quad \Delta = \frac{\eta}{h_0}$$

where  $\lambda$  is the ocean wave length,  $\omega = 2\pi/T$  and  $T$  is the ocean wave period.

Substituting the previous non-dimensional variables in (4), result

$$\begin{aligned} & (1 + \beta Z) \frac{\partial^2 Z}{\partial \tau^2} + \gamma_1 \left\{ \frac{1}{(1 - \gamma_2 Z)^k} - 1 \right\} + \\ & \quad + \frac{1}{2} \beta (1 + K) \left( \frac{\partial Z}{\partial \tau} \right)^2 \gamma_3 Z = \\ & = \Gamma_j \frac{\partial^2 \Delta}{\partial \tau^2} (1 + \Gamma_i \Delta) + \frac{1}{2} \Gamma_j \Gamma_i \left( \frac{\partial \Delta}{\partial \tau} \right)^2 + \Gamma_j \gamma_3 \Delta \end{aligned} \quad (5)$$

in the previous equation

$$\beta = \lambda / W, \quad \gamma_1 = (P_1 / \rho) / (\lambda \omega^2 W), \quad \gamma_2 = g / (\omega^2 W), \\ \gamma_3 = (4H^2 \lambda) / V_0, \quad \Gamma_i = h_0 / W, \quad \Gamma_j = h_0 / \lambda$$

For the evaluation of variable  $\Delta$  in (5) we take into account a small ocean wave,  $h_0 \square \eta$ , propagating in a long channel of variable cross section with width much less than the channel longitudinal length scale,  $L \square b(x)$ , the friction effect on the bottom and the shear stresses on the water surface due to the wind are not considered.

$$\frac{\partial U(x,t)}{\partial t} = -g \frac{\partial \eta(x,t)}{\partial x} \quad (6)$$

$$\frac{\partial \eta(x,t)}{\partial t} + h_0 \left( \frac{\partial U(x,t)}{\partial x} \right) = 0 \quad (7)$$

where  $U$  is the mean velocity.

The equation (7) can be expressed as a function of the cross section channel area  $A(x)$

$$\frac{\partial \eta}{\partial t} = \frac{\partial}{\partial x} \left[ \frac{UA(x)}{b(x)} \right] \quad (8)$$

where  $A(x) = b(x)h_0$  and  $b(x) = x^2 / (2a) + 2H$ .

Taking the time derivative of (8) and combining it with (6), the next equation is obtained

$$\left( \frac{x^2}{2a} + 2H \right) \frac{\partial^2 \eta}{\partial t^2} = gh_0 \left( \frac{x^2}{2a} + 2H \right) \frac{\partial^2 \eta}{\partial x^2} + g \frac{h_0}{a} x \frac{\partial \eta}{\partial x} \quad (9)$$

Equation (9) describes the unsteady oscillations of the water surface level into the parabolic channel. The above equation can be simplified even further. In the following section, we present a non- dimensionless version to characterize the presence of the dimensionless parameters that affect the different solutions.

Additionally the non-dimensional variable,  $\chi = x / L$ , is introduced, thus (9) in non-dimensional form result

$$\left( 1 + \frac{\alpha}{2} \chi^2 \right) \frac{\partial^2 \Delta}{\partial \tau^2} = F_r \left( 1 + \frac{\alpha}{2} \chi^2 \right) \frac{\partial^2 \Delta}{\partial \chi^2} + F_r \alpha \chi \frac{\partial \Delta}{\partial \chi} \quad (10)$$

with the initial and boundary nondimensional conditions

$$\tau = 0: \Delta = 0, \quad (11)$$

$$\chi = 0: \frac{\partial \Delta}{\partial \chi} = 0, \quad (12)$$

$$\chi = 1: \Delta = \beta_i \cos(\tau) \quad (13)$$

where the dimensionless parameters are defined as  $F_r = gh_0 / (\omega L)^2$ ,  $\alpha = L^2 / (2aH)$  and  $\beta_i = \eta_i / h_0$ ,  $\eta_i$  is the physical incident ocean wave.

Equation (10) can be reduced to an ordinary differential equation considering that the surface level  $\Delta$  has a periodic response that is modulated only by the variable amplitude; thus

$$\Delta = \delta(\chi) \cos(\tau) \quad (14)$$

where  $\delta(\chi)$  is the nondimensional wave amplitude in any cross section along the nondimensional coordinate  $\chi$ .

Substituting (14) in (10), we obtain

$$\begin{aligned} & F_r \left( 1 + \alpha \frac{\chi^2}{2} \right) \frac{d^2 \delta(\chi)}{d\chi^2} + (Fr)(\alpha) \chi \frac{d\delta(\chi)}{d\chi} + \\ & \quad + \left( 1 + \frac{\alpha}{2} \chi^2 \right) \delta(\chi) = 0 \end{aligned} \quad (15)$$

with boundary conditions

$$\chi = 0: \frac{d\delta(\chi)}{d\chi} = 0 \quad (16)$$

$$\chi = 1: \delta(1) = \beta_i \quad (17)$$

Equation (15) is normalized by using the nondimensional variable  $\varphi = \delta(\chi) / \beta_l$ , as follows:

$$F_r \left( 1 + \alpha \frac{\chi^2}{2} \right) \frac{d^2 \varphi}{d\chi^2} + (Fr)(\alpha) \chi \frac{d\varphi}{d\chi} + \left( 1 + \frac{\alpha}{2} \chi^2 \right) \varphi = 0 \quad (18)$$

together with the boundary conditions

$$\chi = 0: \quad \frac{d\varphi}{d\chi} = 0 \quad (19)$$

$$\chi = 1: \quad \varphi(1) = 1 \quad (20)$$

In equation (5), the variable  $\Delta$  and its derivatives are calculated using the normalized non-dimensional surface elevation  $\phi = \delta(\chi) / \beta_l$ , thus (5) is wrote as

$$\begin{aligned} & (1 + \beta Z) \frac{\partial^2 Z}{\partial \tau^2} + \gamma_1 \left[ \frac{1}{(1 - \gamma_3 Z)^k} - 1 \right] + \\ & + \frac{1}{2} \beta (1 + K) \left( \frac{\partial Z}{\partial \tau} \right)^2 + \gamma_3 Z = \\ & = -\Gamma_j [\varphi \beta_l \cos(\tau)] [1 + \Gamma_i \varphi \beta_l \cos(\tau)] + \\ & + \frac{1}{2} \Gamma_j \Gamma_i \varphi^2 \beta_l^2 \sin^2(\tau) + \Gamma_j \gamma_3 \varphi \beta_l \cos(\tau) \end{aligned} \quad (21)$$

Taking into account (23) and considering that the internal free surface in the air chamber remain horizontal, whose entails that there is only oscillation in the  $Z$  direction, from (21) a non-linear second ordinary differential equations is obtained

$$\begin{aligned} & (1 + \beta Z) \frac{d^2 Z}{d\tau^2} + \gamma_1 \left[ \frac{1}{(1 - \gamma_3 Z)^k} - 1 \right] + \frac{1}{2} \beta (1 + K) \left( \frac{dZ}{d\tau} \right)^2 + \\ & + \gamma_3 Z = -\Gamma_j [\bar{\varphi} \beta_l \cos(\tau)] [1 + \Gamma_i \bar{\varphi} \beta_l \cos(\tau)] + \\ & + \frac{1}{2} \Gamma_j \Gamma_i \bar{\varphi}^2 \beta_l^2 \sin^2(\tau) + \Gamma_j \gamma_3 \bar{\varphi} \beta_l \cos(\tau) \end{aligned} \quad (22)$$

where  $\bar{\varphi}$  is the mean value of the non-dimensional ocean wave, which is evaluated in the next form

$$\bar{\varphi} = \frac{\int_0^1 \varphi(\chi) d\chi}{\int_0^1 d\chi} \quad (23)$$

For simplicity, (22) must be solved with the following initial conditions

$$Z(\tau = 0) = 0 \quad \text{and} \quad \frac{dZ(\tau = 0)}{d\tau} = 0 \quad (24)$$

The equation (22) represents a coupled system between the structure OWC and the parabolic open channel, whose describe the performance of oscillation water column inside the OWC as a function of nine non-dimensional parameters.

### III. RESULTS

In order to obtain the values of the variable  $Z$ , the equation (22) is solved in two parts, first the value of  $\varphi$  is obtained solving the boundary value problem (18), with a fourth Runge Kutta method, combined with an iterative shooting method. Substituting the values of  $\varphi$  in (23), from (22) the variable  $Z$  is obtained.

In order to investigate the effects of the parabolic channel in the ocean wave surface elevation, Fig. 3, show different numerical results for the nondimensional water surface elevation  $\varphi$  as a function of the nondimensional coordinate  $\chi$ . In this case we are considering values of  $F_r = (0.017, 0.018, 0.019)$  with a constant value of  $\alpha = 5$ . The values of  $\varphi$  increase seven times in the case of  $F_r = 0.017$  compared with the boundary conditions, if the values of  $F_r$  increase, the variable  $\varphi$  tends to diminish, this prove that a channel with this geometry, perturb efficiently the ocean waves.

Figs. 4 and 5 show the oscillation of the water column  $Z$ , considering different values of  $F_r = (0.017, 0.018)$  and  $F_r = (0.019, 0.020)$ . In the case of  $F_r = 0.017$  the values of  $Z$  decrease, reaching a maximum and minimum values of  $Z = 0.0025$  and  $Z = -0.0025$ , respectively. On the other hand if  $F_r$  increase  $Z$  tend to diminish. The frequency oscillation of  $Z$  is almost equal for all Froude numbers

The pressure inside the air chamber is function of the  $Z$  elevation, which as was discussed previously is highly influenced by the parameter  $F_r$ , the influence of the parameter  $F_r$  in the nondimensional pressure relation  $P_2 / P_1$ , is shown in Fig. 6, as is appreciated the maximum pressure reached is around 4% greater than the atmosphere pressure  $P_1$ , with a minimum pressure of 4% less than the atmospheric pressure, this results show that as  $F_r$  decrease the pressure inside the air chamber increase

In analogous form the nondimensional volume variation has an opposed behavior of the pressure, if  $P_2 / P_1$  increase the volumen relation  $V_2 / V_0$  decrease, Fig. 7.

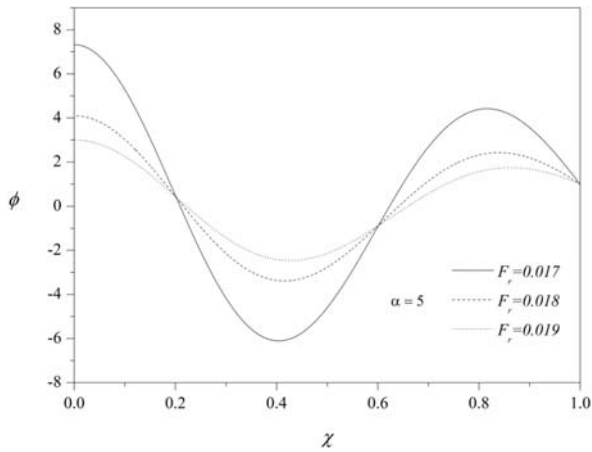


Fig. 3. Numerical solution of the non-dimensional free water surface oscillation  $\phi$ , as a function of the non-dimensional coordinate  $\chi$ , considering values of  $\alpha = 5$  and  $F_r = 0.017, 0.018, 0.019$ .

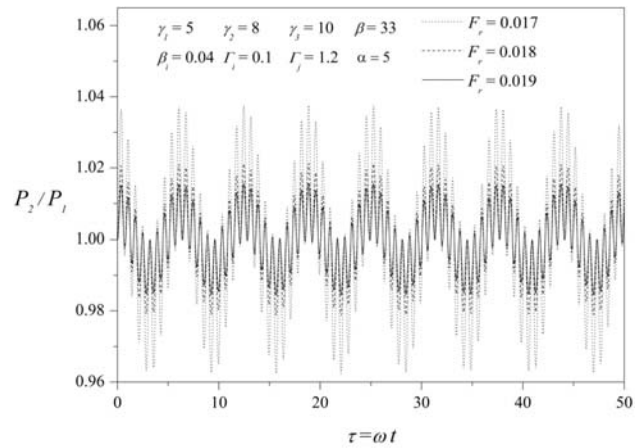


Fig. 6. Numerical solution of the mean non-dimensional pressure  $P_2 / P_1$  inside the air chamber of the OWC, for the non-dimensional channel parameters  $F_r = (0.017, 0.018, 0.019)$ .

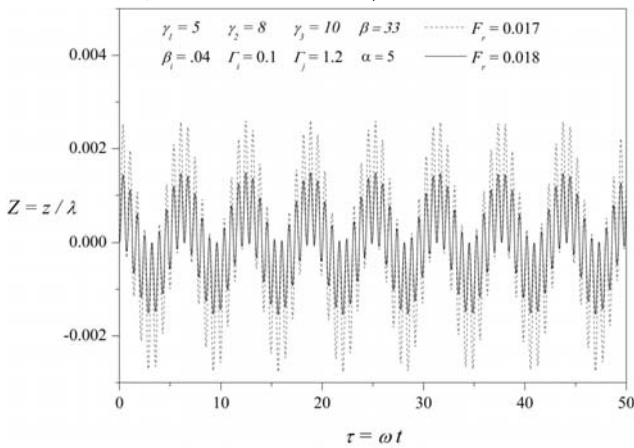


Fig. 4. Numerical solution of the mean heave motion of the OWC,  $Z = z / \lambda$ , for the non-dimensional channel parameters  $F_r = (0.017, 0.018)$ .

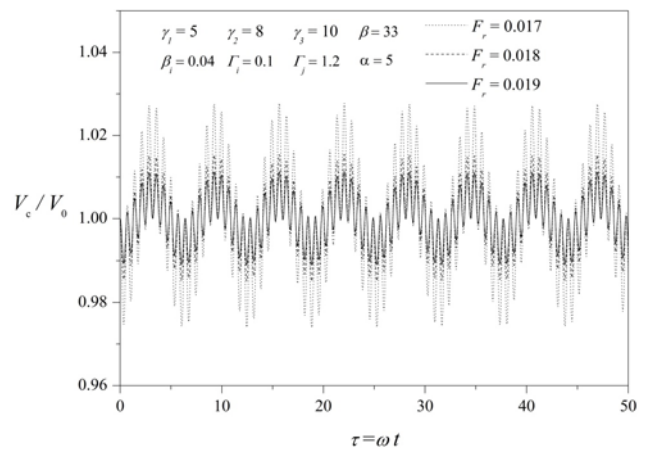


Fig. 7. Numerical solution of the mean non-dimensional volume  $V_c / V_0$  inside the air chamber of the OWC, for the non-dimensional channel parameters  $F_r = (0.017, 0.018, 0.019)$ .

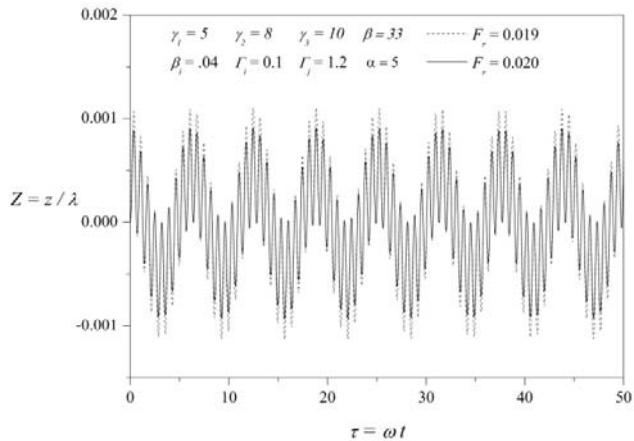


Fig. 5. Numerical solution of the mean heave motion of the OWC,  $Z = z / \lambda$ , for the non-dimensional channel parameters  $F_r = (0.019, 0.020)$ .

#### IV. CONCLUSIONS

A wave energy conversion coupled OWC-Channel system, which has potential for the purpose to extract wave energy is presented. The shallow water equations and the wave energy equation were fixed in order to show the wave influence in the hydrodynamic of the oscillation water column, under different geometric conditions of the OWC and the parabolic channel. To use the heave motion of the OWC governing equation into physical variables, imply several possible combinations, for this reason the governing equation was established in a non-dimensional form, which permit to examine the phenomena under different magnitude orders of physic variables. As was appreciated the nondimensional parameter  $F_r$  of the channel, relates the competition between the gravitational forces and inertial forces, which has inside the parabolic channel a significance influence in the ocean wave propagation. This mathematical model can be used as a first tool in order to analyze the effect of the ocean waves in an OWC structure in regions where the ocean waves are weakly.

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