

Design of Extremum Seeking Control with PD Accelerator and its Application to Monod and Williams-Otto Models

Hitoshi Takata, Tomohiro Hachino, Masaki Horai, and Kazuo Komatsu

Abstract—In this paper, we are concerned with the design and its simulation studies of a modified extremum seeking control for nonlinear systems. A standard extremum seeking control has a simple structure, but it takes a long time to reach an optimal operating point. We consider a modification of the standard extremum seeking control which is aimed to reach the optimal operating point more speedily than the standard one. In the modification, PD acceleration term is added before an integrator making a principal control, so that it enables the objects to be regulated to the optimal point smoothly. This proposed method is applied to Monod and Williams-Otto models to investigate its effectiveness. Numerical simulation results show that this modified method can improve the time response to the optimal operating point more speedily than the standard one.

Keywords—Extremum seeking control, Monod model, Williams-Otto model, PD acceleration term, Optimal operating point.

I. INTRODUCTION

NONLINEAR control methods have been studied in various fields for many years. An extremum seeking control problem is classified in a category of adaptive control problems [1-10]. Mainstream methods of adaptive control deal only with regulation to known set points or reference trajectories. However, extremum seeking controls are designed so as to operate at unknown set points that optimize the value of an evaluation function. There are lots of studies such as optimizing the yield of a product in chemical engineering and biotechnology [1-3, 6-8], adjusting the spark ignition angle of an automotive engine [4], and controlling a chip refiner motor [5]. In all applications, it is desirable to have rapid response to the optimal operating point.

In this paper, we consider a modification of the standard type method for the extremum seeking control problems. It is aimed at shortening a time response to the optimal operating point. This modification is designed by adding a PD (Proportional and Derivative) acceleration term in front of an integrator which makes a principal control, so its structure is quite simple. This proposed method is applied to

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Monod model of bioreactor [1, 2] and Williams-Otto model of chemical reactor [6]. Simulation results show that this method enables to regulate the objects around the optimal operating point more speedily.

II. STATEMENT OF PROBLEM

We consider a nonlinear control system of single-input-single-output:

$$\dot{x}(t) = f(x(t), \alpha(t), u(t)), \quad (1)$$

$$y(t) = h(x(t), \alpha(t), u(t)) + w(t), \quad (2)$$

$$J(t) = J(x(t), \alpha(t), u(t)), \quad (3)$$

where $\bullet = d/dt$, $x \in R^n$ is the state vector, $\alpha \in R^m$ is the unknown parameter, $u \in R$ is the control, $y \in R$ is the output, $f \in R^n$ and $h \in R$ are the unknown nonlinear functions, $J \in R$ is the evaluation function, and $w \in R$ is the noise.

The aim of this problem is to design an extremum seeking control scheme which achieves speedily to the optimal operating point. That is, this approach enables the given system to operate around the optimal operating point of the performance speedily, without requiring the knowledge of the functions f and h , and the parameter α .

In the following sections, we consider the standard extremum seeking control (SESC) and its modified extremum seeking control (MESOC).

III. STANDARD EXTREMUM SEEKING CONTROL

The standard extremum seeking control (SESC) is designed as shown in Fig. 1, which has a high-pass filter (HPF) $s/(s + \omega_h)$, a low-pass filter (LPF) $\omega_l/(s + \omega_l)$, an integrator k/s , and a compulsory perturbation term of sine wave $\beta \sin \omega t$ [1]. This consists of a feedback scheme without requiring the knowledge of a plant dynamic equation from the concept of frequency domain. In the following, the SESC method is described in detail.

It is impossible to conclude that a certain point is a maximum without visiting the neighborhood on both sides of the maximum. For this reason, this scheme employs a compulsory perturbation term of sine wave $\beta \sin \omega t$ which is added to the principal control signal \hat{u} . The persistent nature of $\beta \sin \omega t$ may be undesirable, but it is necessary to maintain a maximum even if the functions of a plant are changed.

This SESC scheme may be analyzed as follows. The compulsory perturbation term $\beta \sin \omega t$ creates a periodic response of output y . The HPF eliminates the DC component of y . And

then, the product of the sine wave $\beta \sin \omega t$ produces $(\beta^2/2) \times (1 - \cos 2\omega t)$, and its DC component $\xi \propto \beta^2/2$ is extracted by the LPF. The sign of this ξ provides the direction to the integrator $\hat{u} = \xi k/s$ moving \hat{u} toward the optimal operating point u^* . Due to this, the output y gradually approaches the maximum output value $y^* = J(u^*)$.

Although it has the merit of easy implementation to practical systems, the SESC method usually takes a long time to reach the optimal operating point u^* , namely, the maximum output value y^* . Therefore, we consider a modification of the SESC to shorten a time response in the next section.

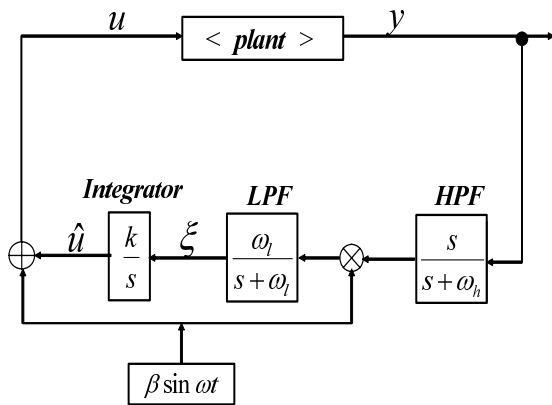


Fig. 1. SESC scheme.

IV. MODIFIED EXTREMUM SEEKING CONTROL

A modified extremum seeking control (MESC) is added a PD term on the SESC as shown in Fig. 2, which is aimed at shortening a time response for an optimal operating point.

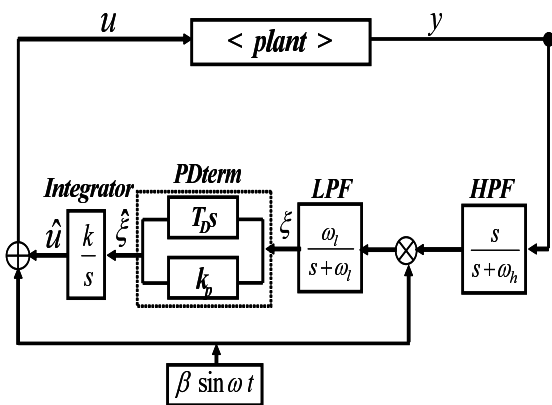


Fig. 2. MESC scheme.

The derivative of the principal control action \hat{u} is $\hat{\xi} = (k_p + T_D s)\xi$ where $\hat{u} = \hat{\xi}k/s$, so that the derivative values of the control function u could increase than those by the SESC (see Fig. 3). The additive term $T_D s$ should be exchanged by $T_D s / (\tau_D s + 1)$ in noisy cases. It needs to properly select the parameters k_p , T_D , k , and τ_D as well as ω_h , ω_l , and β when applied to practical systems.

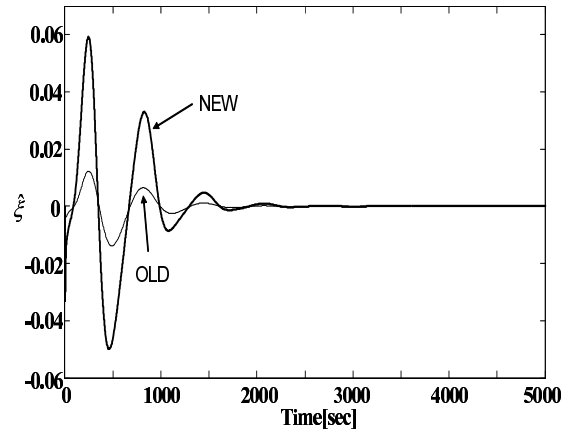


Fig. 3. Time responses of $\hat{\xi}$.

V. NUMERICAL SIMULATIONS

A. Monod model

We consider the problem of optimizing the yield for a bioreactor which is described by Monod model [1, 2]:

$$\dot{x}_1 = f_1(x, \alpha, u) = x_1 \left(\frac{x_2}{\alpha + x_2} - u \right), \quad (4)$$

$$\dot{x}_2 = f_2(x, \alpha, u) = u(1 - x_2) - \frac{x_1 x_2}{\alpha + x_2}, \quad (5)$$

$$y = h(x, \alpha, u) + w = x_1 u + w, \quad (6)$$

where $x = [x_1, x_2]^T$ and $0 \leq u \leq 1$.

The steady state output (evaluation function) is

$$J = \frac{u(1 - (1 + \alpha)u)}{1 - u} \quad (7)$$

which is derived by substituting a solution (x, u) of $\dot{x}_1 = \dot{x}_2 = 0$ in Eqs. (4) and (5) to Eq. (6), where α is fixed.

The x_1 is biomass concentration and the initial value is $x_1(0) = 0$. The x_2 is substrate concentration and the initial value is $x_2(0) = 0$. The u is dilution rate and the initial value is $u(0) = 0.4$. The α is saturation constant. The y is biomass production rate.

We should note that Eqs. (4) ~ (7) are unknown during experiments. The unknown parameter α is initially set to $\alpha = 0.02$, but it is changed to $\alpha = 0.1$ repeatedly. The corresponding optimal operating points are the broken lines as shown in Figs. 4~7, but they are unknown during the experiments.

The purpose of this problem is to follow the output y to the optimal operating points.

We set the parameters as follows. The cutoff frequencies of HPF and LPF: $\omega_h = 0.2$ and $\omega_l = 0.02$, the gains: $k_p = 0.2$, $T_D = 8$ and $k = 5$, the compulsory perturbation term: $\beta \sin \omega t = 0.03 \sin 0.08t$. Two types of simulations of 2500 and 10000[sec] are carried out here. w is white noise being negligible small.

The results of the numerical simulations of control (u) and output (y) are shown in Figs. 4~5 when $0 \sim 2500$ [sec] and Figs. 6~7 when $0 \sim 10000$ [sec], respectively. OLD means SESC, and NEW does MESC. Each figure indicates that this

NEW approaches the optimal operating point more speedily than OLD.

B. Williams-Otto model

We consider the problem of optimizing the yield for a chemical reactor which is described by Williams-Otto model [6]:

$$\dot{x}_1 = \frac{F_A}{W} - \frac{F_A + F_B}{W}x_1 - z_1, \quad (8)$$

$$\dot{x}_2 = \frac{F_B}{W} - \frac{F_A + F_B}{W}x_2 - (z_1 + z_2), \quad (9)$$

$$\dot{x}_3 = -\frac{F_A + F_B}{W}x_3 + (2z_1 - 2z_2 - z_3), \quad (10)$$

$$\dot{x}_4 = -\frac{F_A + F_B}{W}x_4 + 2z_2, \quad (11)$$

$$\dot{x}_5 = -\frac{F_A + F_B}{W}x_5 + 1.5z_3, \quad (12)$$

$$\dot{x}_6 = -\frac{F_A + F_B}{W}x_6 + (z_2 - 0.5z_3), \quad (13)$$

$$y = (F_A + F_B)(125.91x_4 + 5554.1x_6) - (370.3F_A + 555.42F_B) + w, \quad (14)$$

where $z_1 = k_1x_1x_2$, $z_2 = k_2x_2x_3$, $z_3 = k_3x_3x_6$, and

$$k_1 = 1.6599 \times 10^6 e^{-6666.7/(T+273.15)}, \quad (15)$$

$$k_2 = 7.2117 \times 10^8 e^{-8333.3/(T+273.15)}, \quad (16)$$

$$k_3 = 2.6745 \times 10^{12} e^{-11111/(T+273.15)}. \quad (17)$$

The steady state output (evaluation function) is

$$J = (F_A + F_B)(125.91x_4 + 5554.1x_6) - (370.3F_A + 555.42F_B), \quad (18)$$

in which x_4 and x_6 are solutions of $\dot{x} = 0$.

This model has a more complex structure than Monod model.

This model corresponds to Eqs. (1) ~ (3).

$x = [x_1, x_2, x_3, x_4, x_5, x_6]^T$, $\alpha = [F_A, F_B]^T$, $u = T$, and w is white noise being negligible small.

The x is system state function of mass fractions of six chemical components and the initial value is $x(0) = 0$. The α is fraction per unit time of source materials A and B which are flown in a reactor. The relation between F_A [kg/sec] and F_B [kg/sec] is assumed by Ref.[6] to be approximated as

$$F_B = \frac{14F_A + 3}{6}. \quad (19)$$

The u is temperature (T degree centigrade) of the internal reactor and the initial value is $u(0) = 85.1$. The W [kg] is mass content of all the internal reactor of $W = 2104.7$.

The purpose of this problem is to follow the output y to the optimal operating points. y [\$/sec] stands for production per unit time.

We set the parameters as follows. The cutoff frequencies of HPF and LPF: $\omega_h = 0.4$ and $\omega_l = 0.01$, the gains: $k_p = 4.25$, $T_D = 42.5$ and $k = 0.4$, the compulsory perturbation term: $\beta \sin \omega t = 0.004 \sin 0.12t$, and the simulation time: 5×10^4 [sec].

The results of the numerical simulations of control (u) and output (y) are shown in Figs. 8~10, where Fig. 9 is an enlarged figure 8 during the first 5×10^3 [sec]. OLD means SESC, and NEW does MESC. Figure 10 indicates that this NEW approaches the optimal operating point more speedily than OLD.

VI. CONCLUSIONS

This paper has proposed a modification of the standard extremum seeking control so as to regulate to the optimal operating point more speedily. It is added the PD acceleration term before the integrator. Thus the structure of the MESC is quite simple like the SESC. The results of the numerical simulations indicate that this modified extremum seeking control approach enables these systems of Monod and Williams-Otto models to the optimal operating point more speedily than the standard extremum seeking control. This new approach shall be studied for the applications to other systems, selection of better parameters, improvement of transient states, stability proofs, and so on, in the future works.

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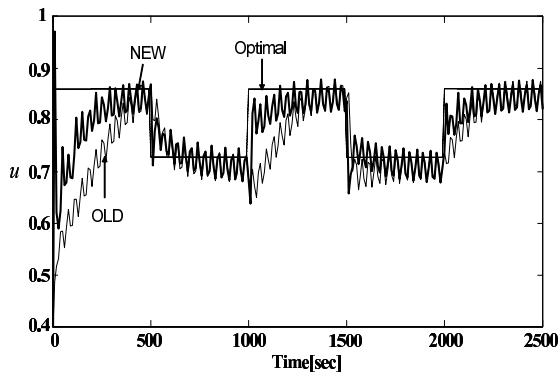


Fig. 4. Time responses of control u (2500[sec]).

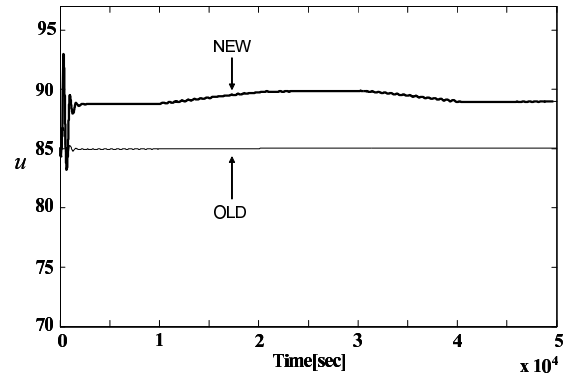


Fig. 8. Time responses of control u .

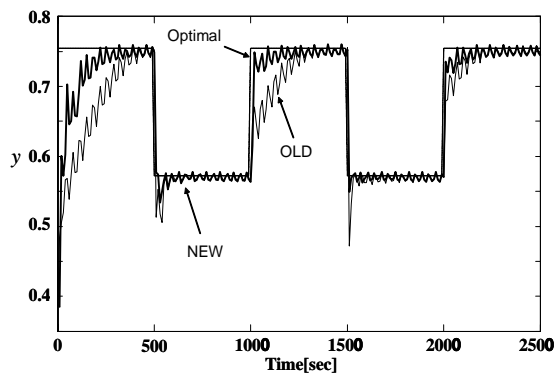


Fig. 5. Time responses of output y (2500[sec]).

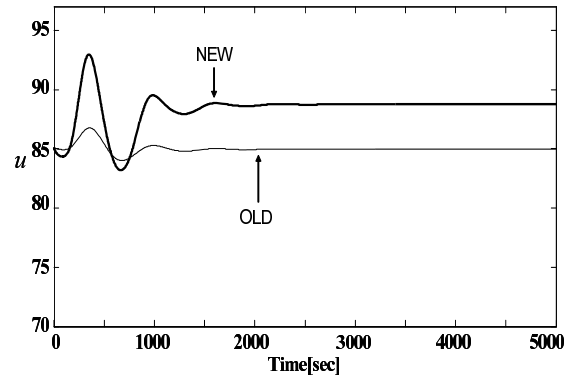


Fig. 9. Time responses of control u (Enlargement).

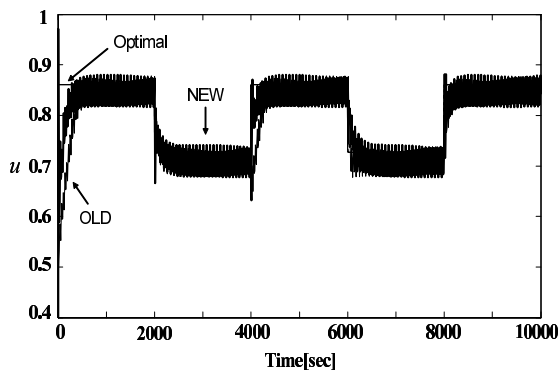


Fig. 6. Time responses of control u (10000[sec]).

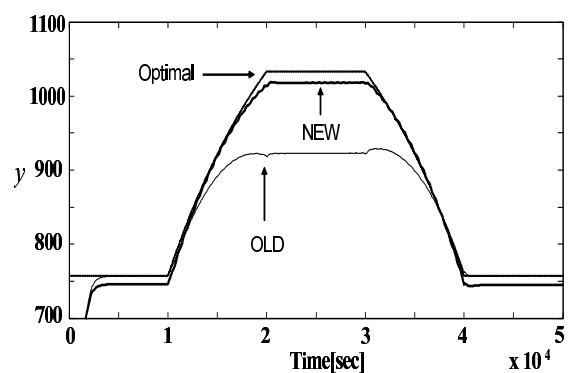


Fig. 10. Time responses of output y .

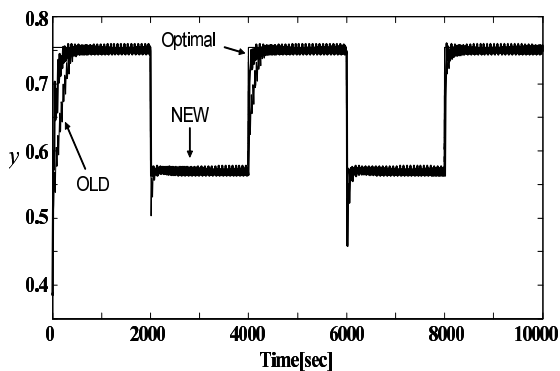


Fig. 7. Time responses of output y (10000[sec]).

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