

Numerical Grid Generation of Oceanic Model for the Andaman Sea

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Abstract—The study of the Andaman Sea can be studied by using the oceanic model; therefore the grid covering the study area should be generated. This research aims to generate grid covering the Andaman Sea, situated between longitudes $90^\circ E$ to $101^\circ E$ and latitudes $1^\circ N$ to $18^\circ N$. A horizontal grid is an orthogonal curvilinear with 87×217 grid points. The methods used in this study are cubic spline and bilinear interpolations. The boundary grid points are generated by spline interpolation while the interior grid points have to be specified by bilinear interpolation method. A vertical grid is sigma coordinate with 15 layers of water column.

Keywords—Sigma Coordinate, Curvilinear Coordinate, Andaman Sea.

I. INTRODUCTION

Andaman Sea is a body of water to the southeast of the Bay of Bengal, south of Myanmar, west of Thailand and east of the Andaman Islands, India; it is part of the Indian Ocean. It is roughly 1,200 km (north-south) and 650 km wide (east-west), with an area of 797,700 square kilometers. Its average depth is 870 m, and the maximum depth is 3,777 m (Fig. 1).

At its southeastern reaches, the Andaman Sea narrows to form the Straits of Malacca, which separate the Malay Peninsula from the island of Sumatra. The International Hydrographic Organization [2] defines the limits of the “Andaman or Burma Sea” as follows:

- On the Southwest. A line running from Oedjong Raja ($5^\circ 32' N, 95^\circ 12' E$) in Sumatra to Poeloe Bras (Breueh) and on through the Western Islands of the Nicobar Group to Sandy Point in Little Andaman Island, in such a way that all the narrow waters appertain to the Burma Sea.
- On the Northwest. The Eastern limit of the Bay of Bengal [A line running from Cape Negrais ($16^\circ 03' N$) in Burma through the larger islands of the Andaman group, in such a way that all the narrow waters between the islands lie to the Eastward of the line and are excluded from the Bay of Bengal, as far as a point in Little Andaman Island ($10^\circ 48' N, 92^\circ 24' E$).
- On the Southeast. A line joining Lem Voalan ($7^\circ 47' N$) in Thailand, and Pedropunt ($5^\circ 40' N$) in Sumatra.

The numerical ocean modeling technologies have been developed in many countries. Three dimensional general ocean circulations models have become an important tool to study

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the state of the ocean. Blumberg and Mellor [1] developed a three dimensional coastal ocean circulation model with a free surface, which named as Princeton Ocean Model (POM). Now POM model is widely used in many countries. This model describes in the rectangular coordinate system. The study of the Andaman Sea can be studied by using the oceanic model; therefore the grid covering the study area should be generated. This research aims to generate grid covering the Andaman Sea.

II. THE VERTICAL SIGMA COORDINATE

The sigma coordinate is a function of density where the density is a function of temperature and Salinity. In order to separate water in Andaman Sea in several layers, the vertical sigma coordinate is used.

In the sigma coordinate system, the governing equations from z -coordinate (x, y, z, t) are transformed to the vertical sigma coordinate (x^*, y^*, σ, t^*) with relationship

$$x^* = x, y^* = y, \sigma = \frac{z - \eta}{H + \eta} \quad \text{and} \quad t^* = t, \quad (1)$$

where $H(x, y)$ is the bottom topography and η is the sea surface elevation. σ ranges from $\sigma = 0$ at $z = \eta$ to $\sigma = -1$ at $z = -H(x, y)$.

III. INTERPOLATION METHODS

A. Cubic Spline Interpolation

The fundamental idea behind cubic spline interpolation is based on the engineer's tool used to draw a smooth curve through a number of data points. Using this method, a series of unique cubic polynomials is fitted between each of data points, with the stipulation that the curve obtained be continuous and appear smooth.

The essential data is to fit a piecewise function of the form

$$S(x) = \begin{cases} s_1(x) & \text{if } x_1 \leq x < x_2 \\ s_2(x) & \text{if } x_2 \leq x < x_3 \\ \dots & \dots \\ s_{n-1}(x) & \text{if } x_{n-1} \leq x < x_n \end{cases}, \quad (2)$$

where $s_i(x)$ is the third degree polynomial defined by

$$s_i(x) = a_i(x - x_i)^3 + b_i(x - x_i)^2 + c_i(x - x_i) + d_i, \quad (3)$$

for $i = 1, 2, \dots, n - 1$. There are four free cubic coefficients a_i, b_i, c_i and d_i in one cubic polynomial, so it needs four conditions.

The first and second derivatives of these $n - 1$ equations are

$$s'_i(x) = 3a_i(x - x_i)^2 + 2b_i(x - x_i) + c_i, \quad (4)$$

$$s''_i(x) = 6a_i(x - x_i) + 2b_i, \quad (5)$$

for $i = 1, 2, \dots, n - 1$.

The cubic spline will need to conform to the following stipulations:

- 1) The piecewise function $S(x)$ will interpolate all data points;
- 2) $S(x)$ will be continuous on the interval $[x_1, x_n]$;
- 3) $S'(x)$ will be continuous on the interval $[x_1, x_n]$; and
- 4) $S''(x)$ will be continuous on the interval $[x_1, x_n]$.

From the above stipulations, there are four conditions to make the curve smooth across the interval.

- 1) $S(x_i) = y_i$;
- 2) $s_i(x_{i+1}) = s_{i+1}(x_{i+1})$;
- 3) $s'_i(x_{i+1}) = s'_{i+1}(x_{i+1})$;
- 4) $s''_i(x_{i+1}) = s''_{i+1}(x_{i+1})$.

Using these conditions, the cubic coefficients are

$$a_i = \frac{M_{i+1} - M_i}{6h_i}, \quad (6)$$

$$b_i = \frac{M_i}{2}, \quad (7)$$

$$c_i = \frac{y_{i+1} - y_i}{h_i} - \left(\frac{M_{i+1} + 2M_i}{6} \right) h_i, \quad (8)$$

$$d_i = y_i, \quad (9)$$

where M_i represents $s''_i(x_i)$ and $h_i = x_{i+1} - x_i$. Therefore the cubic polynomial may be written as

$$s_i(x) = \left(\frac{M_{i+1} - M_i}{6h_i} \right) (x - x_i)^3 + \frac{M_i}{2} (x - x_i)^2 + \left(\frac{y_{i+1} - y_i}{h_i} - \frac{M_{i+1} + 2M_i}{6} h_i \right) (x - x_i) + y_i, \quad (10)$$

for $i = 1, 2, \dots, n - 1$. The second derivative of the curve given by

$$\begin{aligned} & \frac{h_i}{6} M_{i+1} + \left(\frac{h_i + h_{i-1}}{3} \right) M_i + \frac{h_{i-1}}{6} M_{i-1} \\ & = \frac{y_{i+1}}{h_i} - \left(\frac{1}{h_i} + \frac{1}{h_{i-1}} \right) y_i + \frac{y_{i-1}}{h_{i-1}}. \end{aligned} \quad (11)$$

B. Bilinear Interpolation

The bilinear interpolation is analogous to linear interpolation. A weighted average of the four surrounding grid points is used to determine the interpolated value. Suppose the values of function $f(x, y)$ are given on a grid of (x, y) namely (x_i, y_j) . The value on the point (x_i, y_j) denotes as $f_{i,j} = f(x_i, y_j)$. The bilinear interpolation proceeds in two steps, in each of which a one-dimensional interpolation is used.

Suppose one has to estimate the value at a point located in the box defined by $x_{i-1} \leq x \leq x_i$ and $y_{j-1} \leq y \leq y_j$ shown in fig. 2 [4].

The first step is to interpolate the table in the y direction from the two vertically neighboring points by using linear interpolation. The values at points W and E are found as

$$f_W = \frac{y_j - y}{y_j - y_{j-1}} f_{i-1, j-1} + \frac{y - y_{j-1}}{y_j - y_{j-1}} f_{i-1, j}, \quad (12)$$

$$f_E = \frac{y_j - y}{y_j - y_{j-1}} f_{i, j-1} + \frac{y - y_{j-1}}{y_j - y_{j-1}} f_{i, j}. \quad (13)$$

The second step is to interpolate in x direction by using linear interpolation from the two neighboring points f_W and f_E as

$$g(x, y) = \frac{x_i - x}{x_i - x_{i-1}} f_W + \frac{x - x_{i-1}}{x_i - x_{i-1}} f_E. \quad (14)$$

Let $Q = \frac{x - x_{i-1}}{x_i - x_{i-1}}$ and $P = \frac{y - y_{j-1}}{y_j - y_{j-1}}$. Combining the two steps into one equation [3], we obtain

$$g(x, y) = (1 - Q)(1 - P)f_{i-1, j-1} + (1 - Q)Pf_{i-1, j} + Q(1 - P)f_{i, j-1} + QPf_{i, j} \quad (15)$$

IV. COMPUTER PROGRAM

The computer program used to generate grid has the following steps:

- 1) Generate horizontal and vertical grid;
- 2) Read bottom topography and interpolate to grid;
- 3) Write grid for the model; and
- 4) Write grid for the MATLAB plot.

The horizontal grid employs a curvilinear grid system. The data at the interior grid points are interpolated by the bilinear interpolation method because of this method is convenience and sufficiency for the our data. The disadvantage of bilinear interpolation is edges are smoothed and some extremes of data file values are lost, therefore the data at the boundary have to be specified by cubic spline interpolation.

The topography is taken from the NASA Shuttle Radar Topographic Mission (STRM) Digital Elevation Data, which provides ocean depths on 1/120 degree longitude-latitude gridded resolution. This data set was produced by the Consultative Group for International Agriculture Research-Consortium for Spatial Information (CGIAR-CSI).

V. RESULTS

The model has been set up with 87×217 grid points in horizontal and 16 sigma vertical levels covering the Andaman Sea. The horizontal grid employs an orthogonal curvilinear grid system as shown in Fig. 4. It can be generated by using cubic spline for boundary grid points. The diamond nodes represent the local points for cubic spline. The domain is bounded on the east by land, while the west, north and south boundaries are open boundaries. Fig. 3 shows the horizontal grid spacing in the x direction (Δx) and y direction (Δy), in which (Δx) varies in spacing from 2 km near the eastern coast to 45 km at the western boundary, and (Δy) varies in spacing from 2 km in the central part of the Sea to 55 km at the southern boundary.

The vertical increment, which varies in thickness, accommodates more resolution near the surface. The vertical sigma grid

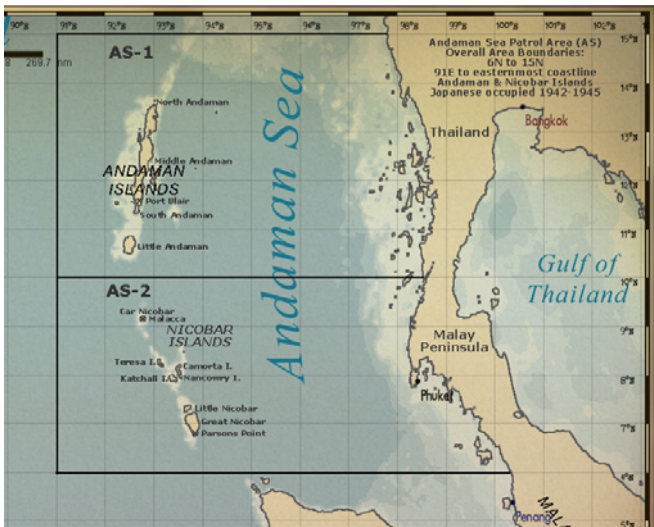


Fig. 1: Andaman Sea [5].

has 16 irregular levels as shown in Table I with σ ranging from $\sigma = 0$ at sea surface to $\sigma = -1$ at the bottom. The quantity σ_k refers to the depth at which the vertical velocity is located, while $\sigma_{k+\frac{1}{2}}$ corresponds to the depth at which horizontal velocity, temperature, salinity and density are defined. The $\Delta\sigma$ is the vertical grid spacing.

TABLE I: The vertical σ -coordinate distribution.

level k	σ_k	$\sigma_{k+\frac{1}{2}}$	$\Delta\sigma_k$	$\Delta\sigma_{k+\frac{1}{2}}$
1	0.0000	-0.0179	0.0357	0.0327
2	-0.0357	-0.0505	0.0357	0.0505
3	-0.0714	-0.1010	0.0714	0.0776
4	-0.1429	-0.1786	0.0714	0.0714
5	-0.2143	-0.2500	0.0714	0.0714
6	-0.2857	-0.3214	0.0714	0.0714
7	-0.3571	-0.3929	0.0714	0.0714
8	-0.4286	-0.4643	0.0714	0.0714
9	-0.5000	-0.5357	0.0714	0.0714
10	-0.5714	-0.6071	0.0714	0.0714
11	-0.6429	-0.6786	0.0714	0.0714
12	-0.7143	-0.7500	0.0714	0.0714
13	-0.7857	-0.8214	0.0714	0.0776
14	-0.8571	-0.8990	0.0714	0.0653
15	-0.9286	-0.9643	0.0714	0.0714
16	-1.0000			

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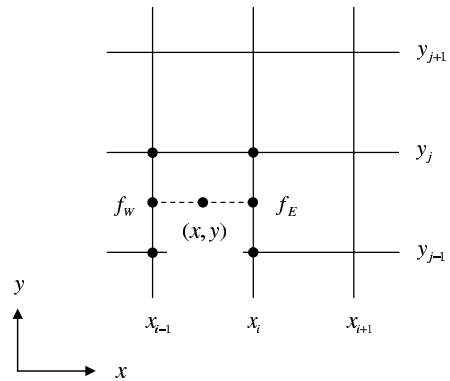


Fig. 2: Bilinear interpolation on a two-dimensional domain.

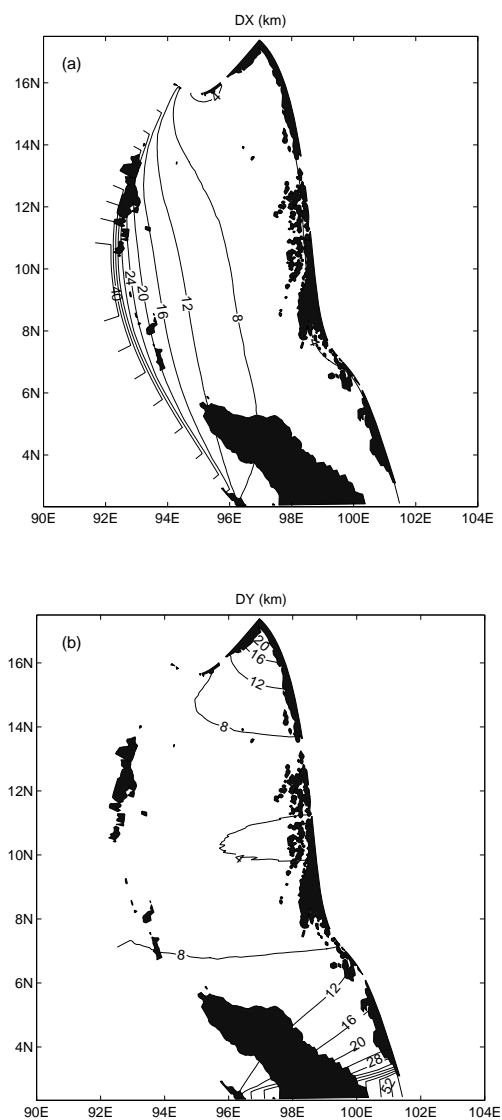


Fig. 3: The horizontal grid spacing in (a) x direction and (b) y direction.

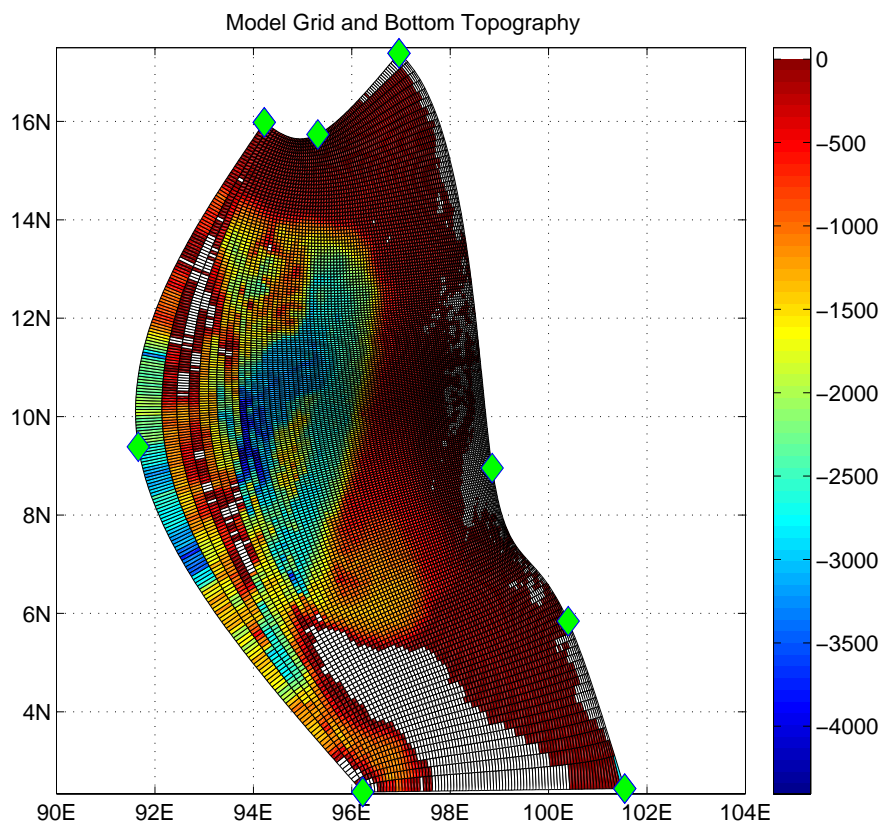


Fig. 4: The curvilinear model grid and the bottom topography (m) in the Andaman Sea.

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