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Quasilinearization–Barycentric approach for numerical investigation of the boundary value Fin problem

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Abstract—In this paper we improve the quasilinearization method by barycentric Lagrange interpolation because of its numerical stability and computation speed to achieve a stable semi analytical solution. Then we applied the improved method for solving the Fin problem which is a nonlinear equation that occurs in the heat transferring. In the quasilinearization approach the nonlinear differential equation is treated by approximating the nonlinear terms by a sequence of linear expressions. The modified QLM is iterative but not perturbative and gives stable semi analytical solutions to nonlinear problems without depending on the existence of a smallness parameter. Comparison with some numerical solutions shows that the present solution is applicable.

Keywords—Quasilinearization method, Barycentric Lagrange interpolation, Nonlinear ODE, Fin problem, Heat transfer

I. Introduction

THE convective heat transfer from a surface to the ambient fluid can be significantly improved by extending the surface and then increasing the heat transfer area. Such extensions into the surrounding fluid are called fins.

In a variety of engineering applications, fins are frequently used to facilitate the dissipation of the heat from a heated wall to the surrounding environment. The heat conducted through a fin body is removed via convective and/or radiative processes. An analysis of this conduction convection/radiation system has been a matter of interest in the heat transfer area due to its practical importance and numerous studies on the fin analysis have been performed for a long time [1], [2], [3].

In cases of constant thermal conductivity and constant heat transfer coefficient, the analytical solution of temperature distribution as well as heat transfer rate can be easily obtained. In general, however, the heat transfer coefficient may be no longer uniform and varies along the fin with the temperature difference between the surface and the adjacent fluid in a nonlinear manner. The former is a typical problem for fins with a forced convection heat transfer, while the latter is common in other heat transfer modes such as natural convection, radiation, boiling and condensation heat transfer. If a large temperature difference exists within the fin, the thermal conductivity may not be constant and its dependence on the temperature should be considered. A considerable amount of research has been conducted to obtain solutions to the nonlinear fin problem with a temperature-dependent thermal conductivity and/or heat transfer coefficient. Due to the nonlinearity of the problem exact solutions are not easy to obtain.

The quasilinearization method (QLM) is a generalization of the Newton-Raphson method [4], [5] for nonlinear differential equations. It was developed originally in the theory of linear programming by Bellman and Kalaba [6], [7], [8]. Quasilinearization techniques are based on the linearization of the high order ordinary differential equation and require the solution of a linear ordinary differential equation at each iteration. Mandelzweig and Tabakin [9] have determined general conditions for the quadratic, monotonic and uniform convergence of the quasilinearization method for solving both initial- and boundary-value problems in nonlinear ordinary differential equations. In particular, they have applied with great success quasilinearization methods to the two-point Thomas-Fermi and Blasius equations and to singular initial-value problems governed by the Lane-Emden equation [10]. Moreover, Recently the quasilinearization method was suggested for solving the Schrödinger equation after conversion to the Riccati equation [9], [11], [12].

The quasilinearization method may be interpreted as a perturbation technique which treats the nonlinear terms as a perturbation about the linear ones, but, unlike perturbation methods, is not based on the existence of a small parameter [10].

Occasionally the linear ordinary differential equation that get from quasilinearization method at each iteration does not solve analytically. Hence we can use barycentric Lagrange interpolation to approximate the solution.

Interpolation [13], [14] is the approximation of function values using evaluations of that function at other points in the domain.

The Lagrange polynomial interpolation formula is widely regarded as being of mainly theoretical interest, as reference to almost any numerical analysis textbook reveals. Yet several authors, including [15], [16], [17], [18], [19], have noted that certain variants of the Lagrange formula are indeed of practical use. [20], have recently collected and explained the attractive features of two modified Lagrange formulas. They argue convincingly that interpolation via a barycentric Lagrange formula ought to be the standard method of polynomial interpolation [21].

Barycentric interpolation is a variant of Lagrange polynomial interpolation. we prefer barycentric Lagrange polynomial

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interpolation [20], [21], over other methods for its numerical stability, more significantly and its speed. Using n nodes, evaluations of the interpolant require $O(n)$ operations, and all other operations requiring more than $O(n)$ operations (excluding function evaluations at the nodes) are function independent and hence precomputable [22].

The primary criterion for polynomial interpolation schemes is that the nodes have the asymptotic density $1/\sqrt{(1-x^2)}$ as $n \to \infty$. Node schemes satisfying this density are typically the roots of orthogonal polynomials, such as two well-known sets of the Legendre and Chebyshev polynomials [14], [22], [23].

This work is arranged as follows: In Section II we explain the formulation of the Fin problem which is a nonlinear equation that occurs in the heat transferring. In Section III we describe how to use barycentric Lagrange interpolation to improve the quasilinearization method. In the next Section the proposed method is applied to solve the Fin equation and then a comparison is made between the obtained results and the exist exact solutions in some cases. The conclusions are described in the final Section.

II. Problem Formulation

Consider a straight fin of length L with a constant cross-sectional area A, and perimeter P. The fin surface is exposed to a convective environment at temperature T_a and the local heat transfer h along the fin surface is assumed to exhibit a power–law–type dependence on the local temperature difference between the fin and the ambient fluid as

$$h = C(T - T_a)^m, \tag{1}$$

where C is a constant derived from natural convection theory, T is the local temperature on the fin surface and the exponent m depends on the heat transfer mode. The value of m can vary in a wide range [24], [25]. For example, the exponent m may take the values -4, -0.25, 0, 2 and 3, indicating the fin subject to transition boiling, laminar film boiling or condensation, convection, nucleate boiling and thermal radiation, respectively.

The Fourier heat–conduction equation, combined with the condition that the state is steady, leads to the following differential equation:

$$Ak\frac{d^2T}{dX^2} = pC(T - T_a)^{m+1}, \tag{2}$$

where k is the thermal conductivity.

For one-dimensional steady state heat conduction, the equation in terms of dimensionless variables

$$\theta = \frac{T - T_a}{T_b - T_a}, \quad x = \frac{X}{L}, \tag{3}$$

$$N^2 = \frac{h_b P L^2}{Ak} = \frac{PCL^2}{Ak}(T_b - T_a)^m, \tag{4}$$

can be written as:

$$\frac{d^2\theta}{dx^2} - N^2\theta^{m+1} = 0, \tag{5}$$

where h_b and T_b are the heat transfer coefficient and temperature at fin base, the axial distance x is measured from the fin tip and N is the convective–conductive parameter of the fin. For simplicity, assume the fin tip is insulated and the boundary conditions to Eq. (5) can be expressed as:

$$\theta'(0) = 0, \tag{6}$$

$$\theta(1) = 1. \tag{7}$$

III. Modify the Quasilinearization Method

A. *The Quasilinearization Method(QLM)*

The aim of the quasilinearization method (QLM) of Bellman and Kalaba [6], [7], [26] based on the Newton-Raphson method [4], [5] is to solve a nonlinear nth order ordinary or partial differential equation in N dimensions as a limit of a sequence of linear differential equations. This goal is easily understandable since there is no useful technique for obtaining the general solution of a nonlinear equation in terms of a finite set of particular solutions, in contrast to a linear equation which can often be solved analytically or numerically in a convenient fashion using superposition. In addition, the QL sequence should be constructed to assure quadratic convergence and, if possible, monotonicity [9].

In this section, we present the main features of the quasilinearization approach [27]. Consider the nonlinear ordinary differential equations (NODE)

$$L^{(n)}y(x) = \frac{d^n y(x)}{dx^n} = \tag{8}$$

$$f\left(x, y(x), \frac{dy}{dx}(x), \ldots, \frac{d^{n-1}y}{dx^{n-1}}(x)\right)$$

$$x \in [0, b] \tag{9}$$

with boundary conditions

$$g_k\left(y(0), \frac{dy}{dx}(0), \ldots, \frac{d^{n-1}y}{dx^{n-1}}(0)\right) = 0 \quad k = 1, \ldots, l, \tag{10}$$

$$g_k\left(y(b), \frac{dy}{dx}(b), \ldots, \frac{d^{n-1}y}{dx^{n-1}}(b)\right) = 0 \quad k = l{+}1, \ldots, n. \tag{11}$$

Here $L^{(n)}$ is a linear nth order ordinary differential operator, f and $g_1, g_2, \ldots, g_n$ are nonlinear functions of $y(x)$ and $(n-1)$ derivatives $\frac{d^s y}{dx^s}(x), s = 1, \ldots, n-1$.

The QLM prescription [9] determines the $(r+1)$th iterative approximation $y^{(r+1)}(x)$ to the solution of problem (8) - (11) as the solution of the linear equation

$$L^{(n)}y^{(r+1)}(x) = f\left(x, y^{(r)}(x), \frac{dy^{(r)}}{dx}(x), \ldots, \frac{d^{n-1}y^{(r)}}{dx^{n-1}}(x)\right) + \sum_{s=0}^{n-1}\left(\frac{d^s y^{(r+1)}}{dx^s}(x) - \frac{d^s y^{(r)}}{dx^s}(x)\right) \times f_{y^s}\left(x, y^{(r)}(x), \frac{dy^{(r)}}{dx}(x), \ldots, \frac{d^{n-1}y^{(r)}}{dx^{n-1}}(x)\right) \tag{12}$$

with linearized two-point boundary conditions for k =

$1, \ldots, l$

$$\sum_{s=0}^{n-1}\left(\frac{d^s y^{(r+1)}}{dx^s}(0)-\frac{d^s y^{(r)}}{dx^s}(0)\right) \times g_{k_{y^s}}\left(0, y^{(r)}(0), \frac{dy^{(r)}}{dx}(0), \ldots, \frac{d^{n-1}y^{(r)}}{dx^{n-1}}(0)\right), \quad (13)$$

and for $k = l+1, \ldots, n$

$$\sum_{s=0}^{n-1}\left(\frac{d^s y^{(r+1)}}{dx^s}(b)-\frac{d^s y^{(r)}}{dx^s}(b)\right) \times g_{k_{y^s}}\left(b, y^{(r)}(b), \frac{dy^{(r)}}{dx}(b), \ldots, \frac{d^{n-1}y^{(r)}}{dx^{n-1}}(b)\right). \quad (14)$$

Here $f_{y^s} = \partial^s f/\partial y^s$ and $g_{k_{y^s}} = \partial^s g_k/\partial y^s$, $s = 0, 1, \ldots, n-1$ are partial derivatives of the functions $f(x, y(x), \frac{dy}{dx}(x), \ldots, \frac{d^{n-1}y}{dx^{n-1}}(x))$ and $g_k(x, y(x), \frac{dy}{dx}(x), \ldots, \frac{d^{n-1}y}{dx^{n-1}}(x))$, respectively.

The initial guess $y^{(0)}(x)$ is chosen from mathematical or physical concepts. Let

$$\delta y^{(r+1)}(x) \equiv y^{(r+1)}(x) - y^{(r)}(x), \quad r = 0, 1, \ldots \quad (15)$$

be the difference between two subsequent iterations.

Under some assumptions on the input data of problem (8) - (11) in [9] the following estimate is established

$$\|\delta y^{(r+1)}\| \leq k\|\delta y^{(r)}\|, \quad (16)$$

where k is a constant independent of r. It is also shown that the difference between exact solution and the rth iteration

$$\triangle y^{(r)}(x) = y^{(r)}(x) - y(x), \quad (17)$$

is quadratically decreasing as well as:

$$\|\triangle y^{(r+1)}\| \leq \|\triangle y^{(r)}\|^2. \quad (18)$$

A simple induction of (16) shows that $\|\delta y^{(r+1)}(x)\|$ for an arbitrary $l < 2$ satisfies the inequality

$$\|\delta y^{(r+1)}\| \leq \frac{1}{k}(k\|\delta y^{(l+1)}\|)^{2^{r-1}}, \quad (19)$$

or, for $l = 0$ [27], they related the $(r+1)$th order result to the 1st iterate by

$$\|\delta y^{(r+1)}\| \leq \frac{1}{k}(k\|\delta y^{1}\|)^{2^{r}}. \quad (20)$$

Therefore the convergence depends on the quantity $k \parallel y^{(1)} - y^{(0)}\|$ where as it has been mentioned above, the initial guess $y^{(0)}(x)$ should be chosen from physical and mathematical concepts. Usually, it is advantageous that $y^{(0)}(x)$ would satisfy at least one of the boundary conditions [9], [27].

B. Barycentric Lagrange Interpolation

In this section we first introduce Lagrange polynomial and then present barycentric interpolation [20].

Let $n+1$ distinct interpolation points (nodes) x_j , $j = 0, \ldots, n$, be given, together with corresponding numbers f_j , which may or may not be samples of a function f. Unless stated otherwise, we assume that the nodes are real, although most of our results and comments generalize to the complex plane. Let $\prod_n$ denote the vector space of all polynomials of degree at most n. The classical problem addressed here is that of finding the polynomial $p \in \prod_n$ that interpolates f at the points x_j , i.e.,

$p(x_j) = f_j, \qquad j = 0, \ldots, n$

The problem is well-posed; i.e., it has a unique solution that depends continuously on the data. Moreover, as explained in virtually every introductory numerical analysis text, the solution can be written in Lagrange form [20], [28]:

$$p(x) = \sum_{j=0}^{n} f_j l_j(x), \qquad l_j(x) = \frac{\prod_{k=0,k\neq j}^{n}(x-x_k)}{\prod_{k=0,k\neq j}^{n}(x_j-x_k)}. \quad (21)$$

The Lagrange polynomial l_j corresponding to the node x_j has the property

$$l_j(x_k) = \begin{cases} 1, & j = k \\ 0, & otherwise \end{cases} \quad (22)$$

for $j, k = 0, \ldots, n$.

Now if we have $l(x) = (x-x_0)(x-x_1)\ldots(x-x_n)$, we can rewrite the Lagrange basis polynomial as

$$l_j(x) = \frac{l(x)}{x-x_j}\frac{1}{\prod_{k=0,k\neq j}^{n}(x_j-x_k)}. \quad (23)$$

If define the barycentric weights by

$$w_j = \frac{1}{\prod_{k\neq j}(x_j-x_k)} \qquad j = 0, \ldots, n, \quad (24)$$

so we can simply write

$$l_j(x) = l(x)\frac{w_j}{x-x_j}, \quad (25)$$

which is commonly referred to as the first form of the barycentric interpolation formula.

The advantages of this representation is that the interpolation polynomial may now be evaluated as

$$p(x) = l(x)\sum_{j=0}^{n}\frac{w_j}{x-x_j}f_j. \quad (26)$$

Suppose we interpolate, besides the data f_j, the constant function $f(x) = 1$, whose interpolant is of course itself. Inserting into (26), we get

$$1 = \sum_{j=0}^{n} l_j(x) = l(x)\sum_{j=0}^{n}\frac{w_j}{x-x_j}. \quad (27)$$

Dividing (26) by this expression and cancelling the common factor $l(x)$, we obtain the barycentric formula for p [20]:

$$p(x) = \frac{\sum_{j=0}^{n}\frac{w_j}{x-x_j}f_j}{\sum_{j=0}^{n}\frac{w_j}{x-x_j}} \quad (28)$$

where w_j is still defined by (24). Rutishauser [16] called (28) the second (true) form of the barycentric formula.

For certain special sets of nodes x_j , one can give explicit formulas for the barycentric weights w_j .

The simplest examples of clustered point sets are the families of Chebyshev points, obtained by projecting equally

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spaced points on the unit circle down to the unit interval $[-1, 1]$.

The Chebyshev points of the first kind are given by

$$x_j = \cos \frac{(2j+1)\pi}{2n+2}, \qquad j = 0, \ldots, n. \tag{29}$$

In this case after cancelling factors independent of j we find [15]

$$w_j = (-1)^j \sin \frac{(2j+1)\pi}{2n+2}. \tag{30}$$

Note that these numbers vary by factors $O(n)$, not exponentially, reflecting the good distribution of the points.

For this set of Chebyshev points, if the interval $[-1, 1]$ is linearly transformed to $[a, b]$, the weights as defined by (24) all get multiplied by $2^n(b-a)^{-n}$. However, as this factor cancels out in the barycentric formula, there is again no need to include it [20].

C. Quasilinearization – Barycenric Lagrange Interpolation Method

In fact the linearized equations (12) - (14) (i.e. the QLM) for some nonlinear problems such as Fin problem does not solve analytically in practice the problem (8) - (11). So we can follow the below steps while get the semi analytical solution of our problem:

Initialize: Linearize the nonlinear ordinary differential equation and its boundary conditions with QLM (12) - (14).

In rth iteration do this (r begins from 0 and $y^{(0)}(x)$ is initial guess):

Step 1: Subsitute $y^{(r)}(x)$ and its derivations in the linear equations that obtained from initialization, to get new ordinary differential equation (ODE).

Step 2: Solve numerically the ODE by using ODE boundary value problem solver package (bvp4c) in MATLAB software.

Step 3: Apply barycentric Lagrange interpolation (28) via Chebyshev points to approximate $y^{(r+1)}(x)$ in linear equation.

Step 4: If gain the sightly results, stop; else go to step 1.

IV. Solution of Fin Problem

In this section we apply the present method for solving the Fin problem and compare the results with the exist exact solutions in some cases.

It is enough to follow the steps mentioned in previous section:

Initialize: The quasilinearized form of Fin equation(5) is:

$$\frac{\mathrm{d}^2\theta^{(n+1)}}{\mathrm{d}x^2} = N^2(\theta^{(n)})^{m+1} + (\theta^{(n+1)} - \theta^{(n)}) \times \left[N^2(m+1)(\theta^{(n)})^m\right],$$

$$\theta'(0) = 0,$$
$$\theta(1) = 1. \tag{31}$$

The simplest initial guess satisfying at least one of the boundary conditions so can be set $\theta^{(0)}(x) = x$ for $m \geq 0$ and $\theta^{(0)}(x) = x + 1$ for $m < 0$.

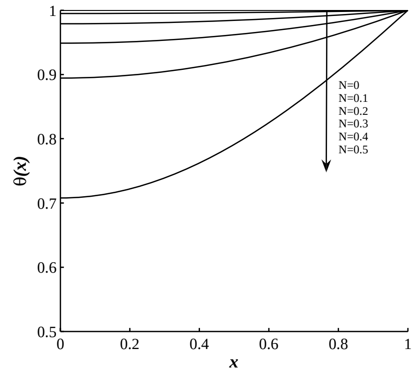

Fig. 1 Plot of $\theta(x)$ for $m = -4$

Since the Fin problem is defined on $[0, 1]$ we use the Chebyshev points in barycentric interpolation by the map function $\phi(x) = (x + 1)/2$.

Now, Executing the algorithm yields the approximate solutions.

In Figures 1, 2, 3, 4, 5, the plot of $\theta(x)$ for $m = -4$, -0.25, 0, 2 and 3 and various N is shown respectively.

The temperature at the fin tip (θ_0), as a function of N has been plotted in Figures 6, 7 for $m = -4$ and -0.25 respectively. Figure 8 shows the fin tip (θ_0), as a function of N for $m = 0$, 2 and 3. These graphs are symmetric with respect to the origin. Note that this problem does not admit any solution for some values of N [1].

Figures 9, 10 show the logarithmic graphs of absolute difference between two subsequent iterations in modified QLM as $\delta\theta^{(r+1)}(0)$ and $\delta d\theta^{(r+1)}(1)$ for $m = 2, N = 5$ and $m = 3, N = 5$ respectively. Note that the average number of QLM iterations in this problem is 5, They shows that the convergence to the exact solutions is very fast.

For $m = 0$, the exact solution is [1]

$$\theta(x) = \frac{\cosh Nx}{\cosh N}. \tag{32}$$

A comparison of the quasilinear solutions corresponding to the first iteration with the numerically computed exact solutions for $m = 0$ and $N = 1$, 3 and 5 are given in Table I.

Tables II and III shows the comparison of the two principal quantities of engineering interest, namely, $\theta(0)$ and $\theta'(1)$ for various m and N, from the method proposed in this paper and exact solutions obtained by Abbasbandy et. al. [1]. It is found from these Table that the parameter $\theta'(1)$ increases with N for all the values of m. However, for fixed N, $\theta'(1)$ decreases as m increases.

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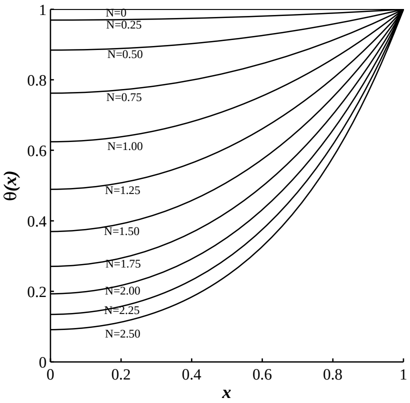

Fig. 2. Plot of $\theta(x)$ for $m = -0.25$.

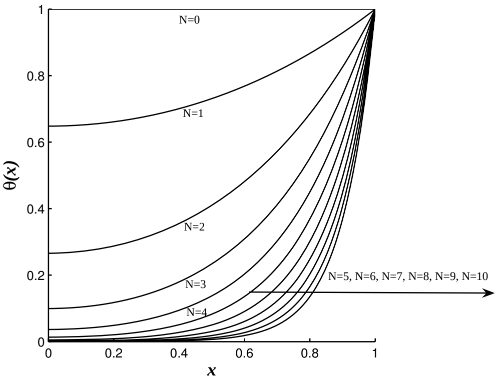

Fig. 3 Plot of $\theta(x)$ for $m = 0$

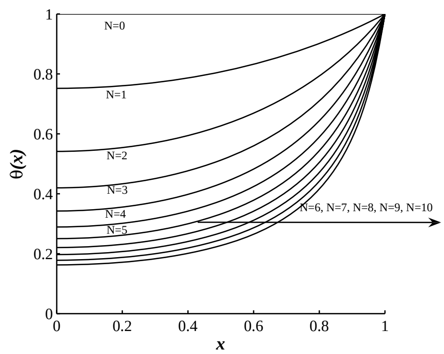

Fig. 4 Plot of $\theta(x)$ for $m = 2$

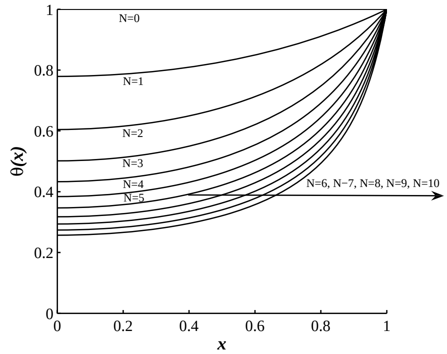

Fig. 5. Plot of $\theta(x)$ for $m = 3$.

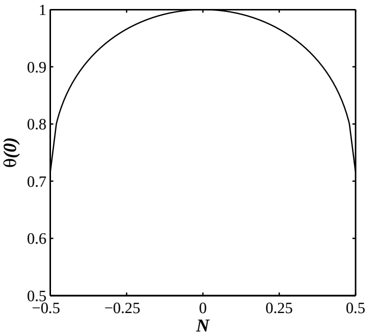

Fig 6 Temperature at the fin tip (θ_0), as a function of N for $m = -4$

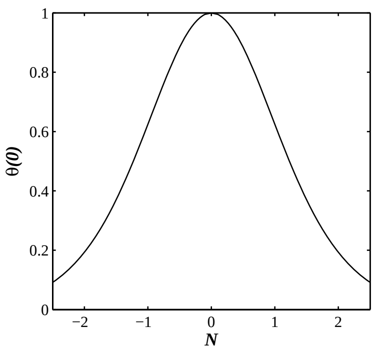

Fig. 7 Temperature at the fin tip (θ_0), as a function of N for $m = -0.25$

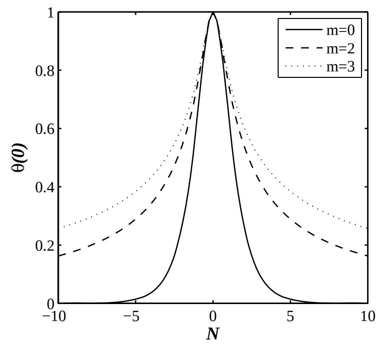

Fig. 8 Temperature at the fin tip (θ_0), as a function of N for $m = 0, 2, 3$

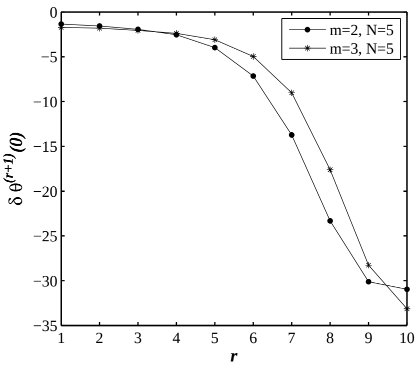

Fig. 9 Logarithmic graph of absolute $\delta\theta^{(r+1)}(0)$ for $m = 2, N = 5$ and $m = 3, N = 5$

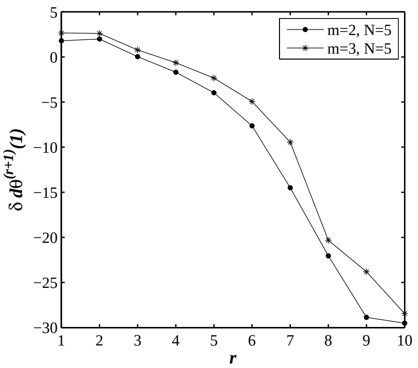

Fig. 10 Logarithmic graph of absolute $\delta d\theta^{(r+1)}(1)$ for $m = 2, N = 5$ and $m = 3, N = 5$

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TABLE I
COMPARISON BETWEEN THE NUMERICAL RESULTS OF $\theta(x)$ FOR $m = 0$ AND EXACT VALUES

	$N=1$		$N=3$		$N=5$	
x	Numerical	Exact	Numerical	Exact	Numerical	Exact
0.0	0.64805427	0.64805427	0.09932793	0.09932793	0.01347528	0.01347528
0.1	0.65129725	0.65129725	0.10383131	0.10383131	0.01519508	0.01519508
0.2	0.66105862	0.66105862	0.11774980	0.11774980	0.02079345	0.02079345
0.3	0.67743609	0.67743609	0.14234550	0.14234550	0.03169938	0.03169938
0.4	0.70059357	0.70059357	0.17984866	0.17984866	0.05069665	0.05069665
0.5	0.73076283	0.73076283	0.23365997	0.23365997	0.08263433	0.08263433
0.6	0.76824580	0.76824580	0.30865887	0.30865887	0.13566459	0.13566459
0.7	0.81341764	0.81341764	0.41164604	0.41164604	0.22332349	0.22332349
0.8	0.86673043	0.86673043	0.55196004	0.55196004	0.36798614	0.36798614
0.9	0.92871776	0.92871776	0.74232415	0.74232415	0.60657797	0.60657797
1.0	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000	1.00000000

TABLE II
COMPARISON OF $\theta(0)$ FOR VARIOUS m AND N BETWEEN PROPOSED METHOD AND EXACT VALUES GIVEN BY ABBASBANDY [1]

m	N	QLM	Exact	N	QLM	Exact	N	QLM	Exact
-4	0.1	0.99493615	0.99493615	0.2	0.97890632	0.97890631	0.4	0.89442778	0.89442719
-0.25	1	0.62416849	0.62416888	1.75	0.27079441	0.27080076	2	0.19307678	0.19307750
0	1	0.64805427	0.64805427	3	0.09932793	0.09932793	5	0.01347528	0.01347528
2	1	0.75162414	0.75162201	3	0.41962917	0.41960158	5	0.28901585	0.28901262
3	1	0.77914785	0.77914516	3	0.50128429	0.50128247	5	0.38389490	0.38389178

TABLE III
COMPARISON OF θ (1) FOR VARIOUS m AND N BETWEEN PROPOSED METHOD AND EXACT VALUES GIVEN BY ABBASBANDY [1]

m	N	QLM	Exact	N	QLM	Exact	N	QLM	Exact
-4	0.1	0.01010205	0.01010206	0.2	0.04174243	0.04174243	0.4	0.19999919	0.20000000
-0.25	1	0.80120868	0.80120816	1.75	1.77319515	1.77318936	2	2.07710091	2.07710000
0	1	0.76159416	0.76159416	3	2.98516426	2.98516426	5	4.99954602	4.99954602
2	1	0.58345481	0.58345853	3	2.08816267	2.08818205	5	3.52311162	3.52317865
3	1	0.53398401	0.53398921	3	1.86707228	1.86709663	5	3.14898258	3.14906707

V. CONCLUSION

The quasilinearization method(QLM) is very applicable and it treats nonlinear terms by a series of nonperturbative iterations and is not based on the existence of some kind of small parameter. At every iterative stage, the differential operator changes significantly to account for the nonlinearity, which is the major way that the QLM differs from other approximative techniques.

In this paper we improved the quasilinearization method by barycentric Lagrange interpolation for solving Fin problem as a nonlinear equation that occurs in the heat transferring. In every iterative stage, we solve $y^{(r)}(x)$ numerically by using ODE boundary value problem solver package (bvp4c) in MATLAB software and then appleid the barycentric Lagrange via Chebyshev points to approximate $y^{(r+1)}(x)$. This interpolation is very stabile, accurate and easy to compute. In the end, we compared this method with the other methods and showed the solutions are accurate, numerically stable and fast convergence. So it is applicable for such problems.

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