Free Convection in an Infinite porous Dusty Medium induced by Pulsating Point Heat Source

K. Kannan and V. Venkataraman

Abstract—Free convection effects and heat transfer due to a pulsating point heat source embedded in an infinite, fluid saturated, porous dusty medium are studied analytically. Both velocity and temperature fields are discussed in the form of series expansions in the Rayleigh number, for both the fluid and particle phases based on the mean heat generation rate from source and on the permeability of the porous dusty medium. This study is carried out by assuming the Rayleigh number small and the validity of Darcy's law. Analytical expressions for both phases are obtained for second order mean in both velocity and temperature fields and evolution of different wave patterns are observed in the fluctuating part. It has been observed that, at the vicinity of the origin, the second order mean flow is influenced only by relaxation time of dust particles and not by dust concentration.

Keywords—pulsating point heat source, azimuthal velocity, porous dusty medium, Darcy's law.

I. INTRODUCTION

THERE have been numerous theoretical and experimental studies of heat and mass transfer induced by natural convection in fluids of porous and non-porous media. These studies have many applications in physical systems (where heat transport by buoyancy induced convective motion to take place) such as chemical reactor, nuclear reactor, combustion systems, pneumatic transport etc. Similarly, the study of free convection in fluid-saturated porous media in the presence of point heat sources has many important geothermal and engineering applications. For example, the sub-sea bed disposal of nuclear waste material is of prime concern for the nuclear waste management since the energy released by the waste material can cause an upward migration of interstitial pore water into water columns which in turn is responsible for the transport of radio nuclide. In some of these applications, the fluid may contain inert, suspended Stokesian solid particles.

Investigations on heat transfer by natural convection of two phase fluids in non-porous media have been made by several authors [1]–[13]. However, not many studies on natural convection problem of two phase fluids in porous media are reported in available literature.

When the dimension of a convective system in a saturated porous medium are sufficiently great, diffusion effects can be neglected except in the regions where the gradients of the fluid properties are very large. Wooding [14] dealt with the steady state high Rayleigh number behavior of flow due to a point heat source placed on the base of a semi-infinite porous medium and found that the flow is similar to a round laminar

K. Kannan is with Department of Mathematics, Sastra University, Thanjavur–613 401, INDIA. E-mail: kkannan@maths.sastra.edu
V. Venkataraman is with Department of Mathematics, SASTRA University, Thanjavur–613 401, INDIA. Email: mathvvr@yahoo.co.in

jet. The buoyancy induced transient and steady state natural convection with a concentrated heat source embedded in an infinite porous medium had been analyzed by Bejan [15]. Yamamoto [16] had analyzed the natural convection about a heated sphere in the porous medium. Hickox and Watts [17] obtained numerical solution for axe-symmetric free convection from concentrated heat sources and their analysis is valid for a wide range of values of the Rayleigh number. Source solutions for a variety of heat sources were presented by Hickox [18] with special emphasis on their application in the analysis of sub-sea bed disposal of nuclear waste material. Hiremath [19] analyzed the natural convection flow and heat transfer induced by a pulsating point heat source embedded in an unbounded fluid saturated porous medium, assuming the flow is governed by Darcy's law and that the thermal Rayleigh number is small.

In this present study, Free convective heat transfer in an infinite porous dusty medium due to a pulsating point heat source has been considered. It is assumed that the source strength, which induces the free convection, being a function of time can be expressed in a Fourier series in which the second and higher harmonics are negligible. Only the small Rayleigh number behavior of the flow is considered with the assumption of Darcy's law. To express the buoyancy force field, Boussinesq approximation is invoked. Perturbation techniques are used to obtain the analytical expressions for second order mean flow of both fluids and particles phases in both velocity and temperature fields.

II. MATHEMATICAL FORMULATION AND FUNDAMENTAL CONCEPTS

In this section, Mathematical models are developed for the description of, axe symmetric free convection in a rigid, infinite, homogeneous and isotropic porous dusty medium of low permeability with a point heat source of strength

$$q(t) = Q(1 + \varepsilon \cos at) \qquad (1)$$

where Q is the mean heat generation rate from the source, a, the frequency oscillation in a source strength and ε, a small positive parameter. In both velocity and temperature fields, amplitudes of fluctuations are assumed to be small. Since the density changes are accounted for only in the buoyancy term, in the equation of motion, the fluid is assumed to be Boussinesq incompressible with density temperature relation

$$\rho = \rho_\infty \left[1 - \beta \left(T - T_\infty \right) \right] \qquad (2)$$

Here ρ is the fluid density, β, coefficient of thermal expansion, and T, the temperature. The subscript ∞ denotes the reference state. It is also assumed that fluid and matrix are in thermal

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equilibrium and fluid motion can be adequately described by Darcy's law. Viscosity, effective thermal diffusivity, and coefficient of thermal expansion are assumed to be constant. Dispersion effects are neglected.

The steady state equations of continuity for fluid and particle phase are

$$\text{div } \vec{q} = 0 \qquad (3)$$

$$\text{div } \vec{q}_p = 0 \qquad (4)$$

Equations of momentum for fluid and particle phase are

$$\frac{\mu}{K}\vec{q} = -\text{grad }(p + \rho g h) + \frac{KN_0}{\rho}(\vec{q}_p - \vec{q}) \qquad (5)$$

$$\vec{q}_p = -\frac{KN_0}{\rho}(\vec{q}_p - \vec{q}) \qquad (6)$$

and Energy equation for fluid and particle phase are

$$\vec{q} \cdot \text{grad } T = \alpha \text{div } [\text{grad } T] + \rho_p C_s \left[\frac{T_p - T}{\tau_T}\right] \qquad (7)$$

$$\vec{q}_p \cdot \text{grad } T_p = \left[\frac{T_p - T}{\tau_T}\right] \qquad (8)$$

where $\vec{q}, q_p, K, \mu, \alpha, p, g, k, N_0, \tau_T$ and T_p are respectively, the velocity vector of fluid, particle phase, medium permeability, dynamic viscosity, effective thermal diffusivity, pressure, acceleration due to gravity, Stokes resistance coefficient, number density of particle phase, thermal relaxation time of particles and temperature of particle phase. The elevation h is measured vertically upward and g is oppositely directed.

Here thermal energy is released continuously at a finite rate from a point source. Hence in the absence of any bounding surfaces which can inhibit motion, any deviation from an isothermal state will result in fluid motion.

Here the volume fraction and viscosity of the pseudo-fluid of solid particles have been neglected. The subscript p in the equations denotes corresponding entities of particle phase. C_s is the specific heat of particles. If Reynolds number based on the relative velocity of the particle is less than unity, then the force accelerating the particle to the fluid speed is given by Stokes law which is $6\pi\mu r_p (\vec{q}_p - \vec{q})$ where r_p is the radius of the particle. If N_0 is assumed to be the number density of particles, the total fluid- particle interaction force per unit volume is given by

$$F_p = 6\pi N_0 r_p \mu (q_p - q) = \frac{\rho_p(q_p - q)}{\tau_m}$$

$$\tau_m = \frac{m}{6\pi\mu r_p};$$

is called the relaxation time during which the velocity of the particle phase relative to the fluid is reduced to $\left(\frac{1}{e}\right)$ times its initial value and m is the mass of each particle.

Similarly the total thermal interaction between the fluid and particle phase per unit volume is given by $Q_p = \frac{pC_s(T_p - T)}{\tau_T}$ and $\tau_T = \frac{mC_s}{4\pi K_1 r_p}$ is thermal relaxation time of particle phase. i.e., τ_T, the temperature of the particle phase relative to the fluid is $\left(\frac{1}{e}\right)$ times the initial value, where K_1 is the thermal conductivity of the fluid.

In most of the studies of dusty fluid flows, certain simplifying assumptions are usually made for dilute suspensions. In this study the following assumptions have been made:

1) The inert dust particles are assumed to be spherical in shape all having the same radius and mass and undeformable.
2) The number density N_0 of particles is constant.
3) The solid particles are sparsely distributed and they are non- interacting, so that the pressure locally have same velocity vector and temperature. Due to this assumption of lack of randomness in local particle motion, the pressure associated with the particle cloud is negligible. Then the fluid pressure p will be the same as the total pressure of the mixture.

Here we wish to consider the axe-symmetric flow induced by a point source of strength Q (energy generated per unit time) situated at the origin and the axis $\phi = 0$ is taken vertically upwards. It is convenient for our analysis to write equations (3)–(8) in spherical polar coordinates (r, ϕ) with associated radial and transverse velocity components of the fluid phase as (u, v) and for particle phase (u_p, v_p).

The relationship between the coordinates system is illustrated in Fig. 1.
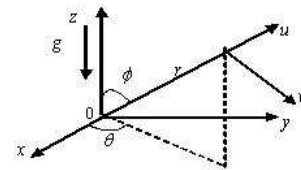

Fig. 1. A spherical polar coordinate system with the point heat source located at the centre.

Following Darcy flow model [19] the equations of motion describing the conservation of mass, momentum for both the phases get reduced to:

$$\frac{\partial}{\partial r}\left[r^2 u \sin\phi\right] + \frac{\partial}{\partial \phi}\left[rv \sin\phi\right] = 0 \qquad (9)$$

$$\frac{\partial}{\partial r}\left[r^2 u_p \sin\phi\right] + \frac{\partial}{\partial \phi}\left[rv_p \sin\phi\right] = 0 \qquad (10)$$

$$u = \frac{-K}{\mu}\left[\frac{\partial P}{\partial r} + \rho g \cos\phi + K_2 N_0 (u_p - u)\right] \qquad (11)$$

$$v = \frac{-K}{\mu}\left[\frac{1}{r}\frac{\partial P}{\partial \phi} - \rho g \sin\phi + K_2 N_0 (v_p - v)\right] \qquad (12)$$

where $K_2 = \frac{K}{\rho}$

$$u_p = -\frac{1}{\tau_p}(u_p - u) \qquad (13)$$

$$v_p = -\frac{1}{\tau_p}(v_p - v) \qquad (14)$$

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and

$$\sigma\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + \frac{v}{r}\frac{\partial T}{\partial \phi} = \alpha\left[\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right)\right.$$
$$\left.+ \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T}{\partial\phi}\right)\right]$$
$$+ \frac{\rho_p C_s}{\tau_T}[T_p - T] \qquad (15)$$

$$\sigma\frac{\partial T_p}{\partial t} + u_p\frac{\partial T_p}{\partial r} + \frac{v_p}{r}\frac{\partial T_p}{\partial\phi} = \frac{[T_p - T]}{\tau_T} \qquad (16)$$

is the heat capacity ratio given by

$$\sigma = \left[\frac{\lambda(\mathrm{Q}\,C_p)f_m + (1 - \lambda(\mathrm{Q}C_p)s_m)}{(\mathrm{Q}\,C_p)f_m}\right] \qquad (17)$$

where f_m, s_m and λ refer to fluid and particle matrix and porosity respectively.

The boundary conditions necessary for the completion of mathematical formulation are

$$u, v, v_p, T, T_p \to 0 \quad \text{as} \quad r \to \infty$$
$$v = \frac{\partial u}{\partial\phi} = \frac{\partial T}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi$$
$$v_p = \frac{\partial u_p}{\partial\phi} = \frac{\partial T_p}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi \qquad (18)$$

Further, the origin being the location of point heat source, is a singular point for both the velocity and temperature fields and hence u, v and T vary as $\frac{1}{r}$ in the limit $r \to 0$. However, for the temperature field, this behavior is described by a heat balance over a spherical surface of radius approaching zero containing the origin:

$$\lim_{r\to 0}\left[-K_1\left(4\pi r^2\frac{\partial T}{\partial r}\right)\right] = Q(1 + \varepsilon\cos at) \qquad (19)$$

Taking advantage of the continuity equations (9) and (10) for both the phases we define stream functions ψ and ψ_p such that

$$u = \frac{1}{r^2\sin\phi}\frac{\partial\psi}{\partial\phi} \quad \text{and} \quad v = -\frac{1}{r\sin\phi}\frac{\partial\psi}{\partial r} \qquad (20)$$

$$u_p = \frac{1}{r^2\sin\phi}\frac{\partial\psi_p}{\partial\phi} \quad \text{and} \quad v_p = -\frac{1}{r\sin\phi}\frac{\partial\psi_p}{\partial r} \qquad (21)$$

using (13), (20) in (9) and (14), (21) in (10), we get

$$\frac{1}{r^2\sin\phi}\frac{\partial\psi}{\partial\phi} = -\frac{K}{\mu}\left[\frac{\partial P}{\partial r} + \rho g\cos\phi - \tau_p K_2 N_0 u_p\right] \qquad (22)$$

$$-\frac{1}{r\sin\phi}\frac{\partial\psi}{\partial r} = -\frac{K}{\mu}\left[\frac{1}{r}\frac{\partial P}{\partial\phi} + \rho g\sin\phi - \tau_p K_2 N_0 v_p\right] \qquad (23)$$

Eliminating the pressure terms in equations (22) and (23) by cross differentiation and introducing the non dimensional variables R, τ, ψ, ψ_p, H, H_p defined by

$$R = \frac{r}{\sqrt{k}}, \quad \tau = \frac{\alpha t}{K\sigma}, \quad \Psi = \frac{\psi}{\alpha\sqrt{K}}, \quad \Psi_p = \frac{\psi_p}{\alpha\sqrt{K}}$$
$$H = \frac{(T - T_\infty)\,K_1\sqrt{K}}{\mathrm{Q}}, \quad H_p = \frac{(T_p - T_\infty)\,K_1\sqrt{K}}{\mathrm{Q}}$$

where K_1 is the thermal conductivity of the fluid/porous matrix, we obtain the equations of the conservation of momentum and energy for fluid and particle phase as follows:

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left(\frac{\partial\Psi}{\partial\phi} + \lambda_1\frac{\partial\Psi_p}{\partial\phi}\right)\right]$$
$$+ \frac{1}{\sin\phi}\left(\frac{\partial^2\Psi}{\partial R^2} + \lambda_1\frac{\partial^2\Psi_p}{\partial R^2}\right)$$
$$= Ra\left[\cos\phi\frac{\partial H}{\partial\phi} + R\sin\phi\frac{\partial H}{\partial R}\right] \qquad (24)$$

$$\frac{\partial\Psi_p}{\partial\phi} = \frac{1}{(1 + \tau_p)}\frac{\partial\Psi}{\partial\phi} \qquad (25)$$

$$\frac{\partial H}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi, H)}{\partial(\phi, R)}\right]$$
$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left[R^2\frac{\partial H}{\partial R}\right] + \frac{1}{R^2\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H}{\partial\phi}\right]$$
$$+ \frac{2f}{3\Lambda}(H_p - H) \qquad (26)$$

$$\frac{\partial H_p}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_p, H_p)}{\partial(\phi, R)}\right] = -\frac{2}{3}\frac{1}{\gamma\Lambda}(H_p - H) \qquad (27)$$

where $\Lambda = \frac{\mu\tau_p}{K}$, $f = \frac{\rho_p}{\alpha\tau_p}\gamma = \frac{c_s}{c_p}$, $\lambda_1 = \frac{KK_1\tau_p N_0}{\mu}$, $Ra = \frac{\beta\rho_\infty gkQ}{\alpha\mu K_1}$ is the Rayleigh number based on the mean source strength and medium permeability.

The non-dimensional form of velocity components are given by

$$(U, V) = \frac{\sqrt{K}}{\alpha}(u, v) \qquad (28)$$

$$(U_p, V_p) = \frac{\sqrt{K}}{\alpha}(u_p, v_p) \qquad (29)$$

Accordingly, the boundary conditions (18) and (19) become $U, V, U_p, V_p, H, H_p \to 0$ as $R \to \infty$.

$$V = \frac{\partial U}{\partial\phi} = \frac{\partial H}{\partial\phi} = 0 \text{ and } V_p = \frac{\partial U_p}{\partial\phi} = \frac{\partial H_p}{\partial\phi} = 0$$
$$\text{at } \phi = 0, \pi \qquad (30)$$

$$\lim_{R\to 0}\left[-4\pi R^2\frac{\partial H}{\partial R}\right] = 1 + \varepsilon\cos\omega\tau \qquad (31)$$

$$\text{Also} \quad \lim_{R\to 0}\left[-4\pi R^2\frac{\partial H_p}{\partial R}\right] = 1 + \varepsilon\cos\omega\tau \qquad (32)$$

where $\omega = \frac{\sigma aK}{\alpha}$ is the non-dimensional frequency parameter.

III. Perturbation Analysis

In view of the boundary conditions and the assumptions of small magnitudes of oscillations ($\varepsilon < 1$), We seek solutions

for Ψ, H, Ψ_p and H_p of the form

$$\Psi = \Psi_0(R,\phi) + \sum_{n=1}^{\infty} \varepsilon^n \Psi(R,\phi,\tau) \tag{33}$$

$$H = H_0(R,\phi) + \sum_{n=1}^{\infty} \varepsilon^n H_n(R,\phi,\tau) \tag{34}$$

$$\Psi_p = \Psi_{p_0}(R,\phi) + \sum_{n=1}^{\infty} \varepsilon^n \Psi_{p_n}(R,\phi,\tau) \tag{35}$$

$$H_p = H_{p_0}(R,\phi) + \sum_{n=1}^{\infty} \varepsilon^n H_{p_n}(R,\phi,\tau). \tag{36}$$

with similar expressions for U, U_p, V and V_p to satisfy the boundary conditions (30). In the expansions of (33) and (34), Ψ_0, H_0, Ψ_{p_0} and H_{p_0} refer to basic steady state whereas the rest of the coefficients refer to the transient state in which the effect of fluctuations can be seen. We substitute equations (33), (34), (35), (36) in (24)–(27) respectively and compare the terms with like powers of ε. The terms of the zero th power in ε yield the equations for the solution of Ψ_0, H_0, Ψ_{p_0} and H_{p_0}. Terms of first power of ε yield the equations for the determination of Ψ_1, H_1, Ψ_{p_1} and H_{p_1} and so on. The appropriate boundary conditions are also obtained from (30) with the help of (33)–(36).

A. Basic steady state

The functions Ψ_0 and H_0 are found from the solutions of equations:

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left\{\frac{\partial\Psi_0}{\partial\phi} + \lambda\frac{\partial\Psi_{p_0}}{\partial\phi}\right\}\right]$$
$$+ \frac{1}{\sin\phi}\left\{\frac{\partial^2\Psi_0}{\partial R^2} + \lambda\frac{\partial^2\Psi_{p_0}}{\partial R^2}\right\}$$
$$= Ra\left[\cos\phi\frac{\partial H_0}{\partial\phi} + R\sin\phi\frac{\partial H_0}{\partial R}\right] \tag{37}$$

$$\frac{\partial\Psi_{p_0}}{\partial\phi} = \frac{1}{1+\tau_p}\frac{\partial\Psi_0}{\partial\phi} \tag{38}$$

$$\frac{1}{R^2\sin\phi}\left[\frac{\partial\Psi_0}{\partial\phi}\frac{\partial H_0}{\partial R} - \frac{\partial H_0}{\partial\phi}\frac{\partial\Psi_0}{\partial R}\right]$$
$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left[R^2\frac{\partial H_0}{\partial R}\right] + \frac{1}{R^2\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_0}{\partial\phi}\right]$$
$$+ \frac{2f}{3\Lambda}\left(H_{p_0} - H_0\right) \tag{39}$$

$$\frac{1}{R^2\sin\phi}\left[\frac{\partial\Psi_{p_0}}{\partial\phi}\frac{\partial H_{p_0}}{\partial R} - \frac{\partial H_{p_0}}{\partial\phi}\frac{\partial\Psi_{p_0}}{\partial R}\right]$$
$$= -\frac{2}{3\gamma\Lambda}\left(H_{p_0} - H_0\right) \tag{40}$$

subject to the boundary conditions

$$U_0, \quad V_0, \quad H_0, \quad U_{p_0}, \quad V_{p_0}, \quad H_{p_0} \to 0 \quad \text{as} \quad R \to \infty;$$
$$V_0 = \frac{\partial U_0}{\partial\phi} = \frac{\partial H_0}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi;$$
$$V_{p_0} = \frac{\partial U_{p_0}}{\partial\phi} = \frac{\partial H_{p_0}}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi. \tag{41}$$

$$\lim_{R \to 0}\left[-4\pi R^2\frac{\partial H_0}{\partial R}\right] = 1 \quad \text{and}$$
$$\lim_{R \to 0}\left[-4\pi R^2\frac{\partial H_{p_0}}{\partial R}\right] = 1 \tag{42}$$

We perform a perturbation analysis about small Rayleigh number in terms of powers of Ra.

$$\Psi_0 = \Psi_{00} + Ra\Psi_{01} + (Ra)^2\Psi_{02} + \cdots;$$
$$\Psi_{p_0} = \Psi_{p_{00}} + Ra_{\Psi_{p_{01}}} + (Ra)^2\Psi_{p_{02}} + \cdots. \tag{43}$$
$$H_0 = H_{00} + RaH_{01} + (Ra)^2 H_{02} + \cdots;$$
$$H_{p_0} = H_{p_{00}} + RaH_{p_{01}} + (Ra)^2 H_{p_{02}} + \cdots, \tag{44}$$

with similar expressions for U_0, U_{p_0}, V_0, and V_{p_0} satisfying the boundary conditions (41).

As $\Psi_{00}, \Psi_{p_{00}}, H_{00}$ and $H_{p_{00}}$ correspond to the state of pure diffusion, we take $\Psi_{00} = 0, \Psi_{p_{00}} = 0$ from (38) and from (41) and (42) we obtain

$$H_{00} = H_{p_{00}} = \frac{1}{4\pi R} \tag{45}$$

I order in Ra: Equations (37) and (38) yield

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left\{\frac{\partial\Psi_{01}}{\partial\phi} + \lambda\frac{\partial\Psi_{p_{01}}}{\partial\phi}\right\}\right]$$
$$+ \frac{1}{\sin\phi}\left\{\frac{\partial^2\Psi_{01}}{\partial R^2} + \lambda\frac{\partial^2\Psi_{p_{01}}}{\partial R^2}\right\}$$
$$= Ra\left[\cos\phi\frac{\partial H_{00}}{\partial\phi} + R\sin\phi\frac{\partial H_{00}}{\partial R}\right] \tag{46}$$

$$\frac{\partial\Psi_{p_{01}}}{\partial\phi} = \frac{1}{1+\tau_p}\frac{\partial\Psi_{01}}{\partial\phi} \tag{47}$$

Separation of variables is achieved by setting

$$\Psi_{01} = f(R)\sin^2\phi;$$
$$\Psi_{p_{01}} = g(R)\sin^2\phi \tag{48}$$

The function $f(R)$ satisfies the ordinary differential equation:

$$R^2 f''(R) - 2f(R) = -\frac{R}{4\pi A} \tag{49}$$

and (47) yield

$$g(R) = \frac{f(R)}{1+\tau_p} \tag{50}$$

where $A = 1 + \frac{\lambda}{(1+\tau_p)}$.

The solution of (49) gives

$$f(R) = c_1 R^2 + \frac{c_2}{R} + \frac{R}{8\pi A}$$

Applying boundary condition (41) as $R \to \infty$ we find $c_2 = 0$. Since in the limit $R \to 0$ the velocities u and v increase as $\left(\frac{1}{R}\right)$, we find upon examining (20) that in the limit, Ψ must be proportional to R, hence $c_1 = 0$. We conclude that

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$f(R) = \frac{R}{8\pi A}$ and in view of (50)

$g(R) = \frac{R}{8\pi A(1+\tau_p)}$ and hence

$$\Psi_{01} = \frac{R\sin^2\phi}{8\pi A} \quad \text{and} \quad \Psi_{p_{01}} = \frac{R\sin^2\phi}{8\pi A(1+\tau_p)} \qquad (51)$$

In a similar way, substituting (43) and (44) in (39) and (40), comparing the first order in Ra, and achieving the separation of variables by setting $H_{01} = f_1(R)\cos\phi$ and $H_{p_{01}} = g_1(R)\cos\phi$, we get an ordinary differential equation $\inf_1(R)$. Solving this differential equation and applying boundary conditions, We find that

$$H_{01} = \frac{(1+f\gamma)\cos\phi}{32\pi^2 RA} \quad \text{and}$$

$$H_{p_{01}} = \left[\frac{(1+f\gamma)}{32\pi^2 RA} + \frac{3\gamma\Lambda}{32\pi^2 R^3 A}\right]\cos\phi \qquad (52)$$

The other non-vanishing coefficients are also found by applying the same procedure as above. For the sake of brevity, we present only the solutions of the second order non vanishing coefficients

$$\Psi_{02} = \frac{(1+f\gamma)R\sin^2\phi\cos\phi}{96\pi^2 A^2} \quad \text{and}$$

$$\Psi_{p_{02}} = \frac{(1+f\gamma)R\sin^2\phi\cos\phi}{96\pi^2 A^2(1+\tau_p)} \qquad (53)$$

These solutions were also obtained by Bejan [15] for clean fluid and for steady state. In terms of velocity components (for both the phases) we have

$$
\begin{aligned}
U_{01} &= \frac{\cos\phi}{4\pi AR}, \\
U_{p_{01}} &= \frac{\cos\phi}{4\pi AR(1+\tau_p)}, \\
V_{01} &= -\frac{\sin\phi}{8\pi AR}, \\
V_{p_{01}} &= -\frac{\sin\phi}{8\pi AR(1+\tau_p)}, \\
U_{02} &= \frac{(1+f\gamma)}{192\pi^2 A^2 R}[1+3\cos 2\phi], \\
U_{p_{02}} &= \frac{(1+f\gamma)}{192\pi^2 A^2 R(1+\tau_p)}[1+3\cos 2\phi], \\
V_{02} &= -\frac{(1+f\gamma)\sin\phi\cos\phi}{96\pi^2 A^2 R}, \\
V_{p_{02}} &= -\frac{(1+f\gamma)\sin\phi\cos\phi}{96\pi^2 A^2 R(1+\tau_p)}.
\end{aligned}
\qquad (54)
$$

B. Fluctuating part

Substituting (33)–(36) in (24)–(27) we obtain for the first order in ε.

$$
\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left\{\frac{\partial\Psi_1}{\partial\phi} + \lambda\frac{\partial\Psi_{p_1}}{\partial\phi}\right\}\right]
$$
$$
+ \frac{1}{\sin\phi}\left\{\frac{\partial^2\Psi_1}{\partial R^2} + \lambda\frac{\partial^2\Psi_{p_1}}{\partial R^2}\right\}
$$
$$
= Ra\left[\cos\phi\frac{\partial H_1}{\partial\phi} + R\sin\phi\frac{\partial H_1}{\partial R}\right] \qquad (55)
$$

$$
\frac{\partial H_1}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_0, H_1)}{\partial(\phi, R)} + \frac{\partial(\Psi_1, H_0)}{\partial(\phi, R)}\right]
$$
$$
= \frac{1}{R^2}\frac{\partial}{\partial R}\left[R^2\frac{\partial H_1}{\partial R}\right]
$$
$$
+ \frac{1}{R^2\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_1}{\partial\phi}\right] + \frac{2f}{3\Lambda}(H_{p_1} - H_1) \qquad (56)
$$

$$
\frac{\partial\Psi_{p_1}}{\partial\phi} = \frac{1}{(1+\tau_p)}\frac{\partial\Psi_1}{\partial\phi} \qquad (57)
$$

$$
\frac{\partial H_{p_1}}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_{p_0}, H_{p_1})}{\partial(\phi, R)} + \frac{\partial(\Psi_{p_1}, H_{p_0})}{\partial(\phi, R)}\right]
$$
$$
= -\frac{2}{3}\frac{1}{\gamma\Lambda}(H_{p_1} - H_1) \qquad (58)
$$

The associated boundary conditions are

$$U_1, \quad V_1, \quad H_1, \quad U_{p_1}, \quad V_{p_1}, \quad H_{p_1} \to 0 \quad \text{as} \quad R \to \infty$$

$$V_1 = \frac{\partial U_1}{\partial\phi} = \frac{\partial H_1}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi;$$

$$V_{p_1} = \frac{\partial U_{p_1}}{\partial\phi} = \frac{\partial H_{p_1}}{\partial\phi} = 0 \quad \text{at} \quad \phi = 0, \pi;$$

$$\lim_{R \to 0}\left[-4\pi R^2\frac{\partial H_1}{\partial R}\right] = \cos\omega\tau \qquad (59)$$

Let $\quad \Psi_1 = \Psi_{10} + Ra\Psi_{11} + (Ra)^2\Psi_{12} + \cdots$;

$\quad \Psi_{p_1} = \Psi_{p_{10}} + Ra\Psi_{p_{11}} + (Ra)^2\Psi_{p_{12}} + \cdots$;

$\quad H_1 = H_{10} + RaH_{11} + (Ra)^2 H_{12} + \cdots$;

$\quad H_{p_1} = H_{p_{10}} + RaH_{p_{11}} + (Ra)^2 H_{p_{12}} + \cdots \qquad (60)$

As $\Psi_{10}, \Psi_{p_{10}}, H_{10}$ and $H_{p_{10}}$ correspond to the state of pure diffusion, we have

$$\Psi_{10} = 0, \Psi_{p_{10}} = 0.$$

From (18),

$$
\left.\begin{aligned}
H_{10} &= \frac{e^{-pR}}{4\pi R}[\cos(\omega\tau - pR)] \\
H_{p_{10}} &= \frac{1}{(1+\tau_p)}H_{10}
\end{aligned}\right\} \qquad (61)
$$

where $p = \sqrt{\frac{\omega}{2}}$.

This shows that the heat transport due to conduction over the mean thermal distribution is in the form of a traveling thermal wave in the radial direction attenuating at a distance of order $\sqrt{\frac{2}{\omega}}$.

Substituting (60) in (55) and collecting terms of first order in Ra, We get equation for Ψ_{11} in which the variables can be

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separated by setting

$$\Psi_{11} = \frac{\sin^2\phi\, e^{i\omega\tau}f(\eta)}{4\pi}; \quad \eta = (1+i)pR \qquad (62)$$

This yields an ordinary differential equation

$$\eta^2 f''(\eta) - 2f(\eta) = -\frac{\eta(\eta+1)e^{-\eta}(1-i)}{2p} \cdot \frac{1}{1+\frac{\lambda}{1+\tau_p}} \qquad (63)$$

whose general solution is

$$f(\eta) = c_1\eta^2 + \frac{c_2}{\eta} - \frac{(\eta+1)e^{-\eta}(1-i)}{2p\eta}\cdot\frac{1}{1+\frac{\lambda}{1+\tau_p}} \qquad (64)$$

The requirements that U and V approach zero as $R \to \infty$, imply $c_1 = 0$ and that U and V vary as $\frac{1}{R}$ in the limit $R \to 0$ imply $c_2 = \frac{1-i}{2p}\cdot\frac{1}{1+\frac{\lambda}{1+\tau_p}}$. Substituting the values of c_1 and c_2 in (64), and (62) and simplifying we get,

$$\Psi_{11} = \frac{\sin^2\phi}{8\pi p^2 R}\left[\sin\omega\tau - \frac{e^{-pR}}{1+\frac{\lambda}{1+\tau_p}}\right.$$
$$\left. \{(1+pR)\sin(\omega\tau - pR) + pR\cos(\omega\tau - pR)\}\right] \qquad (65)$$

yielding,

$$U_{11} = \frac{\cos\phi}{4\pi p^2 R^3}\left[\sin\omega\tau - \frac{e^{-pR}}{1+\frac{\lambda}{1+\tau_p}}\right.$$
$$\left. \{(1+pR)\sin(\omega\tau - pR) + pR\cos(\omega\tau - pR)\}\right] \qquad (66)$$

$$V_{11} = -\frac{\sin\phi}{4\pi pR^2}\left[\frac{e^{-pR}}{1+\frac{\lambda}{1+\tau_p}}\cos(\omega\tau - pR)\right] \qquad (67)$$

(62) yields

$$\Psi_{p_{11}} = \frac{\sin^2\phi}{8\pi p^2 R(1+\tau_p)}$$
$$\times\left[\sin\omega\tau - \frac{e^{-pR}}{1+\frac{\lambda}{1+\tau_p}}\right.$$
$$\left.\times\{(1+pR)\sin(\omega\tau - pR) + pR\cos(\omega\tau - pR)\}\right] \qquad (68)$$

$$U_{p_{11}} = \frac{\cos\phi}{4\pi p^2 R^3(1+\tau_p)}$$
$$\times\left[\sin\omega\tau - \frac{e^{-pR}}{1+\frac{\lambda}{1+\tau_p}}\right.$$
$$\left.\times\{(1+pR)\sin(\omega\tau - pR) + pR\cos(\omega\tau - pR)\}\right] \qquad (69)$$

$$V_{p_{11}} = -\frac{\sin\phi}{4\pi pR^2}\left[\frac{e^{-pR}}{\lambda+1+\tau_p}\{\cos(\omega\tau - pR)\}\right] \qquad (70)$$

As in the case of clean fluid, in two phase fluid also, the solution for Ψ_{11} and $\Psi_{p_{11}}$ show that the fluctuations over the mean velocity due to the first convective correction consist of a traveling wave form in the radial direction coupled with an oblique wave form, both attenuating at a distance of order $\sqrt{\frac{2}{\omega}}$, fluctuations of the dusty fluid is slightly less than that of the clean fluid.

They are super imposed over an azimuthal wave form which persists even beyond the distance of order $\sqrt{\frac{2}{\omega}}$. In addition, the solution exhibits the influence of pure oscillations which is essentially due to the source. In both the expressions Ψ_{11} and $\Psi_{p_{11}}$, the azimuthal wave form is due to the combined effect of the source and the basic state; the traveling wave in the radial direction arises on account of the interaction with the diffusion state. The oblique wave form is due to the source, basic steady state and pure diffusion H_{10} and H_{p_0} and there is a phase difference of $\frac{\pi}{2}$ between the waves, in both the expressions H_{10} and H_{p_0}.

Collecting the coefficients of Ra after substituting for H_1, Ψ_1, H_{p_1} and Ψ_{p_1} in equations (56) and (58) and using the expressions for Ψ_0, Ψ_{p_0}, H_0 and H_{p_0} which occur in the basic steady state, We have the first convective correction to the temperature field as obtained from solution of equation:

$$\frac{\partial H_{11}}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_{01},H_{10})}{\partial(\phi,R)} + \frac{\partial(\Psi_{11},H_{00})}{\partial(\phi,R)}\right]$$
$$= \frac{1}{R^2}\frac{\partial}{\partial R}\left[R^2\frac{\partial H_{11}}{\partial R}\right] + \frac{1}{R^2\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_{11}}{\partial\phi}\right]$$
$$+ \frac{2f}{3\Lambda}(H_{p_{11}} - H_{11}) \qquad (71)$$

$$\frac{\partial H_{p_{11}}}{\partial\tau} + \frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_{p_{01}},H_{p_{11}})}{\partial(\phi,R)} + \frac{\partial(\Psi_{p_{11}},H_{p_{01}})}{\partial(\phi,R)}\right]$$
$$= -\frac{2}{3}\frac{1}{\gamma\Lambda}(H_{p_{11}} - H_{11}) \qquad (72)$$

in which the separation of variables can be achieved by setting

$$H_{11} = \frac{(1+i)p\cos\phi\, e^{i\omega\tau}g_1(\eta)}{16\pi^2} \quad \text{and}$$
$$H_{p_{11}} = \frac{(1+i)p\cos\phi\, e^{i\omega\tau}g_2(\eta)}{16\pi^2} \qquad (73)$$

Substituting (73) and using (51), (68) and (52) in (72) we have

$$g_2(\eta) = \frac{1}{1+\tau_p}g_1(\eta) \qquad (74)$$

Using (73) and (74) in (71), we have an ordinary differential equation for g_1:

$$\eta^2 g_1'' + 2\eta g_1' - (2+\eta^2)g_1 = -\frac{1}{\eta^3}\left[e^{-\eta}(\eta+1)(\eta^2-2) + 2\right] \qquad (75)$$

whose complete integral is derived to be

$$g_1(\eta) = \frac{c_1 e^\eta(\eta-1)}{\eta^2} + \frac{c_2 e^{-\eta}(\eta+1)}{\eta^2} + F(\eta) \qquad (76)$$

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where

$$F(\eta) = \frac{e^{\eta}(\eta - 1)}{2\eta^2} \left[\int\limits_{-\infty}^{2\eta} \frac{e^{-u}}{u} du - \Gamma + \frac{1}{4}\left[\int\limits_{-\infty}^{\eta} \frac{e^{-u}}{u} du - \Gamma \right] \right]$$

$$+ \frac{e^{-\eta}(\eta + 1)}{2\eta^2} \left[\frac{1}{4}\left\{ \overline{\int\limits_{-\infty}^{\eta} \frac{e^{-u}}{u} du} \right\} - \frac{\Gamma}{4} + \log \eta + \frac{1}{\eta} \right]$$

$$- \frac{1}{2\eta^3} \qquad (77)$$

Making use of boundary conditions, We obtain the expressions for H_{11} and $H_{p_{11}}$ as:

$$H_{11} = \frac{p \cos \phi}{64\pi^2}$$
$$\times \Big[\{\cos(\omega\tau + pR)F_1(\eta) + \sin(\omega\tau + pR)F_2(\eta)\} e^{pR}$$
$$+ \{\cos(\omega\tau - pR)F_3(\eta) + \sin(\omega\tau - pR)F_4(\eta)\} e^{-pR}$$
$$+ \sin \omega\tau F_5(\eta) \Big] \qquad (78)$$

$$H_{p_{11}} = \left[\frac{1}{(1 + \tau_p)} \right] H_{11} \qquad (79)$$

where

$$F_1(\eta) = \frac{1}{p^2 R^2} \Big[(2pR - 1)\{c(2\eta) - \log 2 - \Gamma\} - S(2\eta)$$
$$+ \frac{1}{4}(2pR - 1)\{c(\eta) - \Gamma\} - \frac{1}{4}S(\eta) \Big]$$

$$F_2(\eta) = -\frac{1}{p^2 R^2} \Big[\{c(2\eta) - \log 2 - \Gamma\} + (2pR - 1)S(2\eta)$$
$$+ \frac{1}{4}\{c(\eta) - \Gamma\} - \frac{1}{4}(2pR - 1)S(\eta) \Big]$$

$$F_3(\eta) = \frac{1}{p^2 R^2} \Big[\frac{\pi}{4} + \{\log(\sqrt{2}pR)\}$$
$$+ 1 + \frac{1}{4}(2pR + 1)\{c(\eta) - \Gamma\} - \frac{1}{4}S(\eta) \Big]$$

$$F_4(\eta) = \frac{1}{p^2 R^2} \Big[-\frac{\pi}{4} + \{\log(\sqrt{2}pR)\} + (1 + pR)$$
$$+ \frac{1}{4}\{c(\eta) - \Gamma\} + \frac{1}{4}(1 + 2pR)S(\eta) \Big]$$

$$F_5(\eta) = -\frac{1}{p^3 R^3}$$

$c(t) = $ R.P. $E_1(t)$, $S(t) = $ I.P. $E_1(t)$. R.P. stands for 'Real part of', I.P. for 'Imaginary part of $\Gamma = $ Euler's constant, 0.57721.

These solutions were also obtained by Hiremath [19]. Here, we find that H_{11} also exhibits (like Ψ_{11} and $\Psi_{p_{11}}$) the emergence of different wave forms, a few of which attenuating at a radial distance of order $\sqrt{\frac{2}{\omega}}$ and the remaining persist even beyond. However, unlike Ψ_{11} and $\Psi_{p_{11}}$, both H_{11} and $H_{p_{11}}$ do not exhibit influence of pure oscillation.

In order to avoid algebraic complexity, higher order corrections to the velocity and temperature fields of both the phases are not obtained.

C. Second order mean flow

The coefficients of ε^2 in (33), (35), (34) and (36) can be split into two parts, one dealing with the second order mean flow and other the unsteady second harmonic solution.

Or if,

$$\Psi_2 = \Psi_m(R, \phi) + \Psi_f(R, \phi, \tau);$$
$$H_2 = H_m(R, \phi) + H_f(R, \phi, \tau);$$
$$\Psi_{p_2} = \Psi_{p_m}(R, \phi) + \Psi_{pf}(R, \phi, \tau); \qquad (80)$$
$$H_{p_2} = H_{p_m}(R, \phi) + H_{pf}(R, \phi, \tau).$$

with similar expressions for U_2, V_2, U_{p_2} and V_{p_2} then, Ψ_m, H_m, Ψ_{p_m} and H_{p_m} are found to satisfy the equations

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left\{ \frac{\partial\Psi_m}{\partial\phi} + \lambda\frac{\partial\Psi_{p_m}}{\partial\phi} \right\} \right]$$
$$+ \frac{1}{\sin\phi}\left[\frac{\partial^2\Psi_m}{\partial R^2} + \lambda\frac{\partial^2\Psi_{p_m}}{\partial R^2} \right]$$
$$= Ra\left[\cos\phi\frac{\partial H_m}{\partial\phi} + R\sin\phi\frac{\partial H_m}{\partial R} \right] \qquad (81)$$

$$\frac{\partial\Psi_{p_m}}{\partial\phi} = \frac{1}{1 + \tau_p}\frac{\partial\Psi_m}{\partial\phi} \qquad (82)$$

$$\frac{1}{\sin\phi}\left[\frac{\partial(\Psi_0, H_m)}{\partial(\phi, R)} + \frac{\partial(\Psi_m, H_0)}{\partial(\phi, R)} \right.$$
$$+ \frac{1}{4}\left[\frac{\partial(\Psi_1, \overline{H_1})}{\partial(\phi, R)} + \frac{\partial(\overline{\Psi_1}, H_1)}{\partial(\phi, R)} \right] \Big]$$
$$= \frac{\partial}{\partial R}\left[R^2\frac{\partial H_m}{\partial R} \right] + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_m}{\partial\phi} \right]$$
$$+ \frac{2fR^2}{3\Lambda}[H_{p_m} - H_m] \qquad (83)$$

$$\frac{1}{R^2\sin\phi}\left[\frac{\partial(\Psi_{p_0}, H_{p_m})}{\partial(\phi, R)} + \frac{\partial(\Psi_{p_m}, H_{p_0})}{\partial(\phi, R)} \right.$$
$$+ \frac{1}{4}\left[\frac{\partial(\Psi_{p_1}, \overline{H_{p_1}})}{\partial(\phi, R)} + \frac{\partial(\overline{\Psi_{p_1}}, H_{p_1})}{\partial(\phi, R)} \right] \Big]$$
$$= -\frac{2}{3\gamma\Lambda}[H_{p_m} - H_{p_1}] \qquad (84)$$

where over bars denote complex conjugate of the functions below them.

Equations (81), (82), (83), and (84) are to be solved subject to the boundary conditions

$$U_m, \quad V_m, \ H_m, \ U_{p_m}, V_{p_m} \text{ and } H_{p_m} \to 0 \text{ as } R \to \infty$$
$$\frac{\partial U_m}{\partial\phi} = V_m = \frac{\partial H_m}{\partial\phi} = 0 \quad \text{at} \quad \theta = 0, \pi$$
$$\frac{\partial U_m}{\partial\phi} = V_m = \frac{\partial H_m}{\partial\phi} = 0 \quad \text{at} \quad \theta = 0, \pi$$
$$\frac{\partial U_{p_m}}{\partial\phi} = V_{p_m} = \frac{\partial H_{p_m}}{\partial\phi} = 0 \quad \text{at} \quad \theta = 0, \pi$$
$$\lim_{R \to 0}\left[-4\pi R^2\frac{\partial H_m}{\partial R} \right] = 0 \qquad (85)$$

It is interesting to note both in the case of fluid and particle phases that the higher order solutions contribute to the mean through the first harmonic.

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As seen earlier assuming

$$
\begin{aligned}
\Psi_m &= \Psi_{m0} + Ra\Psi_{m_1} + (Ra)^2\Psi_{m_2} + \cdots; \\
\Psi_{p_m} &= \Psi_{pm0} + Ra\Psi_{pm_1} + (Ra)^2\Psi_{pm_2} + \cdots; \\
H_m &= H_{m0} + RaH_{m_1} + (Ra)^2 H_{m_2} + \cdots; \\
H_{p_m} &= H_{pm0} + RaH_{pm_1} + (Ra)^2 H_{pm_2} + \cdots.
\end{aligned}
\tag{86}
$$

With similar expressions for U_m, V_m, U_{p_m} and V_{p_m} the first non-vanishing convective contributions to the second order mean in both the velocity and temperature fields are found from the solutions of

$$
\frac{1}{R^2}\frac{\partial}{\partial\phi}\left[\frac{1}{\sin\phi}\left\{\frac{\partial\Psi_{m_2}}{\partial\phi} + \lambda\frac{\partial\Psi_{pm_2}}{\partial\phi}\right\}\right]
$$
$$
+ \frac{1}{\sin\phi}\left[\frac{\partial^2\Psi_{m_2}}{\partial R^2} + \lambda\frac{\partial^2\Psi_{pm_2}}{\partial R^2}\right]
$$
$$
= \left[\cos\phi\frac{\partial H_{m_1}}{\partial\phi} + R\sin\phi\frac{\partial H_{m_1}}{\partial R}\right]
\tag{87}
$$

$$
\frac{\partial\Psi_{pm_2}}{\partial\phi} = \frac{1}{1+\tau_p}\frac{\partial\Psi_{m_2}}{\partial\phi}
\tag{88}
$$

From (83) we have

$$
\frac{1}{R^2}\left[\frac{\partial}{\partial R}\left(R^2\frac{\partial H_{m_1}}{\partial R}\right)\right] + \frac{1}{R^2\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_{m_1}}{\partial\phi}\right]
$$
$$
+ \frac{2f}{3\Lambda}\left[H_{pm_1} - H_{m_1}\right]
$$
$$
= \frac{\cos\phi\, e^{-pR}}{32\pi^2 p^2 R^5}\left[\frac{pR\cos pR - (1+pR)\sin pR}{1+\frac{\lambda}{1+\tau_p}}\right]
\tag{89}
$$

and

$$
\frac{2}{3\gamma\Lambda}\left[H_{m_1} - H_{pm_1}\right]
$$
$$
= \frac{\cos\phi\, e^{-pR}}{32\pi^2 p^2 R^3(1+\tau_p)}\left[\frac{pR\cos pR - (1+pR)\sin pR}{1+\frac{\lambda}{1+\tau_p}}\right]
\tag{90}
$$

Using (89) in (90), we have

$$
\left[\frac{\partial}{\partial R}\left(R^2\frac{\partial H_{m_1}}{\partial R}\right)\right] + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left[\sin\phi\frac{\partial H_{m_1}}{\partial\phi}\right]
$$
$$
= \frac{\cos\phi\, e^{-pR}}{32\pi^2 p^2 R^3}\left[\frac{pR\cos pR - (1+pR)\sin pR}{1+\frac{\lambda}{1+\tau_p}}\right]
$$
$$
\times\left[1 + R^2\frac{\gamma f}{1+\tau_p}\right]
\tag{91}
$$

Setting $H_{m_1} = \frac{p\cos\phi G(r)}{32\pi^2}$ and $\Psi_{m_2} = \frac{\sin\phi\sin 2\phi H(r)}{64\pi^2 p}$ where $r = pR$, we obtain after separation of variables and applying boundary conditions

$$
r^2 G''(r) + 2rG'(r) - 2G(r)
$$
$$
= \frac{e^{-r}}{r^3}\left[\frac{r(\cos r - \sin r) - \sin r}{1+\frac{\lambda}{1+\tau_p}}\right]
\tag{92}
$$

$$
r^2 H''(r) - 6H(r) = \frac{1}{1+\frac{\lambda}{1+\tau_p}}\left[r^3 G'(r) - r^2 G(r)\right]
\tag{93}
$$

We solve equations (92) and (93) by assuming power series expansion for both H and G in terms of r in the vicinity of the origin as it is important region of thermal activity near the point heat source and we obtain the expressions for H_{m_1}, H_{pm_1}, Ψ_{m_2} and Ψ_{pm_2} as follows:

$$
H_{m_1} = \frac{p\cos\phi}{32\pi^2}\cdot\frac{1}{1+\frac{\lambda}{1+\tau_p}}
$$
$$
\times\left[\frac{1}{2pR} - \frac{1}{3} - \frac{p^2 R^2}{30} + \frac{p^3 R^3}{180} - \frac{p^4 R^4}{1890} + o(p^6 R^6)\right]
\tag{94}
$$

$$
H_{pm_1} = \frac{p\cos\phi}{32\pi^2}\cdot\frac{1}{\lambda + 1 + \tau_p}
$$
$$
\times\left[\frac{1}{2pR} - \frac{1}{3} - \frac{p^2 R^2}{30} + \frac{p^3 R^3}{180} - \frac{p^4 R^4}{1890} + o(p^6 R^6)\right]
\tag{95}
$$

$$
\Psi_{m_2} = \frac{pR\sin\phi\sin 2\phi}{384\pi^2 p}\cdot\frac{1}{1+\frac{\lambda}{1+\tau_p}}
$$
$$
\times\left[1 - \frac{pR}{2} - \frac{p^3 R^3}{30} + \frac{p^4 R^4}{210} - \frac{p^5 R^5}{2520} + o(p^7 R^7)\right]
\tag{96}
$$

$$
\Psi_{pm_2} = \frac{1}{1+\tau_p}\Psi_{m_2}
\tag{97}
$$

Equations (91) and (92) yield

$$
U_{m_2} = \frac{1}{1+\frac{\lambda}{1+\tau_p}}\cdot\frac{p(1+3\cos 2\phi)}{384\pi^2 pR}
$$
$$
\times\left[1 - \frac{pR}{2} - \frac{p^3 R^3}{30} + \frac{p^4 R^4}{210} - \frac{p^5 R^5}{2520} + o(p^7 R^7)\right]
\tag{98}
$$

$$
U_{pm_2} = \frac{1}{\lambda + 1 + \tau_p}\cdot\frac{p(1+3\cos 2\phi)}{384\pi^2 pR}
$$
$$
\times\left[1 - \frac{pR}{2} - \frac{p^3 R^3}{30} + \frac{p^4 R^4}{210} - \frac{p^5 R^5}{2520} + o(p^7 R^7)\right]
\tag{99}
$$

$$
V_{m_2} = \frac{1}{1+\frac{\lambda}{1+\tau_p}}\cdot\frac{(-p)\sin 2\phi}{384\pi^2 pR}
$$
$$
\times\left[1 - pR - \frac{2p^3 R^3}{15} + \frac{p^4 R^4}{42} - \frac{p^5 R^5}{420} + o(p^7 R^7)\right]
\tag{100}
$$

$$
V_{pm_2} = \frac{1}{\lambda + 1 + \tau_p}\cdot\frac{(-p)\sin 2\phi}{384\pi^2 pR}
$$
$$
\times\left[1 - pR - \frac{2p^3 R^3}{15} + \frac{p^4 R^4}{42} - \frac{p^5 R^5}{420} + o(p^7 R^7)\right]
\tag{101}
$$

The second order corrections to the mean velocity components for both fluid and particle phases are depicted in Fig. 2. From Figs. 2 and 3 we find that as radial velocity increases, azimuthal velocity decreases and this is true for both the phases and near the vicinity of the origin, dust velocity is slightly greater than the fluid velocity. This may be due to the presence of heat source at origin. From Figs. 4 and 5 we find that an increase in τ_p causes an increase in radial velocity and decrease in azimuthal velocity. It is interesting to note that near the vicinity of the plate, the dust concentration has no effect on the mean velocity, where as the increase in relaxation parameter of dust particles slightly reduces the
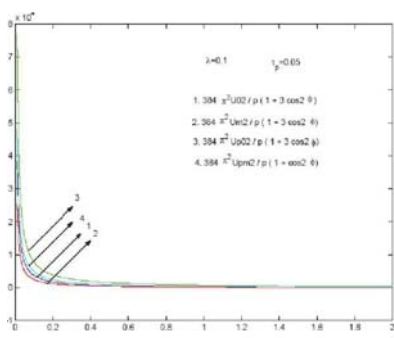
Fig. 2.   Second order mean velocity components U_{02}, U_{m_2}, $U_{p_{02}}$ and $U_{p_{m2}}$.
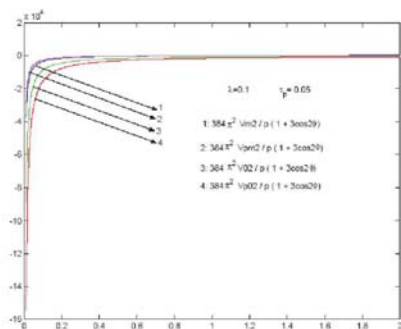

Fig. 3.   Second order mean velocity components V_{m_2}, $V_{p_{m2}}$, V_{02} and $V_{p_{02}}$.
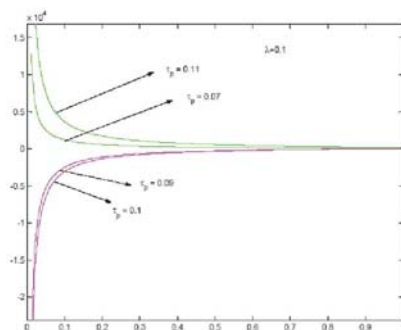

Fig. 4.   Second order mean velocity components for different values of τ_p-particle phase ($U_{p_{m2}}$, $V_{p_{m2}}$).
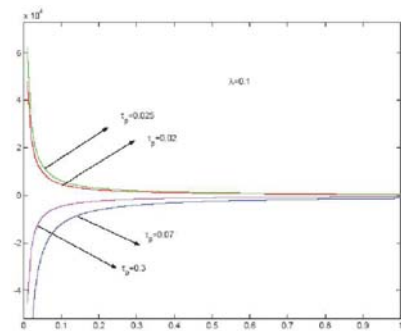

Fig. 5.   Second order mean velocity components for different values of τ_p-particle phase ($U_{p_{02}}$, $V_{p_{02}}$).

azimuthal velocity and increases the radial velocity of the fluid and dust.

IV. Conclusions

The heat transfer and free convection effects in porous dusty medium due to the presence of point heat source is studied. Series expansions in the Rayleigh number have been derived to study the velocity and temperature fields. Expressions for second order mean flow in temperature and velocity fields for both phases have been derived assuming Darcy's law. (1)

1) Evolution of different wave patterns are observed in both the fluid and particle phases in fluctuating part.
2) In both fluid and particle phases as the radial velocity increases, azimuthal velocity decreases.
3) An increase in relaxation parameter causes increase in radial velocity and decrease in azimuthal velocity for both the phases.
4) At the vicinity of the origin, where the heat source is present, the second order mean flow is influenced only by relaxation time of dust particles and not by dust concentration.

Acknowledgement

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