

Production Throughput Modeling under Five Uncertain Variables using Bayesian Inference

Amir Azizi, Amir Yazid B. Ali, Loh Wei Ping

Abstract—Throughput is an important measure of performance of production system. Analyzing and modeling of production throughput is complex in today's dynamic production systems due to uncertainties of production system. The main reasons are that uncertainties are materialized when the production line faces changes in setup time, machinery break down, lead time of manufacturing, and scraps. Besides, demand changes are fluctuating from time to time for each product type. These uncertainties affect the production performance. This paper proposes Bayesian inference for throughput modeling under five production uncertainties. Bayesian model utilized prior distributions related to previous information about the uncertainties where likelihood distributions are associated to the observed data. Gibbs sampling algorithm as the robust procedure of Monte Carlo Markov chain was employed for sampling unknown parameters and estimating the posterior mean of uncertainties. The Bayesian model was validated with respect to convergence and efficiency of its outputs. The results presented that the proposed Bayesian models were capable to predict the production throughput with accuracy of 98.3%.

Keywords— Bayesian inference, Uncertainty modeling, Monte Carlo Markov chain, Gibbs sampling, Production throughput

I. INTRODUCTION

THROUGHPUT analysis is an important and efficient way to control and match the production output with the ordered demands. Mostly the throughput of production line does not meet the required demand on the shop floor of production especially in presence of product mix and multi stages of production line. Many variables can affect on the throughput degradation of each stage for example break down of machine, lead time of manufacturing, and scrap, which caused maybe by error of machines, material, and workers. Changes in demand in terms of type and volume also affect the throughput because of changing of customer needs and interests. On the other hand, the company requires having innovation on design of new products in order to survive in today's competitive manufacturing world.

[10] Emphasized to have the right demand quantity estimating for surviving in a constantly fluctuating business environment. Managing random variables of production by making a robust estimating maximize the profitability. Production line uncertainty is taken to attention recently because of needs to handle the uncertainty using development of new methodology and computational approaches.

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Study on production uncertainty is an opportunity for new research and development [6]. Analyzing and estimating throughput is being crucial because the efficiency of the production system is usually measured using throughput [14]. Managing production in terms of supply and demand requires forecasting of both time delivery and quantity [10], [11]. Different strategies and approaches are proposed to overcome the production uncertainties. However there is not a formal strategy or standard approach [10]. It is still under development and optimization. The empirical study on the real production line and actual data from industry are required overcoming to the production uncertainties.

Discrete event simulation and stochastic planning are among the famous approaches. Simulation can be applied for any production system to estimate the throughput, however it is not robust and this becomes a computational chore when the number of alternatives to be examined is large [14]. Stochastic planning indeed is difficult to solve and impractical because it uses assumed scenarios. The main problem with the stochastic planning is how to make sure that the assumed scenarios will be exactly observed in future. This paper offers a robust procedure for modeling the throughput under popular production uncertainties. This paper is organized in five sections. Section II presents literature review. Section III presents the methodology of Bayesian regression modeling. Section IV shows the results and Bayesian model. Section VI shows the conclusion of this paper.

II. LITERATURE REVIEW

Throughput is considered for analysis and modeling as an important measure of production line performance [15]-[17]. [19] Provided a review paper of models under uncertainty for production planning. He highlighted that the models for production planning, which consider the uncertainty can make superior planning decisions compared to those models that do not present for the uncertainty. On the other hand, [22] have shown using simulation that ignoring uncertainty sources lead to wrong decisions. [3] categorized uncertainties into two groups: (1) environmental uncertainty and (2) system Uncertainty. Before 1990 focusing on uncertainty was more on environmental uncertainty [2]. Investigating about uncertainties on a production line is launched by [1]. [21] compiled all the uncertain factors through different sources, which are system uncertainty, lead time uncertainty, environmental uncertainty, supply uncertainty, operation yield uncertainty, interrelationship between levels, demand uncertainty, probabilistic market demand and product sales price, capacity, breakdown, changing product mix situation, labour hiring and lay-offs, quantity uncertainty, cost parameters, and quality. Many papers worked on throughput analysis using conventional approaches such as simulation and analytical methods [15].

Simulation method and approximation algorithm are applied for analyzing throughput under uncertainty such as unreliable machine and random processing times, for example studied by [23] & [24]. [18] provided an analytical equation for the general case where there are two workstations in a serial production line. In his model, the workstations have unequal processing time, downtime, and buffer size, while [15] considered a serial production line including two workstations with same speed and buffer size. [18] and [17] demonstrated that the processing time and down time affect the throughput or production volume. [22] examined the effects of three uncertainties namely demand, manufacturing delay, and capacity scalability delay. They found that manufacturing delay has highest impact. A recent survey have been performed on material shortage, labor shortage, machine shortage, and scrap to show the association of these uncertainties on the product tardy delivery through analysis of variance, correlation analysis and cluster analysis [20].

[4] proposed to use buffer to manage uncertainty in production system. However they did not make a robust decision by forecasting based on relationship of uncertainties and throughput. Later, [5] studied on supply-demand mismatches. They believe that the long delivery time of throughput to supplier caused because of lead time uncertainty in production system, which leads to lost sales. However in their proposed methodology to manage lead time uncertainty, they did not consider other production uncertainties. And also the rate of demand is assumed to be constant in their work. Approximate method also is used for forecasting throughput, [14] presented an analytical algorithm to analyze and predict the production throughput under unbalanced workstations, where operation times of stations are random. A hybrid combination of autoregressive integrated moving average models and neural network for demand forecasting in supply chain management is presented by [11], [12]. They developed a replenishment system for a Chilean supermarket. The linear regression models for strategy, environmental uncertainty and performance measurement in New Zealand manufacturing firms are formulated by [7].

Recently, a model using ANFIS has been developed for production throughput under uncertain conditions [40]. A data mining approach is utilized for cycle time prediction by [13]. A panel or longitudinal data sets for uncertain demand and price have been considered to evaluate the alternative capacity strategies using simulation [25]. Recently [26] proposed an autoregressive moving average model for throughput bottleneck prediction of a serial production line under production blockage and starvation times. Other new attempts have been carried out using ARIMA or other methods combined with RAIMA to develop the forecasting method in manufacturing area [27], [28]. ARIMA approach was proposed for modelling production uncertainties in a serial production line [38], [39].

Stochastic variables of production lines are studied separately for example on breakdown by [16] and on processing time by [14]. This study is considering more variability into the consideration. Variability can be measured by the coefficient of variation [14]. Therefore the economic uncertainty needs the mathematical Models [10].

III. BAYESIAN INFERENCE REGRESSION MODELING

Bayesian inference use distribution-based approach where the prior probabilities were utilized to quantify uncertainty regarding the occurrences of events. Bayesian inference algorithm is illustrated in Fig. 1.

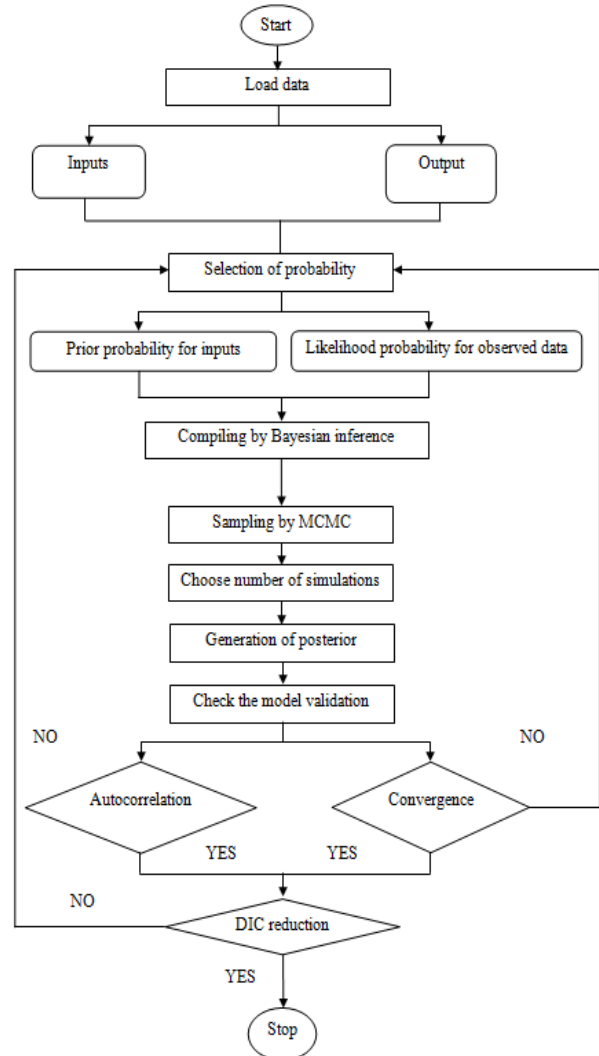


Fig. 1 Flow diagram of computations in Bayesian inference

A. Load data

The data observed for input uncertainties and throughput of production was translated to the BUGS language by inserting them into the R software. The translated data was loaded by importing them to the model programmed in BUGS. A list from the vector of output and each uncertain variable was developed by using a command for reading the data.

B. Selection of probability

Problem formulation with predefined probability levels explicitly considered the stochastic property of the uncertainties. The selection of probability was divided into the prior distributions of inputs and the likelihood distribution for observed data. These two probability distributions were two main input components of Bayesian inference.

C. Prior distribution

Prior distribution refers to the historical behavior of the inputs. Its selection for inputs is done before observing the data. This behavior can be elicited from the experts [29]. The distribution of prior usually is defined in question by the normal distribution with mean of zero and low variance. Unfortunately, as the propagation of uncertainty may change with time, the prior information on the inputs cannot assume true. Therefore, the determination of prior probability distribution is done by the trial and error method.

BUGS can modify the approximate prior by considering the sum of Gaussians centered on each sample generated. The selection of prior probability distribution to express the uncertainty propagation of inputs can be examined with different distribution to see which one is more accurate based on lower error generated.

One way to compare the models with different probability distributions is to use a criterion based on trade-off between the fit of data to the model and the corresponding complexity of the model. A Bayesian model [30] was proposed to compare criterion based on deviance information criterion (DIC). For each uncertain variable, three popular probability distributions were examined: uniform, exponential and normal. The posterior probability distribution function of the model parameters was computed from the defined prior probability distribution function. The best prior probability distribution was based on lower DIC comparison.

D. Likelihood

The purpose of selecting likelihood probability distribution is to identify the best probability function which can fit the observed data. The likelihood function for production throughput was computed using the conditional distributions given the data observed in a tile industry. The probability distributions of normal, exponential, Weibull, and logistic function were tested. The procedure was to maximize the likelihood to fit the data better. Dependencies values between variables were also identified through the conditional probabilities. The predicted values were gained through the equations (1) and (2).

$$p(\hat{y}|y) = \int p(\hat{y}|x) p(x|y) dx \quad (1)$$

$$\Rightarrow \text{for normal distribution} = p(\hat{y}|y) = \int \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\hat{y}-x)^2} \times \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \quad (2)$$

where

\hat{y} = future observation,
 y = observed at given x .

E. Compilation

The compilation process utilizes both prior and likelihood. It synchronizes the information about the uncertainty before observation and the behavior of data after observation. The compiling is to multiply the prior distribution and likelihood probability.

F. Sampling

Various samplings were computed from the joint posterior distribution. Markov chain method is used to obtain sample from full conditional distributions. A vector of unknown parameter was considered to consist of n subcomponents. Then the sampling started choosing the value of unknown parameters from the conditional distribution to find the best value of the beta for the posterior distribution, where the posterior distribution was maximized. Gibbs sampling algorithm was utilized because it is the robust procedure of MCMC. The Gibbs sampling algorithm approximated the posterior distribution function by making random draws from the probability distributions of the input uncertainties and evaluating the model at the resulting values.

G. Quantity of simulations

Four simulation runs of 1000, 5000, 8000, and 10000 for drawing samples were examined to test the model based on DIC. Simulation started from 1000 and was increased until it reached convergence. The amount optimal simulation run was determined by the higher level of convergence and the lower value of DIC.

H. Generation of posterior

The posterior is the product of observation probability (likelihood) and previous information (prior). Different samplings were performed to generate posterior of unknown parameters. Each kernel of the generated sample had weight in term of closeness to the posterior. Kernel is a function of the sample variance. Closer kernels dominated the posterior. Final posterior was obtained by weight-normalizing of sum of kernel products, which had the best posterior mean and variance.

Fig. 2 shows a construction of Bayesian black box diagram. A processor of Bayesian inference engine including rules of probabilities and Bayesian theory to derive the posterior mean and variance of the model is at the centre of the diagram.

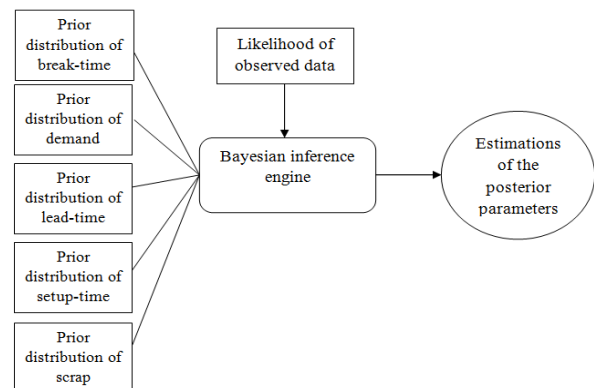


Fig. 2 The construction of Bayesian inference model

Two different sets of prior uncertainty were assigned for each uncertain variable. Two competing models were generated into two chains denoted by M1 and M2 as shown in equation (3). Bayesian inference engine used the Bayes factor (BF) to analyze the model proposed as shown in equations (4) and (5). The data observed for each uncertainty was denoted by X .

$$BF(x) = \frac{\pi(M_1 | x)p(M_1)}{\pi(M_2 | x)p(M_2)} \quad (3)$$

$$M_1: f_1(x | \beta_1) \text{ and } M_2: f_2(x | \beta_2) \quad (4)$$

where

$$\frac{\pi(M_1 | x)}{\pi(M_2 | x)} = \frac{p(M_1)/p(x)}{p(M_2)/p(x)} \times \frac{\int f_1(x | \beta_1)p_1(\beta_1)d\beta_1}{\int f_2(x | \beta_2)p_2(\beta_2)d\beta_2} \quad (5)$$

When the M_1 is as the null model, the possibilities of BF results are as follows.

If $BF(x) \geq 1 \Rightarrow M_1$ supported,

If $1 > BF(x) \geq 10^{-1/2} \Rightarrow$ minimal evidence faced for M_1 ,

If $10^{-1/2} > BF(x) \geq 10^{-1} \Rightarrow$ substantial evidence faced for M_1 ,

If $10^{-1} > BF(x) \geq 10^{-2} \Rightarrow$ strong evidence faced for M_1 ,

If $10^{-2} > BF(x) \Rightarrow$ decisive evidence faced for M_1 .

The error of Monte Carlo (MC) for sampling procedures was calculated for each uncertain parameter by equation (6).

$$MC \text{ error} = \frac{SD}{\sqrt{\text{Number of iterations}}} \quad (6)$$

I. Check the model validation

The model validation was verified through two ways of checking. First checking was by visual inspection of trace/history plots to see if the model is convergence. The model convergence was achieved when the chains were overlapping. The second way of checking was to check the autocorrelation. The convergence graphically presents the distribution of uncertainty. Gelman Rubin statistic (GRS) showed the convergence ratio [31]. The autocorrelation is defined between zero and one. A slow convergence shows the high autocorrelation, indicating validity of model.

IV. RESULTS

A. Model programmed in BUGS

Table I presents the BUGS model expressions. The sign ~ indicates a stochastic relationship, where Tau =1/variance showed precision level. The c function combines objects into a vector, where the variable x was collected by different values that were measured in different period of time.

Expression	Type	Usage
dnorm	Normal distribution	$x \sim \text{dnorm}(\mu, \tau)$
c	Vector of data set	$x = c(x_1, x_2, \dots, x_n)$

B. Probability distribution test

Four popular probability distributions including normal, Weibull, logistic, and exponential were tested. Fig. 3 shows the normal distribution is the best fit for production throughput and Fig. 4 presents the summary of the normal distribution function.

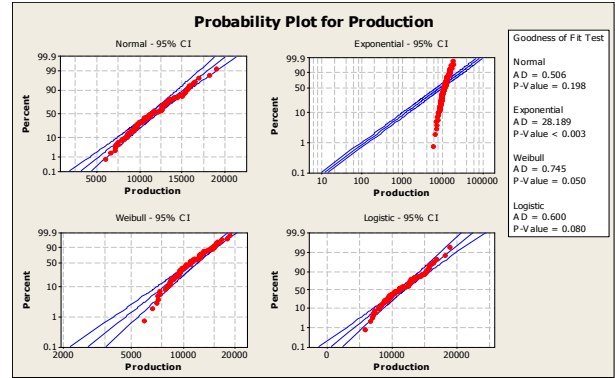


Fig. 3 Testing four popular probability distributions

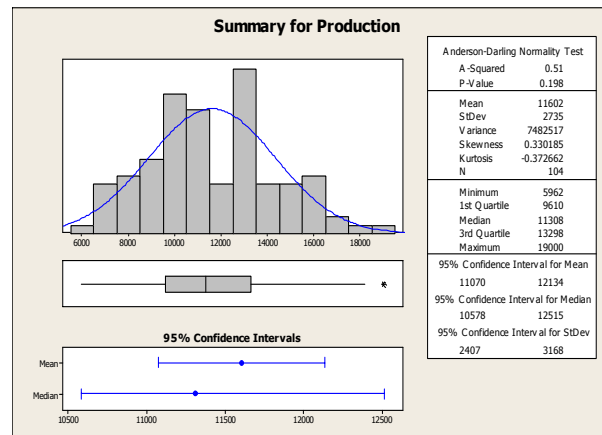


Fig. 4 Anderson-Darling normality test

C. Checking the programmed model

After programming, the model was checked for any completeness and consistency with the data. The initial values were generated by sampling from the prior. The model programmed was proven syntactically correct and compiled.

D. Convergence diagnostics test

Computational results of the lowest MAPE were selected in this section for the Bayesian model. The convergence diagnostics were checked through two chains results. The convergence was achieved because both chains overlapped each other, according to [32]. The dynamic race plots of the stochastic parameters with 10,000 iterations were done to check the convergence on 95% credible interval. Fig. 5 graphically shows the results.

DIC is the summation of goodness of fit and complexity. Deviance is the average of the log likelihoods calculated at the end of iteration in Gibbs Sampler. The definition of deviance is $-2 \times \log(\text{likelihood})$. Likelihood is defined as $p(y|\theta)$, where y comprises all stochastic parameters given values and theta comprises the stochastic parents of y - 'stochastic parents' are the stochastic parameters upon which the distribution of y depends, when collapsing over all logical relationships.

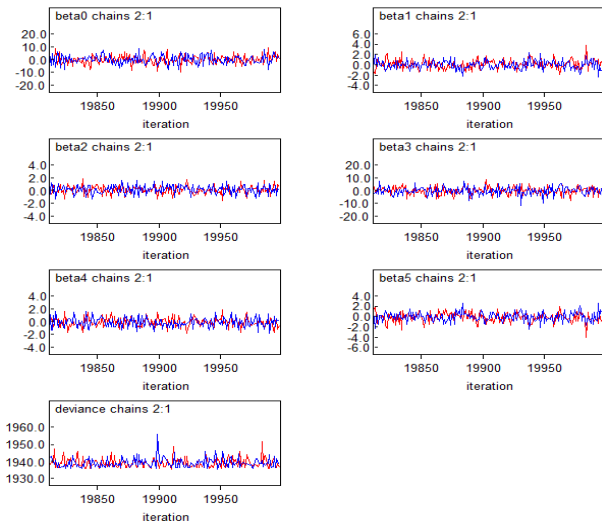


Fig. 5 Dynamic trace plots of uncertain parameters

E. Kernel density

Fig. 6 shows the value of Kernel density for each stochastic parameter was performed on 10000 samples. The diagrams indicated smoothed kernel density estimate. The trends indicated the posterior distribution of each stochastic parameter is normal like prior distribution, thus proving the estimations were robust and logical.

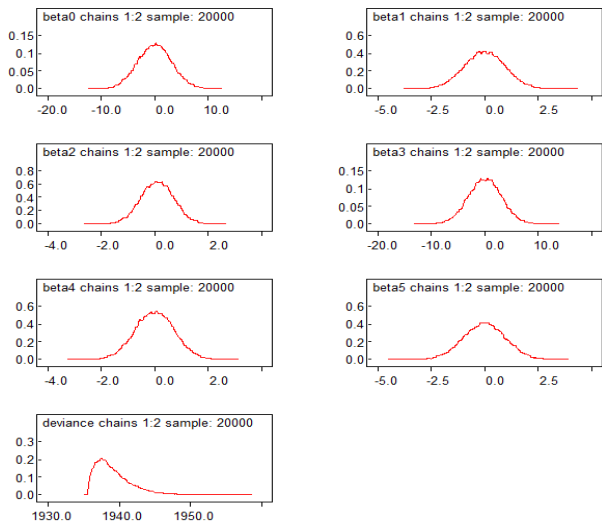


Fig. 6 Kernel density of the uncertain parameters

F. Running quartiles

Running quartiles plot out the running was done for mean with running 95% confidence intervals where 10000 iterations were used. Results are presented in Fig. 7.

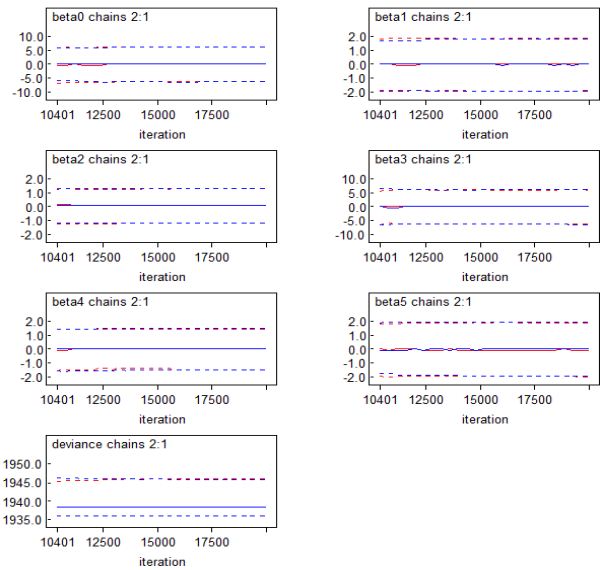


Fig. 7 Running mean of the uncertain parameters

G. Autocorrelation function

The autocorrelation function for the chain of each parameter indicated the dimensions of the posterior distribution were mixing slowly before 20 lags in each case. Slow mixing is often associated with high posterior correlations between parameters.

H. Gelman Rubin statistics

Gelman Rubin statistic (GRS) was performed for all stochastic parameters, which were modified by [32] in equation (7). The idea was to generate the multiple chains starting at over dispersed initial values, and assesses the convergence by comparing within-chain and between-chain variability over the second half of those chains.

$$GRS = A / W \tag{7}$$

Where

A= width of the empirical credible interval based on samples pooled together (2 chains × 10000 iterations).

W= width average of the intervals across the two chains

The GRS is to average the interval widths (shown in red color). It should be 1 if the starting values are suitably over dispersed and the convergence is approached. The blue and green interval lines should be approximately stabilized to constant value (not necessarily 1). It is proven and shown for all five stochastic parameters in Fig. 8.

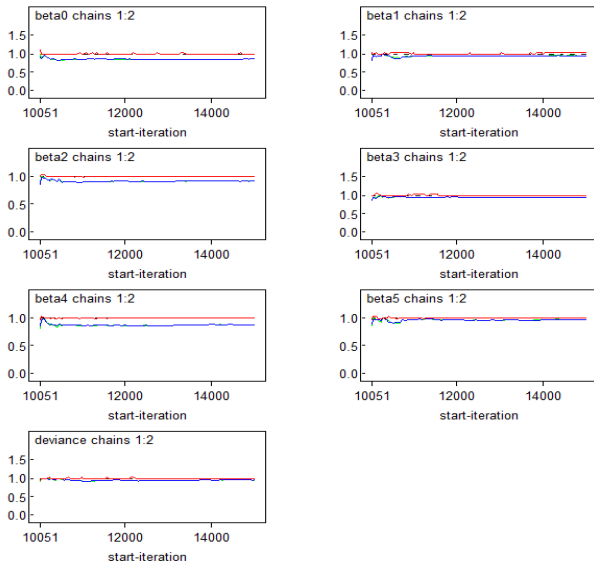


Fig. 8 Gelman Rubin statistic for the uncertain parameters

Where

Green = width of 80% intervals of pooled chains: should be stable

Blue = average width of 80% intervals for chains: should be stable

Red = ratio of pooled/within: should be near 1

I. Box plot of posterior

Box plot of posterior efficiency distributions are presented in Fig.9. The calculated baseline value was 11595.7809089724.

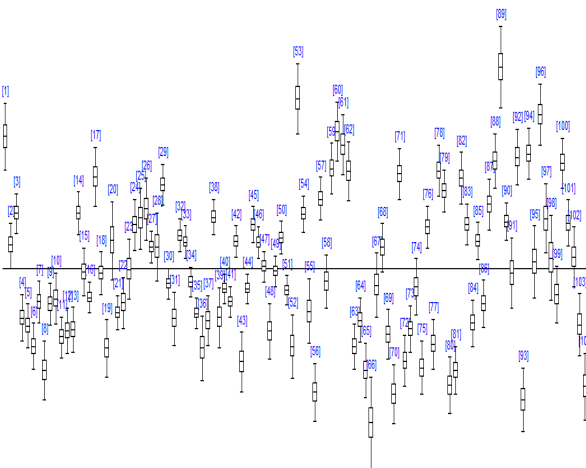


Fig. 9 Box plot of posterior efficiency distributions

J. Model fit

Fitted values were compared with actual values for production output, breakdown, demand, lead time, setup time, and scrap in 95% interval. The results showed production throughput and demand had similar upward trend while breakdown time, lead time, set up time, and scrap had similar downward trend. Fig.10 shows comparison between fitted values to actual value for production throughput, while Fig.11 presents the similar comparison for breakdown time.

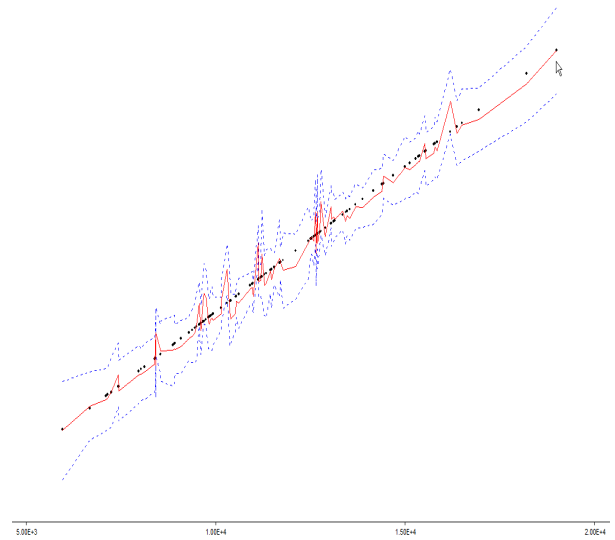


Fig. 10 Fitted value compare with actual values over production throughput observed with 95 % interval

Where

Red = posterior mean of μ_i ,

Blue = 95% interval,

Black dot = observed data

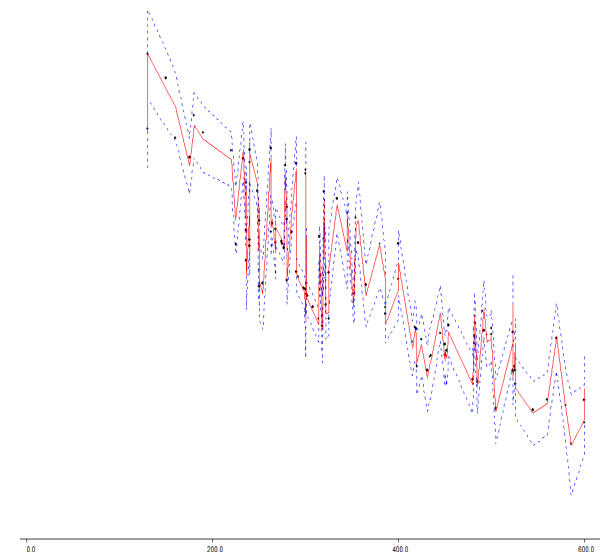


Fig. 11 Fitted value compare with actual values over breakdown time observed with 95 % interval

K. Posterior estimates

The final set of posterior estimates using Gibbs sampling in 95% credible interval was summarized in Table 2. The percentiles of 2.5% and 97.5% of posterior estimates produce an interval, which the parameter lies with probability of 0.95.

Coefficient	mean	Std. Dev.	MC error	median
β_0	0.01343	3.179	0.0242	0.02376
β_1	-0.0849	2.896	0.01872	-0.1016
β_2	0.9585	0.1596	0.001056	0.958
β_3	0.1268	0.6618	0.004444	0.1246

β_4	-0.0458	3.156	0.02213	-0.0614
β_5	-0.1481	0.7179	0.005325	-0.1474
Deviance	1939.0	2.383	0.01624	1939.0

The value of MC error shows an estimate of (σ / \sqrt{N}) . The batch means method outlined by [31] was used to estimate σ .

Finally, the Bayesian model is formulated as presented in equation (8).

$$P(t) \sim 0.01343 - 0.0849 B(t) + 0.9585 D(t) + 0.1268 L(t) - 0.04589 Se(t) - 0.1481 S(t) \quad (8)$$

V. CONCLUSION

This paper modeled the uncertain variables of a serial tile production line consist of demand, break-time, scrap, and lead-time. The contribution of this paper was to consider more uncertainties and propose Bayesian inference regression to model the five uncertain variables with the production throughput. The proposed model can be used to predict the production throughput efficiently, and presents the mathematical relationship between the main production uncertainties and throughput. It provides quick "what-if" comparisons. Other types of production systems and industries are recommended for future studies. The best simulations iterations of MCMC were 10000 and the best prior distributions for stochastic variables were normal distributions for the Bayesian model. The second model proposed was based on Bayesian inference. This approach utilized the prior knowledge on uncertainties and existing information based on data analysis of throughput and uncertainties. The robust Gibbs sampling is applied for MCMC to produces acceptable knowledge on prior events when analytical solutions were unavailable. The Bayesian model results generated the posterior information on propagation of uncertainties and relationship between them and the throughput with 95% credible interval and accuracy of 98.3%.

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