Design of Stable IIR Digital Filters with Specified Group Delay Errors

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Abstract—The design problem of Infinite Impulse Response (IIR) digital filters is usually expressed as the minimization problem of the complex magnitude error that includes both the magnitude and phase information. However, the group delay of the filter obtained by solving such design problem may be far from the desired group delay. In this paper, we propose a design method of stable IIR digital filters with prespecified maximum group delay errors. In the proposed method, the approximation problems of the magnitude-phase and group delay are separately defined, and these two approximation problems are alternately solved using successive projections. As a result, the proposed method can design the IIR filters that satisfy the prespecified allowable errors for not only the complex magnitude but also the group delay by alternately executing the coefficient update for the magnitude-phase and the group delay approximation. The usefulness of the proposed method is verified through some examples.

Keywords—Filter design, Group delay approximation, Stable IIR filters, Successive projection method.

I. Introduction

Finite-impulse response (FIR) digital filters find many applications in image processing, waveform transmission, etc. in which phase distortion becomes a problem, because they can easily realize an exactly linear phase characteristic and are always stable [1]-[4]. However, since the resulting delay at the output of the exactly linear phase FIR filters is half of the filter order, its group delay may become unacceptably large when high-order filters or narrow transition bands are required. On the other hand, it is known that Infinite Impulse Response (IIR) digital filters can realize about the same magnitude response by the lower order compared with the FIR filters. Hence, in order to process a signal processing with high-speed and with high-precision, it is very important to design the IIR filters.

The design problem of the IIR filters is usually expressed as the approximation problem of the magnitude and the phase responses [7]-[12]. However, the group delay response of the filter obtained by those methods tends to become relatively large, especially in the vicinity of the band-edge. This is because that the complex magnitude error, which include both the magnitude and phase responses, is minimized instead of the group delay error. In general, an allpass filter is used to equalize the phase response to make the group delay approximately constant in the interested bandwidth. However, the use of the allpass filter is not necessarily a good policy because the filter coefficients are redundant. Therefore, it is desirable to realize the filter that has an equalized group delay without using the allpass filter. This motivates the investigation on designing the IIR filters with prespecified maximum group delay errors by directly approximating the group delay response.

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By the way, it is desirable to have a unified approach that is able to design various types of digital filters. Recently, semi-definite programming (SDP) has been widely employed designing various types of FIR and IIR filters [6],[12]-[14]. However, because the design method based on the SDP needs to solve a large matrix, it often causes a problem that personal computers cannot be used. On the other hand, the successive projection method (SP) proposed by A. A. -Taleb et. al. [3], which is one of convex projection algorithm, is a simple iterative approximation method, and the amount of computing memory is very small. The SP method also has been applied to the design problem of various types of the filters such as one- and two-dimentional IIR filters, minimum phase FIR filters, the filters with time- and frequency-domain constraints, and so on [3],[15]-[19].That is, the applicability of the SP method is wider than that of the SDP algorithm. Moreover, it is known that the filters designed by the SP method have better filter characteristics than those by other design methods.

In [18], we presented a design method of stable IIR filters using the SP method. However, since this method is also formulated as the approximation problem of the magnitude-phase responses, the group delay response of the filter obtained tends to become a large. So, in this paper, we propose an improved method to design the stable IIR filters with prespecified maximum group delay errors using the SP method. In our proposed method, the approximation problems of the magnitude-phase and the group delay responses are defined separately on the different dimension, and these two approximation problems are solved alternately using the SP method. As a result, the proposed method allows the direct approximation of the group delay response, and it can restrict the group delay response within the preselected allowable errors. The usefulness of the proposed method is verified through some examples.

This paper is organized as follows: in section 2, the iterative algorithm for the approximation of the magnitude-phase responses using the SP method [18] is described. The iterative algorithm to directly approximate the group delay response are described in Section 3. Several design examples are given in Section 4. Conclusions of this work are drawn in Section 5.

II. Magnitude-Phase Approximation [18]

Let the transfer function of an infinite impulse response (IIR) digital filter with numerator degree m and denominator degree n be

$$H(z)=\frac{N(z)}{D(z)}=\frac{\sum_{i=0}^{m} b_i z^{-i}}{\sum_{i=0}^{n} a_i z^{-i}}, \tag{1}$$

where a_i and b_i are the real-valued filter coefficients and $a_0 = 1$. Then, the frequency response $H(\omega)$ of $H(z)$ can be given by

$$H(\omega)=\frac{N(\omega)}{D(\omega)}=\frac{\sum_{i=0}^{m} b_i e^{-ji\omega}}{\sum_{i=0}^{n} a_i e^{-ji\omega}}=|H(\omega)|\, e^{-j\tau(\omega)\omega}, \tag{2}$$

where ω is the normalized angular frequency.

Now, we define a desired frequency response $H_d(\omega)$ as

$$H_d(\omega)=A(\omega)\, e^{-j\tau_d(\omega)\omega}, \tag{3}$$

where $A(\omega)$ is the desired magnitude response and $\tau_d(\omega)$ is the desired group delay response. Then, the filter design problem considered here is to find the filter coefficients a_i and b_i satisfying the following complex magnitude error:

$$|H_d(\omega)-H(\omega)|\le\lambda(\omega) \tag{4}$$

for $\omega\in\Omega_H$, where $\lambda(\omega)$ is the positive maximum allowable deviation from the desired frequency response $H_d(\omega)$ and Ω_H is the interest region, which usually consists of the passband and stopband.

The SP method is an iterative approximation technique. Using the technique presented in [18], the error function $E_H^{[k+1]}(\omega)$ in the $(k+1)$th iteration step can be written as

$$E_H^{[k+1]}(\omega)=\frac{Q^{[k+1]}(\omega)}{R^{[k]}(\omega)}, \tag{5}$$

where

$$\begin{aligned}Q^{[k+1]}(\omega)=A(\omega)\sum_{i=0}^{n} a_i\cos\left(\tau_d(\omega)\omega+i\omega-2\pi t_n^{[k+1]}(\omega)\right)\\ -\sum_{i=0}^{m} b_i\cos\left(i\omega-2\pi t_n^{[k+1]}(\omega)\right)\end{aligned} \tag{6}$$

$$t_n^{[k+1]}(\omega)=-\frac{1}{2\pi}\tan^{-1}\frac{y_n^{[k+1]}(\omega)}{x_n^{[k+1]}(\omega)} \tag{7}$$

$$\begin{aligned}x_n^{[k+1]}(\omega)=A(\omega)\sum_{i=0}^{n} a_i^{[k+1]}\cos(\tau_d(\omega)\omega+i\omega)\\ -\sum_{i=0}^{m} b_i^{[k+1]}\cos(i\omega)\end{aligned} \tag{8}$$

$$\begin{aligned}y_n^{[k+1]}(\omega)=-A(\omega)\sum_{i=0}^{n} a_i^{[k+1]}\sin(\tau_d(\omega)\omega+i\omega)\\ +\sum_{i=0}^{m} b_i^{[k+1]}\sin(i\omega)\end{aligned} \tag{9}$$

$$R^{[k]}(\omega)=\sum_{i=0}^{n} a_i^{[k]}\cos\left(i\omega-2\pi t_d^{[k]}(\omega)\right) \tag{10}$$

$$t_d^{[k]}(\omega)=-\frac{1}{2\pi}\tan^{-1}\frac{y_d^{[k]}(\omega)}{x_d^{[k]}(\omega)} \tag{11}$$

$$x_d^{[k]}(\omega)=\sum_{i=0}^{n} a_i^{[k]}\cos(i\omega) \tag{12}$$

$$y_d^{[k]}(\omega)=-\sum_{i=0}^{n} a_i^{[k]}\sin(i\omega). \tag{13}$$

The filter coefficient update by the SP method is equivalent to solving the following optimization problem:

$$\text{Minimize}\quad \left\|\boldsymbol{a}^{[k+1]}-\boldsymbol{a}^{[k]}\right\|^2+\left\|\boldsymbol{b}^{[k+1]}-\boldsymbol{b}^{[k]}\right\|^2$$

$$\text{Subject to}\quad E_H^{[k+1]}(\omega_M)-\lambda(\omega_M)\le 0$$

where

$$\left\|\boldsymbol{a}^{[k+1]}-\boldsymbol{a}^{[k]}\right\|^2=\sum_{i=0}^{N}\left(a_i^{[k+1]}-a_i^{[k]}\right)^2 \tag{14}$$

$$\left\|\boldsymbol{b}^{[k+1]}-\boldsymbol{b}^{[k]}\right\|^2=\sum_{i=0}^{N}\left(b_i^{[k+1]}-b_i^{[k]}\right)^2, \tag{15}$$

and ω_M is the frequency point at which the specifications are violated the most and satisfies

$$E_H^{[k]}(\omega_M)-\mu(\omega_M)=\max_{\text{all }\omega\in\Omega_H} E_H^{[k]}(\omega)-\mu(\omega). \tag{16}$$

Consequently, for the magnitude-phase response approximation, we can get the following iterative algorithm:

$$a_i^{[k+1]}=a_i^{[k]}-\frac{R^{[k]}(\omega_M)\left\{E_H^{[k]}(\omega_M)-\lambda(\omega_M)\right\}\phi_{1M}^{[k]}}{\sum_{i=0}^{n}(\phi_{1M}^{[k]})^2+\sum_{i=0}^{m}(\phi_{2M}^{[k]})^2} \tag{17}$$

$$b_i^{[k+1]}=b_i^{[k]}+\frac{R^{[k]}(\omega_M)\left\{E_H^{[k]}(\omega_M)-\lambda(\omega_M)\right\}\phi_{2M}^{[k]}}{\sum_{i=0}^{n}(\phi_{1M}^{[k]})^2+\sum_{i=0}^{m}(\phi_{2M}^{[k]})^2}, \tag{18}$$

where

$$\phi_{1M}^{[k]}=A(\omega_M)\cos\left(\tau_d(\omega_M)\omega_M+i\omega_M-2\pi t_n^{[k]}(\omega_M)\right) \tag{19}$$

$$\phi_{2M}^{[k]}=\cos(i\omega_M-2\pi t_n(\omega_M)). \tag{20}$$

In order to obtain the stable IIR filters, in [18], they expanded eq.(17) using Rouché's theorem. Then, the iterative algorithm is

$$a_i^{[k+1]}=a_i^{[k]}-\beta\delta_{i,a}^{[k]}, \tag{21}$$

where

$$\delta_{i,a}^{[k]}=\frac{R^{[k]}(\omega_M)\left\{E_H^{[k]}(\omega_M)-\lambda(\omega_M)\right\}\phi_{1M}^{[k]}}{\sum_{i=0}^{n}(\phi_{1M}^{[k]})^2+\sum_{i=0}^{m}(\phi_{2M}^{[k]})^2}. \tag{22}$$

In eqs.(21) and (22), β $(0<\beta\le 1)$ is chosen as much as possible large value satisfying

$$\beta\left|\sum_{i=1}^{N}\delta_{i,a}^{[k]}z^{-i}\right|<\left|D^{[k]}(z)\right|\quad\text{for}\quad z=\rho e^{j\omega}, \tag{23}$$

where $\rho (0 < \rho < 1)$ is the maximum allowable radius of the poles of the filter obtained.

The detail of this algorithm has been presented in [18].

By the way, the group delay response of the filter obtained by the above iterative algorithm may be distant from the desired group delay because this is the iterative algorithm for the magnitude-phase approximation. In the next section, we consider about the iterative algorithm to approximate directly the group delay response.

III. Group Delay Approximation

The group delay response of the frequency response $H(\omega)$ in eq.(2) is written by

$$\tau(\omega) = \mathrm{Re}\left\{ \frac{\sum_{i=0}^{m} i \cdot b_i e^{-ji\omega}}{\sum_{i=0}^{m} b_i e^{-ji\omega}} + \frac{\sum_{i=0}^{n} i \cdot a_i e^{-ji\omega}}{\sum_{i=0}^{n} a_i e^{-ji\omega}} \right\}. \tag{24}$$

Then, the problem considered here is to find the filter coefficients a_i and b_i satisfying

$$\left| \tau_d(\omega) - \mathrm{Re}\left\{ \frac{\sum_{i=0}^{m} i \cdot b_i e^{-ji\omega}}{\sum_{i=0}^{m} b_i e^{-ji\omega}} + \frac{\sum_{i=0}^{n} i \cdot a_i e^{-ji\omega}}{\sum_{i=0}^{n} a_i e^{-ji\omega}} \right\} \right| \le \mu(\omega) \tag{25}$$

for $\omega \in \Omega_\tau$. Where a nonnegative function $\mu(\omega)$ is the maximum allowable error from the desired group delay response $\tau_d(\omega)$, Ω_τ is the frequency band where the desired group delay is prescribed, which consists of the passband in this paper. However, it is very difficult to solve directly this design problem using the SP method because eq. (25) is the rational function of the filter coefficients. The SP method has a property that the filter coefficients become $a_i^{[n+1]} \simeq a_i^{[n]}$ and $b_i^{[n+1]} \simeq b_i^{[n]}$, i.e. $D^{[n+1]}(\omega) \simeq D^{[n]}(\omega)$ and $N^{[n+1]}(\omega) \simeq N^{[n]}(\omega)$, if the algorithm converges. And so, we consider the following new design formula instead of eq.(25):

$$\left| \tau_d(\omega) - \mathrm{Re}\left\{ \frac{\sum_{i=0}^{m} i \cdot b_i e^{-ji\omega}}{\hat{N}(\omega)} + \frac{\sum_{i=0}^{n} i \cdot a_i e^{-ji\omega}}{\hat{D}(\omega)} \right\} \right| \le \mu(\omega) \tag{26}$$

where

$$\hat{N}(\omega) = \sum_{i=0}^{m} b_i' e^{-ji\omega} = |\hat{N}(\omega)| e^{j\hat{\theta}_N(\omega)}, \tag{27}$$

$$\hat{D}(\omega) = \sum_{i=0}^{n} a_i' e^{-ji\omega} = |\hat{D}(\omega)| e^{j\hat{\theta}_D(\omega)}, \tag{28}$$

and a_i' and b_i' are the previous filter coefficients in each iteration step.

The coefficient update by the SP method is equivalent to solving the following optimization problem:

$$\begin{aligned} &\text{Minimize} \quad \left\| \boldsymbol{a}^{[k+1]} - \boldsymbol{a}^{[k]} \right\|^2 + \left\| \boldsymbol{b}^{[k+1]} - \boldsymbol{b}^{[k]} \right\|^2 \\ &\text{Subject to} \quad \left| E_\tau^{[k+1]}(\omega_M) \right| - \mu(\omega_M) \le 0 \end{aligned}$$

where

$$\left\| \boldsymbol{a}^{[k+1]} - \boldsymbol{a}^{[k]} \right\|^2 = \sum_{i=0}^{N} \left(a_i^{[k+1]} - a_i^{[k]} \right)^2 \tag{29}$$

$$\left\| \boldsymbol{b}^{[k+1]} - \boldsymbol{b}^{[k]} \right\|^2 = \sum_{i=0}^{N} \left(b_i^{[k+1]} - b_i^{[k]} \right)^2 \tag{30}$$

$$\begin{aligned} E_\tau^{[k+1]}(\omega_M) &= \tau_d(\omega_M) \\ &- \frac{\sum_{i=0}^{m} i b_i^{[k+1]} \cos\left(i\omega_M + \hat{\theta}_N(\omega_M) \right)}{|\hat{N}(\omega_M)|} \\ &- \frac{\sum_{i=0}^{n} i a_i^{[k+1]} \cos\left(i\omega_M + \hat{\theta}_D(\omega_M) \right)}{\left| \hat{D}(\omega_M) \right|}, \end{aligned} \tag{31}$$

and ω_M is the frequency point at which the specifications are violated the most and satisfies

$$\left| E_\tau^{[k]}(\omega_M) \right| - \mu(\omega_M) = \max_{\text{all } \omega \in \Omega_\tau} \left| E_\tau^{[k]}(\omega) \right| - \mu(\omega). \tag{32}$$

Below, in order to simplify the designation, $\hat{D}(\omega_M)$, $\hat{N}(\omega_M)$, $E_\tau^{[k]}(\omega_M)$, and $\mu(\omega_M)$ are written as $\hat{D}_M$, $\hat{N}_M$, $E_{\tau,M}^{[k]}$, and μ_M.

Solving the above optimization problem, we can get the following iterative algorithm.

$$a_i^{[k+1]} = a_i^{[k]} - \frac{|\hat{D}_M||\hat{N}_M| \left(\left| E_{\tau,M}^{[k]} \right| - \mu_M \right) \psi_{2M}^{[k]} \mathrm{sign}(E_{\tau,M}^{[k]})}{\sum_{i=0}^{m} (\psi_{1M}^{[k]})^2 + \sum_{i=0}^{m} (\psi_{2M}^{[k]})^2} \tag{33}$$

$$b_i^{[k+1]} = b_i^{[k]} + \frac{|\hat{D}_M||\hat{N}_M| \left(\left| E_{\tau,M}^{[k]} \right| - \mu_M \right) \psi_{1M}^{[k]} \mathrm{sign}(\mathrm{E}_{\tau,M}^{[k]})}{\sum_{i=0}^{m} (\psi_{1M}^{[k]})^2 + \sum_{i=0}^{m} (\psi_{2M}^{[k]})^2}, \tag{34}$$

where

$$\psi_{1M}^{[k]} = |\hat{D}_M| i \cos\left(i\omega_M + \hat{\theta}_N(\omega_M) \right) \tag{35}$$

$$\psi_{2M}^{[k]} = |\hat{N}_M| i \cos\left(i\omega_M + \hat{\theta}_D(\omega_M) \right). \tag{36}$$

And then, in order to design the stable IIR filters, eq.(33) is modified using Rouché's theorem as follows:

$$a_i^{[k+1]} = a_i^{[k]} - \beta \delta_{i,a}^{[k]} \tag{37}$$

where

$$\delta_{i,a}^{[k]} = \frac{|\hat{D}_M||\hat{N}_M| \left(\left| E_{\tau,M}^{[k]} \right| - \mu_M \right) \psi_{2M}^{[k]} \mathrm{sign}(E_{\tau,M}^{[k]})}{\sum_{i=0}^{m} (\psi_{1M}^{[k]})^2 + \sum_{i=0}^{m} (\psi_{2M}^{[k]})^2} \quad (38)$$

and β $(0 < \beta \leq 1)$ is chosen as much as possible large value satisfying

$$\beta \left| \sum_{i=1}^{N} \delta_{i,a}^{[k]} z^{-i} \right| < \left| D^{[k]}(z) \right| \qquad \text{for} \quad z = \rho e^{j\omega}. \qquad (39)$$

In the proposed method, the magnitude-phase approximation algorithm described in section 2 and the group delay approximation algorithm described in this section are carried out alternately. Note, however, that the direct approximation for the group delay response should be executed after the magnitude-phase approximation converged enough.

The design procedure of the proposed method is summarized as follows.

The Design Procedure.

Step 0 Set the filter order m and n, desired frequency response $H_d(\omega)$, magnitude allowable error $\lambda(\omega)$, group delay allowable error $\mu(\omega)$, and an initial filter coefficients $\boldsymbol{a}^{[k]}|_{k=0}$ and $\boldsymbol{b}^{[k]}|_{k=0}$.

Step 1 Calculate the complex magnitude error $E_H^{[k]}(\omega)$ and find the frequency points ω_M satisfying eq. (16).

Step 2 If $E_H^{[k]}(\omega_M) > \lambda(\omega_M)$, then compute the new filter coefficients $h^{[n+1]}$ by eqs. (17)-(23). Otherwise, set $h^{[n+1]} = h^{[n]}$

Step 3 If $(E_H^{[k+1]}(\omega) - \lambda(\omega))/\lambda(\omega) \leq \epsilon$ (≤ 0.1) for all $\omega \in \Omega_H$, then go to next step. Otherwise, go back to Step 1.

Step 4 Calculate the group delay error $|E_\tau^{[n+1]}(\omega)|$ and find the frequency points ω_M satisfying eq. (32).

Step 5 If $E_\tau^{[k+1]}(\omega_M) > \mu(\omega_M)$, then compute the new filter coefficients $h^{[n+2]}$ by eqs. (33)-(39); otherwise, set $h^{[n+2]} = h^{[n+1]}$.

Step 6 If $h^{[n+1]} = h^{[n]}$ in Step 2 and $h^{[n+2]} = h^{[n+1]}$ in Step 4, then terminates. Otherwise, go back to Step 1.

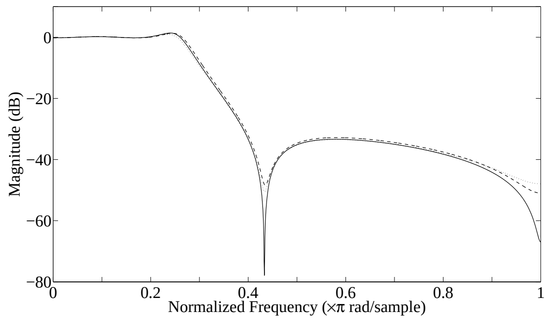
IV. Design Examples

In this section, the examples of the IIR filters with reduced group delay errors are presented to illustrate the effectiveness of the proposed method. In all the following examples, the initial value of the filter coefficients was set to $a_i = 0$ for $i = 1, 2, \cdots, n$ and $b_i = 1$ for $i = 0, 1, 2, \cdots, m$.

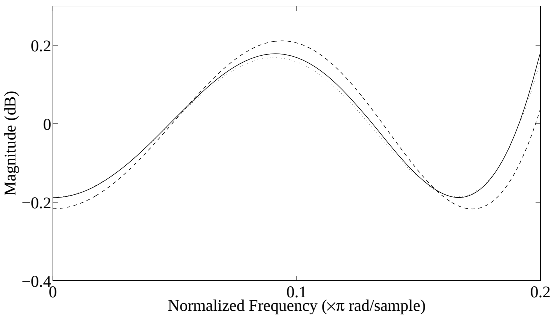
A. Example 1

The specifications are as follows:

- $n = m = 4$
- $H_d(\omega) = \begin{cases} e^{-j5.0\omega} & \text{if } 0 \leq |\omega| \leq 0.2\pi \\ 0 & \text{if } 0.4\pi \leq |\omega| \leq \pi \end{cases}$
- $\lambda(\omega) = \begin{cases} \Delta_p & \text{if } 0 \leq |\omega| \leq 0.2\pi \\ \Delta_s & \text{if } 0.4\pi \leq |\omega| \leq \pi \end{cases}$
- $\mu(\omega) = \Delta_\tau \quad$ if $0 \leq |\omega| \leq 0.2\pi$



(a)



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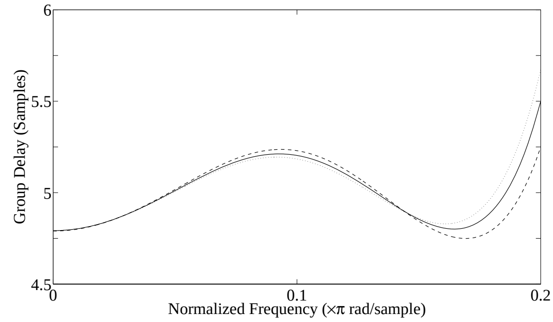


(c)

Fig. 1. Frequency responses of the proposed IIR filters with $\Delta_\tau = 0.500$ (solid lines) and $\Delta_\tau = 0.250$ (dashed lines) and of the previous IIR filter (dotted lines) in Example 1. (a) Overall magnitude response (b) Magnitude response in the passband (c) Group delay response in the passband

The maximum allowable pole radius ρ to guarantee the stability was set to 0.940.

First of all, for comparison, the IIR filter with the above design specifications was designed using the previous SP method [18]. Note that this method [18] is not possible to specify the maximum group delay allowable error $\mu(\omega)$. The resulting maximum group delay error of the filter obtained using this method was 0.630 in passband, and the maximum passband ripple and the maximum stopband ripple were 0.02130 and 0.02130, respectively. And, the maximum pole radius was 0.895.

On the other hand, the main advantage of the proposed method is that it can directly specify the maximum group delay allowable error $\mu(\omega)$. The resulting filters for many different $\mu(\omega)$ are summarized in Table 1. Moreover, the frequency

TABLE I
PERFORMANCE COMPARISON WITH [18] (EX. 1)

	Previous SP [18]	Proposed Method	
Δ_p	0.0213	0.0215	0.0247
Δ_s	0.0213	0.0215	0.0247
Δ_τ	0.6302	0.5000	0.2500
Maximum pole radius	0.8951	0.9028	0.9018

responses of the obtained filter are depicted in Figures. 1(a)-(c). In Figure 1, the dotted line is the frequency response of the filter designed by [18], and the solid line and the dashed line are the frequency response in the case of $\mu(\omega) = 0.500$ and $\mu(\omega) = 0.250$ of the proposed method, respectively. It is seen from Figure 1 and Table 1 that the performance of the filter obtained using the proposed method is much better in group delay response, while the magnitude response is poorer ripples. Also, we can see that the magnitude and the group delay errors have the relation of trade-off.

B. Example 2

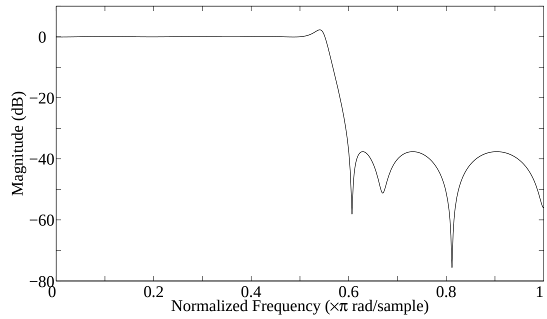
We attempt the design of the IIR filters that satisfy the following specifications:

- $H_d(\omega) = \begin{cases} e^{-j10.0\omega} & \text{if } 0 \leq |\omega| \leq 0.5\pi \\ 0 & \text{if } 0.6\pi \leq |\omega| \leq \pi \end{cases}$
- $\lambda(\omega) = \begin{cases} 0.0132 & \text{if } 0 \leq |\omega| \leq 0.5\pi \\ 0.0132 & \text{if } 0.6\pi \leq |\omega| \leq \pi \end{cases}$
- $\mu(\omega) = 0.250 \quad \text{if } 0 \leq |\omega| \leq 0.5\pi$
- $\rho = 0.960$

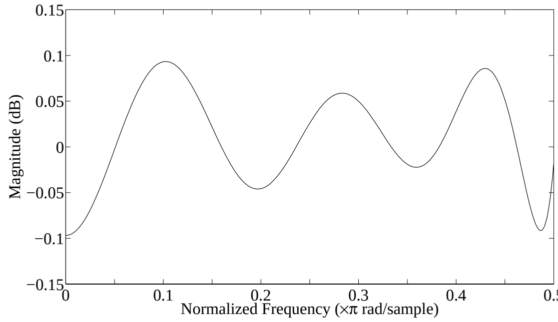
The performance of the resulting filters are summarized in Table 2, and the frequency responses of the IIR filter with $(n, m) = (8, 12)$ obtained by the proposed method are shown in Figure 2. From Table 2, it can see that, in the previous SP method, the passband ripple becomes small with an increase in order n but the group delay response in the passband cannot satisfy the given specification. Whereas, the filter with order $(n, m) = (8, 12)$ obtained by the proposed method satisfies all the given specifications. That is, the proposed method has a possibility that the filter that can not be designed using the previous SP method can be designed.

V. CONCLUSION

In this paper, we have proposed the improved method to design the stable IIR filters with prespecified group delay errors using the successive projection method. In our proposed method, the approximation problems of the magnitude-phase and the group delay responses are defined separately on the different dimension, and these two approximation problems are solved alternately using the successive projection method. With the proposed method, it is possible to directly approximate of the group delay response, and it can restrict the group delay response within the preselected allowable error. As a result, the proposed method can design the filters that can not be obtained using the previous method. Finally, the design examples are demonstrated to illustrate the effectiveness of the proposed method.



(a)



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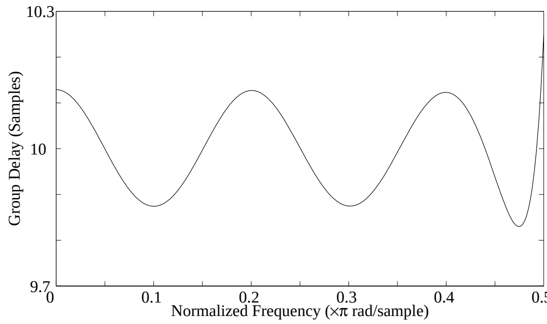

(c)

Fig. 2. Frequency response of the proposed IIR filters with the order $(n, m) = (8, 12)$ in Example 2. (a) Overall magnitude response (b) Magnitude response in the passband (c) Group delay response in the passband

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TABLE II
PERFORMANCE COMPARISON WITH [18] (EX. 2)

	Previous SP [18]				Proposed Method
(n, m)	(6,12)	(7,12)	(8,12)	(9,12)	(8,12)
Δ_p	0.0132	0.0100	0.0079	0.0062	0.0131
Δ_s	0.0132	0.0132	0.0132	0.0132	0.0131
Δ_τ	0.8644	0.7579	0.4804	0.4223	0.2500
Maximum pole radius	0.9463	0.9433	0.9596	0.9599	0.9583

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