

# Phenomenological and Semi-microscopic Analysis for Elastic Scattering of Protons on ${}^{6,7}\text{Li}$

A. Amar, N. Burtebayev, Sh. Hamada, Kerimkulov Zhambul and N. Amangeldy

**Abstract**—Analysis of the elastic scattering of protons on  ${}^{6,7}\text{Li}$  nuclei has been done in the framework of the optical model at the beam energies up to 50 MeV. Differential cross sections for the  ${}^{6,7}\text{Li} + p$  scattering were measured over the proton laboratory-energy range from 400 to 1050 keV. The elastic scattering of  ${}^{6,7}\text{Li}+p$  data at different proton incident energies have been analyzed using single-folding model. In each case the real potential obtained from the folding model was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. Normalization factor  $N_R$  is calculated in the range between 0.70 and 0.84.

**Keywords**—scattering of protons on  ${}^{6,7}\text{Li}$  nuclei, Esis88 Code single-folding model, phenomenological.

## I. INTRODUCTION

In practice it is required to obtain the potential from the experimental data, and this may be done by systematically varying the parameters of the optical potential to optimize the overall fit to the data, using appropriate computer programs. Optical model analysis of the elastic scattering and polarization data cannot give unique values of the all parameters of the potential. Many such analysis of nucleon scattering have now been made and is found that the potentials are quite similar for all nuclei and vary other slowly with the incident energy [1]. The folding model has been used for years to calculate the nucleon-nucleus optical potential and inelastic form factors. It can be seen from the basic folding formulas that this model generates the first-order term of the microscopic optical potential that is derived from Feshbach's theory of nuclear reactions. The success of this approach in describing the observed nucleon-nucleus elastic scattering data for many targets suggests that the first-order term of the microscopic optical potential is indeed the dominant part of the nucleon optical potential [2]. The phenomenological optical model was used to

analyze the elastic nucleon-nucleus scattering in the same region of energy. The potential used in the single folding were obtained in a semi-microscopic way including one nucleon exchange effects and the density dependence of the nucleon-nucleon interaction. In each case the real potential obtained from the folding model was supplemented by a phenomenological imaginary potential, and during the fitting process the real potential was normalized and the imaginary potential optimized. The basic inputs for a single-folding calculation of the nucleon-nucleus potential are the nuclear densities of the target and the effective nucleon-nucleon (NN) interaction. If one has a well tested, realistic effective NN interaction, the folding model is a very useful approach to check the target nuclear densities [3].

## II. SINGLE FOLDING MODEL

The real part of the optical potential for the nucleon-nucleus elastic scattering is given for the single folding model, in the following form:

$$U_F(R) = \int dr_1 \rho_1(r_1) V(r), \quad (1)$$

where  $r = R - r_1$ ,  $\rho_1(r_1)$  is the matter density distribution of the target nucleus,  $V(r)$  is the effective NN-interaction. In the present calculation the effective NN-interaction is taken according to [4] in the form of M3Y-interaction

$$V(R) = 2999 \frac{\exp(-4R)}{4R} - 2134 \frac{\exp(-2.5R)}{2.5R} - 276 \left(1 - \frac{0.005E}{A}\right) \chi(R) \quad (2)$$

The density of the  ${}^6\text{Li}$  target nucleus is considered in the form [5]

$$\rho = \frac{6}{8\pi^{\frac{3}{2}}} \left( \frac{1}{a^3} \exp\left(-\frac{r^2}{4a^2}\right) - \frac{c^2(6b^2 - r^2)}{4b^7} \exp\left(-\frac{r^2}{4b^2}\right) \right), \quad (3)$$

Where  $a$ ,  $b$ , and  $c$  are constants parameters and in the range:  $a := \sqrt{0.87}$  fm,  $b := \sqrt{1.7}$  fm and  $c := \sqrt{0.205}$  fm. And for the density of the  ${}^7\text{Li}$  nucleus is considered in the form [6]

$$\rho(R) = \rho_0 \left( 1 + \alpha \left( \frac{R}{a} \right)^2 \right) \exp\left( - \left( \frac{R}{a} \right)^2 \right), \quad (4)$$

where for harmonic oscillator  $a=1.77$ fm and  $\alpha=0.327$ fm. The analytical form of the real part of the optical potential is obtained by substituting Eqs. (2), (3) and (4) into Eq. (1) and carrying out the required integrations over  $r_1$ .

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### III. RESULTS AND DISCUSSIONS

Measurements of elastic scattering of protons on  ${}^6\text{Li}$  nuclei in low energy region were carried out with using the extracted beam from the complex recharged UKP-2-1 accelerator of the Institute of Nuclear Physics (National Nuclear Center, Republic of Kazakhstan, Almaty, Kazakhstan) in the angular range  $40-170^\circ$ . The proton energy varied in the range  $400 - 1200$  KeV. The beam intensity was  $200 - 300$  nA. Scattered particles were detected using surface-barrier silicon counters. Analysis of the elastic scattering of protons on  ${}^6\text{Li}$  nuclei has been done in the framework of the optical model at the beam energies up to  $50$  MeV. For  ${}^6\text{Li}$  nuclei, the most suitable phenomenological parameters values are  $r_0=1.05\text{fm}$ ,  $r_c=1.3\text{fm}$ ,  $r_D=1.923\text{fm}$ ,  $a_s=0.20\text{fm}$  and  $r_s=1.20$  fm. Optical parameters obtained for protons elastic scattering on  ${}^6\text{Li}$  are shown in table I. As expected the relations between  $V_0$ ,  $W_D$  and  $E_p$  are linear. The strength parameters in table I can be represented by:

$$V_0 = 56.1 - 0.61 E_p, W_D = -0.66 + 0.46 E_p \quad (5)$$

TABLE I  
 THE OPTICAL PARAMETERS FOR PROTONS SCATTERING ON  ${}^6\text{Li}$

$E_p$ (MeV)	$V_0$ (MeV)	$r_0$ (fm)	$a_0$ (fm)	$W_D$ (MeV)	$r_D$ (fm)	$a_D$ (fm)	$V_s$ (MeV)	$r_s$ (fm)	$a_s$ (fm)	$J_R$ (fm)	$J_W$ (MeV.fm <sup>3</sup> )
0.4	59.0	1.05	0.85	0.30	1.92	0.57	9.3	1.08	0.66	490	20.5
0.74	57.2	1.05	0.67	0.35	1.92	0.65	9.3	1.02	0.20	475	22.19
0.97	54.0	1.05	0.52	0.35	1.92	0.57	9.3	1.02	0.20	454	55.19
3.0	52.0	1.05	0.52	0.87	1.92	0.80	9.3	1.02	0.20	437	55.72
5.0	50.0	1.05	0.50	1.18	1.92	0.57	15.6	1.02	0.20	407	75.58
0.0	49.0	1.05	0.65	2.78	1.92	0.49	12.2	1.02	0.77	391	148.3
4.0	46.5	1.05	0.5	6.72	1.92	0.42	9.86	1.02	0.20	378	304.0
5.0	38.0	1.05	0.5	2.80	1.92	0.80	5.57	1.02	0.20	270	309.0
9.9	34.0	1.05	0.67	2.93	1.92	0.80	3.37	1.02	0.20	149	111.0
5.0	34.7	1.05	0.65	2.93	1.92	0.80	3.37	1.02	0.20	142	111.0
5.0	30.0	1.05	0.65	2.63	1.92	0.80	2.33	1.02	0.20	122	100.0
9.0	26.0	1.05	0.65	1.69	1.93	0.80	1.69	1.02	0.20	64.0	106.0

Using SPI GENOA Code [7] we could calculate a set of optical parameters as the volume integral per nucleon pair for the real and imaginary potential,  $J_R$  and  $J_W$ . Figure 1 shows the comparison between calculated using optical model and experimental angular distributions of protons scattered from  ${}^6\text{Li}$ .

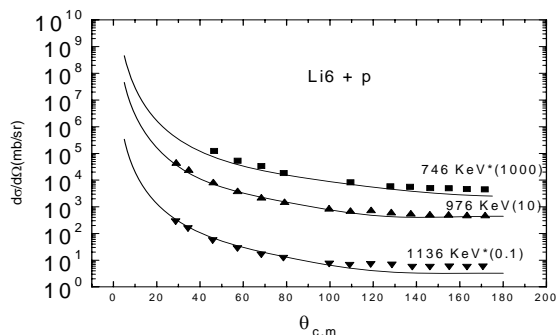


Fig.1 shows the comparison between calculated and experimental angular distribution of protons scattered from  ${}^6\text{Li}$  at low energies

where dots represent experimental data and lines represent the calculated values.

The experimental data of both elastic scattering differential cross section of scattering for proton on  ${}^6\text{Li}$  at low energies between  $0.4$  MeV and  $1$  MeV have been analyzed using single folding model. The numerical calculations have been done using the DEPOT code. In the present calculations, we have derived different analytical expressions for the real part of the optical potential in the frame of single folding model. In the present calculation the effective NN-interaction is taken according to [4] in the form of M3Y-interaction. The variations of the real potential values according to the radius are directly put in to the calculations with the aid of this model, and the imaginary parts are defined by a phenomenological way. To be able to fit the calculations with the experimental data, the normalization factor and the imaginary potential parameters must be adjusted. We will concern here on energies measured by us from  $400$  KeV to  $1$  MeV using semi-microscopic single folding model as shown in table II.

TABLE II  
 THE PARAMETERS OBTAINED USING SINGLE FOLDING MODEL FOR  ${}^6\text{Li}$

$E_p$ (MeV)	$V_0$ (MeV)	$r_0$ (fm)	$a_0$ (fm)	$W_D$ (MeV)	$r_D$ (fm)	$a_D$ (fm)	$V_s$ (MeV)	$r_s$ (fm)	$a_s$ (fm)	$J_{mic}$ (fm)	$N_R$ (MeV.fm <sup>3</sup> )
0.4	39.0	1.74	0.74	3.03	1.92	0.91	7.61	1.45	0.25	420.24	0.70
0.74	39.0	1.74	0.74	5.78	1.92	0.56	8.47	1.28	0.60	436.31	0.70
0.975	39.0	1.74	0.74	5.78	1.92	0.56	8.47	1.28	0.60	436.31	0.70

This agreement might be improved by adjusting the normalization factor and imaginary potential parameters. With comparison of other works for the real part of the optical potential we can introduce the following table which contains a comparison between present and other works. I'd like to mention that when we used another form of nuclear density with the same potential form we obtained  $V_0=46\text{MeV}$  and the same  $a_0=1.58\text{fm}$  and  $r_0=0.56\text{fm}$ , the form of the density and potential NN-interaction are very important.

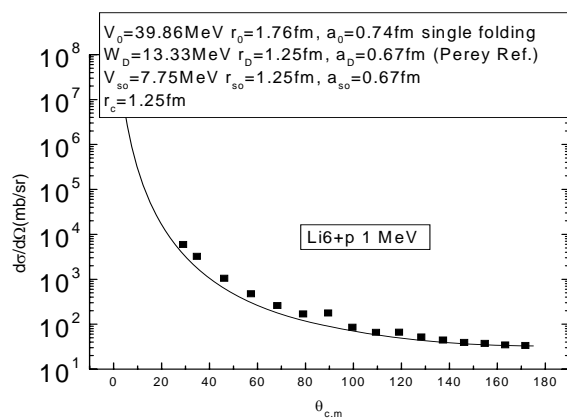


Fig. 2 shows the measured differential cross sections of protons elastically scattering on  ${}^6\text{Li}$  at  $1$  MeV, where dots represent

experimental data and lines represent the calculated values using folding model.

TABLE III  
COMPARISON BETWEEN REAL PART OF OPTICAL POTENTIAL  
OBTAINED FOR  $p+{}^6\text{Li}$  USING SINGLE FOLDING

Parameters	Present work	Wiringa	Electron scattering
$V_0$	39.98	79	34
$r_0$	1.74	1.727	1.81
$a_0$	0.74	0.613	0.478

And for  ${}^7\text{Li}$  we made the same calculations using Ecis88, SPI-GENOA and DEPOT codes in our calculations. Table IV contains optical parameters for protons scattering on  ${}^7\text{Li}$  nuclei (phenomenological).

TABLE IV  
OPTICAL PARAMETERS FOR PROTONS SCATTERING ON  ${}^7\text{Li}$   
NUCLEI (PHENOMENOLOGICAL)

$E_p$ (MeV)	$V_0$ (MeV)	$r_0$ (fm)	$a_0$ (fm)	$W_D$ (MeV)	$r_D$ (fm)	$a_D$ (fm)	$V_s$ (MeV)	$r_s$ (fm)	$a_s$ (fm)	$J_R$ (fm)	$J_W$ (MeV.fm <sup>3</sup> )
.364	56.0	1.17	0.65	0.70	1.80	0.504	12.48	1.17	0.50	228.3	11.0
1.441	62.0	1.17	0.60	0.30	1.80	0.504	12.48	1.17	0.50	220.5	4.10
.991	55.0	1.17	1.04	0.93	1.80	0.87	18.86	1.17	0.74	535.9	32.2
.03	55.0	1.17	1.03	0.93	1.80	0.79	18.8	1.17	0.75	535.9	27.25
.0	49.13	1.17	0.91	2.2	1.80	0.34	11.1	1.17	0.77	367.3	19.84
.2	48.36	1.17	0.94	2.22	1.80	0.21	12.82	1.17	1.05	379.8	10.99
.0	48.96	1.17	0.94	3.79	1.80	0.36	11.69	1.17	0.65	370.8	10.20
0.3	37.3	1.17	0.53	8.55	1.80	0.545	12.86	1.17	0.8	109.4	140.4
9.75	32.9	1.17	0.46	4.28	1.80	0.785	11.26	1.17	0.75	80.8	124.1

The parameters calculated for  ${}^7\text{Li}$  is good agreement with experimental values.

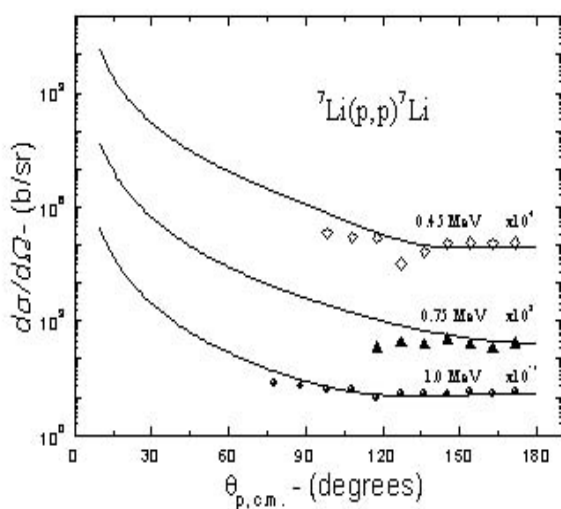


Fig. 3 Measured differential cross sections of protons elastically scattering on  ${}^7\text{Li}$  where dots represent experimental data and lines represent the calculated values.

As shown in figures 3 the differential cross sections calculated using optical parameters and experimental values are close to each other in spite of this situation is not completely true at low energies.

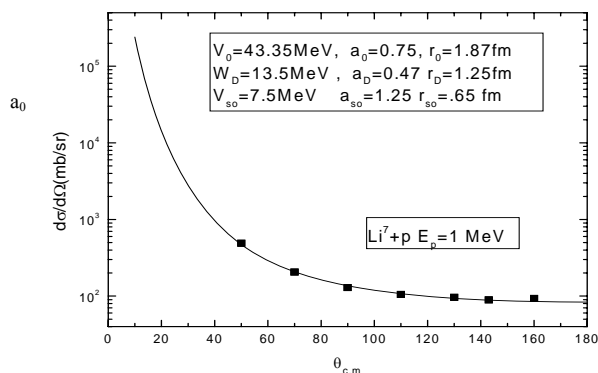


Fig. 4 shows the measured differential cross sections of protons elastically scattering on  ${}^7\text{Li}$  at 1 MeV, where dots represent experimental data and lines represent the calculated values using folding model.

Fig. 4 contains measured differential cross sections and that calculated using single folding model.

#### IV. CONCLUSION

From the above results and discussions we can say that the single folding model gives a reasonable fit to the experimental data for the differential cross section at low energies.

The optical model calculations from Ecis88 give the most suitable fits among these programs used. The present analysis shows that the optical model can give a good description of the general features of nucleon scattering from light nuclei and as our work concentrated at low energies where we can see normal description in case of  ${}^6,{}^7\text{Li}$ .

This evidenced by the fact that, to a fair degree, a single set of energy-dependent parameters is able to reproduce the differential cross sections of elastically scattering protons from 1p-shell nuclei. There is no evidence that our parameters obtained in this analysis are the best.

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