Development of A Jacobean Model for A 4-Axes Indigenously Developed SCARA System

T.C.Manjunath, C.Ardil

Abstract—This paper deals with the development of a Jacobean model for a 4-axes indigenously developed scara robot arm in the laboratory. This model is used to study the relation between the velocities and the forces in the robot while it is doing the pick and place operation.

Keywords—SCARA, Jacobean, Tool Configuration Vector, Computer Control, Visual Basic, Interfacing, Drivers,

I. INTRODUCTION

IMAGINE a day in your life when you wake up in the morning and find a machine walking up to you and saying "GOOD MORNING SIR ! Have a cup of tea". How would you respond to such a situation? With so much progress made in the field of science, engineering and technology, this dream is absolutely realizable in the automation age with the advent of robotization [1].

Robotics is an interdisciplinary filed that mixes various engineering disciplines into one. In this, work a unique 4 axes system was designed and fabricated with indigenous components with a brief tool configuration jacobian analysis of the designed robot. The designed robot was used for some PNP operations without human intervention using sensors and was named as a Selective Compliance Assembly Robot Arm (SCARA). The primary motive behind the work was to develop a modular educational robotic system, the CRUST 2002 (Computerized Robotic Unit with Selective Tractability system) with the help of locally available components and sub-systems as shown in Fig. 1.

The paper is organized as follows. First, a introduction to robotics, robots and the design of the mechanical assembly is presented in section 2. In section 3, the review of the jacobian model is presented. Fourthly, the mathematical representation of the jacobian development is presented in section 4. Section 5 gives the TCJM for the developed robot, In Section 6, the algorithm is presented followed by the conclusions and the references.



Fig. 1 The designed SCARA robot

II. PHYSICAL SYSTEM DESIGN & A BRIEF REPORT

The mechanical design of the developed system is divided into 3 parts, viz., the base assembly, the arm assembly and the gripper assembly. The designed robot has R-R-P (Rotary-Rotary-Prismatic) type of axes [2].

A four axis / four DOF designed SCARA robot arm as shown in Fig. 1. A SCARA robot is a 4 DOF stationary robot arm having base, elbow, vertical extension and tool roll and consisting of both rotary and prismatic joints. There is no yaw and pitch, only roll. There are 4 joints, 4 axis (three major axes - base, elbow, vertical extension and one minor axis - tool roll) [4]. The 4 DOF's are given by Base, Elbow, Vertical Extension and Roll, i.e., there are three rotary joints and one prismatic joint. Since n = 4; 16 KP's are to be obtained and 5 RHOCF's are to be attached to the various joints [3].

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Fig. 2 Roll DOF

The vector of joint variables is a combination of θ and d, i.e., $q = \{ \theta, d \}^{T}$.

Vector of joint variables are $q = \{ \theta_1, \theta_2, d_3, \theta_4 \}^T$. Vector of joint distances are $d = \{ d_1, 0, d_3, d_4 \}^T = \{ 400, 0, d_3, 100 \}^T$ mm. Vector of link lengths are $a = \{ a_1, a_2, 0, 0 \}^T = \{ 250, 200, 0, 0 \}^T$ mm. Vector of link twist angles are $\alpha = \{ \alpha_1, \alpha_2, 0, \alpha_4 \}^T = \{ \pm \pi, 0, 0, 0 \}^T$.

All the 4 joint axes are vertical in nature (all the zaxes can be pointing down or up) as shown in Fig. 3. The first three (B, E, VE) axes are called as the *major axes* and are used for positioning the wrist, while the last one, the *minor axes* (TR) is used to orient the gripper in the direction of the object [5]. The first three major axes determines the shape and size of work envelope. It consists of a L shaped structure, to the end of which the second link is attached. There are two links a_1 and a_2 which move parallel to the work surface; The vertical extension d_3 is variable and moves in a direction \perp^r to the work surface; the length of the gripper or the endeffector (EE) is d_4 [2], [6].

The gripper / EE is permanently pointing down as shown in Fig. 2 and can rotate in a plane \perp^{r} to the work surface plane $x^{0} y^{0}$. The approach vector r^{3} is fixed, i.e., $r^{3} \perp^{r} x^{0} y^{0}$ (work surface) plane ; $r^{3} = -z^{0}$. Because of this reason, our designed and kinematically modeled SCARA robot can do robotic manipulation directly from above the

object when exact perpendicularity is required. The SCARA robot is a minimal representation of any robot. Our SCARA robot is a special type of polar / spherical coordinate robot in which the major axes are R R P [10].



Fig. 3 The computer controlled SCARA robot

III. TOOL CONFIGURATION JACOBIAN MATRIX

In this section, we present a review of the jacobians. Jacobian is defined as a multi-dimensional form of partial derivatives and is a matrix of size $(m \times n)$. Jacobian is the study of both velocities and static forces that leads to a matrix entity [1], [9]. The transformation from the joint velocity to the tool configuration velocity is called tool configuration Jacobian matrix V(q). It gives the relation between the speed at which the joint is moving and the speed at which the tool-tip is moving. Tool configuration Jacobian matrix is not a square matrix. Hence, an inverse of it is obtained in order to find the joint velocity in terms of tool configuration velocity [1], [8].

Trajectory planning is always done in TCS, R^6 . Tool configuration gives p, R of tool in 3D space. Since, we cannot control the tool tip directly, indirectly it is controlled by using joint velocity control [7].



Fig. 4 Relation between TCS and JS

The output of the direct kinematic model of the designed robot is given by [1], [11]

$$T_{0}^{4} = \begin{bmatrix} C_{1-2-4} & S_{1-2-4} & 0 & a_{1}C_{1} + a_{2}C_{1-2} \\ S_{1-2-4} & -C_{1-2-4} & 0 & a_{1}S_{1} + a_{2}S_{1-2} \\ 0 & 0 & -1 & d_{1} - q_{3} - d_{4} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$T_{0}^{4} = \begin{bmatrix} R_{11} & R_{12} & R_{13} & p_{1} \\ R_{21} & R_{22} & R_{23} & p_{2} \\ R_{31} & R_{32} & R_{33} & p_{3} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$
$$w(q) = \begin{bmatrix} w^{1} \\ \dots \\ w^{2} \end{bmatrix}$$
$$w(q) = \begin{bmatrix} w^{1} \\ \left(\frac{w^{1}}{m} \right) \right]$$
$$w(q) = \begin{bmatrix} p \\ \left(\frac{w^{1}}{m} \right) \right]$$
$$w(q) = \begin{bmatrix} p \\ p_{2} \\ p_{3} \\ \dots \\ q_{2} \\ w_{3} \\ \dots \\ w_{4} \\ w_{5} \\ w_{6} \end{bmatrix} = \begin{bmatrix} p \\ p_{2} \\ p_{3} \\ \dots \\ \left\{ exp\left(\frac{q_{4}}{\pi}\right) \right\} R_{13} \\ \left\{ exp\left(\frac{q_{4}}{\pi}\right) \right\} R_{23} \\ \left\{ exp\left(\frac{q_{4}}{\pi}\right) \right\} R_{33} \end{bmatrix}$$

Let the tool configuration vector be represented by [1]

$$\mathbf{x} = \mathbf{w}(\mathbf{q}) = \begin{bmatrix} \mathbf{w}_1 \\ \mathbf{w}_2 \\ \mathbf{w}_3 \\ \cdots \\ \mathbf{w}_4 \\ \mathbf{w}_5 \\ \mathbf{w}_6 \end{bmatrix}$$
(1)

The relationship between the joint space variables and tool configuration variables is given by the direct kinematic equations and this relation is given by the Eq. (1) and is shown diagrammatically in Fig. 4 [1], [12].

- Given: Trajectory x(t) in TCS.To find: Trajectory q(t) in Joint Space, JS.Approach: To invert Eq. (1) and solve
inverse kinematics equations .Drawback: To get a closed form of solutions
to the inverse kinematics is
difficult [13].
- Alternative approach : Differentiate Eq. (1) w.r.t to $q_i (q_i = \theta_i \text{ or } q_i = d_i)$, i^{th} joint variable.

$$\dot{x}(t) = V(q) \dot{q}(t)$$

$$\dot{x} = V(q) \dot{q}$$
(2)

V(q) → (6 × n) matrix of partial derivatives of w w.r.t. q, $\frac{\partial w}{\partial q_i}$

 $\frac{\partial w}{\partial q_i} \rightarrow \text{Called as } TC \text{ Jacobian matrix of the arm [1],}$

Has got 6 rows and n columns, where n is the DOF [1].

 $(6 \times n)$ components of tool configuration jacobian matrix [1], [14].

 $\dot{x} \rightarrow$ Tool configuration velocity.

 $\dot{q} \quad \rightarrow \mbox{ Joint space velocity}\,.$

Note that the differential relationship shown in the Eq. (2) can be used to solve for the joint space trajectory q(t) given the tool configuration trajectory x(t) [1], [15], [40].

IV. MATHEMATICAL REPRESENTATION

The component of V(q) in the k^{th} row and the j^{th} column is the derivative of $w_k(q)$ w.r.t. q_j and is given by [16], [1]

$$V_{ki}(q) = \frac{\partial w_k(q)}{\partial q_i} \quad ; \ 1 \le k \le 6 \quad \text{and} \quad 1 \le i \le n$$
(3)

Definition : For each $q \in \mathbb{R}^n$, the TC Jacobian matrix V(q) is defined *as a linear transformation which maps the instantaneous joint space velocity* \dot{q} *into the instantaneous tool configuration velocity* \dot{x} as shown in the Fig. 5 OR it is a matrix which gives the relation between tool-tip velocity and the joint velocity. Once, a robot starts moving from the pick point to the place point, all the joints moves with a particular velocity [39].

Similarly, the tip of the gripper also starts to move with a particular velocity. The relation between the tip speeds and the joint speeds is given a matrix called as the tool configuration jacobian matrix [17], [1].



Fig. 5 Relation between the inputs and output of the tool configuration jacobian matrix TCJM

The main use of TC Jacobian Matrix is, it is used to find JS trajectory q(t), given TCS trajectory x(t) [1].

V. TOOL CONFIGURATION JACOBIAN MATRIX FOR A 4 AXIS INDIGENOUSLY DEVELOPED SCARA ROBOT

The designed and developed SCARA robot has got 4 DOF, n = 4, Joint variables q_1 to q_4 , joints are combination of prismatic and rotary type, 3 revolute and 1 prismatic joint. The TCV w(q) which is obtained from the output of the direct kinematic problem is given by [1], [18], [38]

$$w(q) = \begin{bmatrix} p \\ \dots \\ R \end{bmatrix}$$
$$= \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \dots \\ w_4 \\ w_5 \\ w_6 \end{bmatrix}$$
$$= \begin{bmatrix} a_1 \cos q_1 + a_2 \cos (q_1 - q_2) \\ a_1 \sin q_1 + a_2 \sin (q_1 - q_2) \\ d_1 - q_3 - d_4 \\ \dots \\ 0 \\ 0 \\ -exp\left(\frac{q_4}{\pi}\right) \end{bmatrix}_{(6 \times 1)}$$

which can be written in short form as [1]

$$w(q) = \begin{bmatrix} a_1C_1 + a_2C_{1-2} \\ a_1S_1 + a_2S_{1-2} \\ d_1 - q_3 - d_4 \\ \dots \\ 0 \\ 0 \\ -exp\left(\frac{q_4}{\pi}\right) \end{bmatrix}_{(6 \times 1)}$$

Note that $q = \theta$ for a rotary joint and q = d for a prismatic joint. The notations 'p' and 'R' in the TCV is nothing but the position of the tip of the gripper and the orientation of the gripper w.r.t. the base which can be further explained as [19],[20], [1]

 $p \rightarrow position of gripper-tip : Given by the Ist three components of TCV, w(q) [21], [22], [1]$

 $R \rightarrow$ orientation of the gripper : Given by the last three components and TCV, w(q) or approach vector r³ scaled by an invertible exponential function given by exp $\left(\frac{q_n}{\pi}\right)$, where q₄ is roll angle [23], [24], [1]

Differentiate Eq. (4) w.r.t. q, we get the TC Jacobian matrix; The resultant TC Jacobian matrix V(q) obtained is a (6×4) matrix, since n = 4 [25], [26], [1]

$$V(q) = [v^{1}(q), v^{2}(q), v^{3}(q), v^{4}(q)]$$
(5)

where,

(4)

 $v^1(q)$ to $v^4(q)$ are the 4 columns of V(q) and [36]

 $v^k(q) \rightarrow k^{th}$ column of TC Jacobian matrix and $1 \le k \le 4$ [37], [1]

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Substitute the equations for $v^1(q)$ to $v^4(q)$ in V(q), we get the TC Jacobian Matrix (TCJM) as [27], [28], [1]



Note: Differential of $\sin = \cos$ and that of $\cos = -\sin$. Observe the number of zeros in V(q), [1]

Finally, V(q) is given by [1]



Note :

z component of p : Depends only on q_3 [31].

From the Eq. (10), we can come to a conclusion that the matrix is a sparse one and this shows that the kinematics of the designed SCARA robot is very very simple because of the number of zeros in the TCJM, i.e., there are 18 zeros in 24 element matrix [32], [1].

The position of the tip of the gripper depends only on the first two joint variables, i.e., the base and the elbow, whereas the z-axis is entirely dependent on the vertical extension parameter and the orientation of the gripper is finally dependent on the roll angle [33], [1].

The velocities of all the joints and the tip of the gripper was also controlled during the pick and place operation [34], [1].

VI. ALGORITHM

The algorithm for obtaining the joint velocities in terms of tool velocities is shown below in the following form as follows [35]

- Input the pick and place points
- Input the geometric link parameters
- Obtain the DK model
- Obtain the IK model
- Differentiate the IK model w.r.t. q₁ only
- Differentiate the IK model w.r.t. q₂ only
- Differentiate the IK model w.r.t. q₃ only
- Differentiate the IK model w.r.t. q₄ only
- Combine the differentiated IK models w.r.t. the different joint variables into the form of a matrix

- Name that matrix as the tool configuration jacobian matrix.
- Use MATLAB interfaced with real time and feedback units to plot the velocities of the 4 joints from the pick point to the place point.
- Observe the experimental results





Fig. 6 Velocities of the 4 joints of the robot w.r.t. time



Fig. 7 Velocity of the tip of the gripper w.r.t. time

VIII. CONCLUSIONS

A indigenously designed and fabricated 4-axes robot system was used to obtain the Jacobian model and the same and was used to perform a successful pick and place task using a user-friendly developed graphical user interface and real time implementation. It was seen that the velocities were properly controlled.

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Plot of the velocities of the joints v/s time

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