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Using Tabu Search to Analyze the Mauritian Economic Sectors

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***Abstract*—**The aim of this paper is to express the input-output matrix as a linear ordering problem which is classified as an NP-hard problem. We then use a Tabu search algorithm to find the best permutation among sectors in the input-output matrix that will give an optimal solution. This optimal permutation can be useful in designing policies and strategies for economists and government in their goal of maximizing the gross domestic product.

***Keywords*—**Input-Output matrix, linear ordering problem, Tabu search.

I. INTRODUCTION

INPUT-output analysis has been widely used by economists during the last four to five decades to explain the impact of the flow of funds from one sector to another. Since the development of this concept by [5], there have been further studies done at both micro and macro level. Bobst [2] has used this approach to develop a procedure to show that the financing of the farm sector is related to other economic sectors. This can be used when considering matters of farm credit policy. Lixon [6] stated that the input-output framework is an adequate tool to measure the impact of climate change policy on the economy. Their work describes the change in the GDP of Canada if it adheres to the Kyoto protocol and adopts carbon permit trading. Mayer [7] explained the importance of this method to study how the sources of energy are utilized either directly or indirectly so as to assess the pressure of economic activities on the environment. Andressen [1] has also studied the money-flow between the sectors by representing the macro-economy as a dynamic system.

Input-output analysis can be coarsely described by a matrix which represents the amount of money flowing between different economic sectors. The way the matrix is represented may not always show which sectors contribute most to the economy and hence we need to triangulate the matrix to determine a hierarchy of the sectors such that the predominant production sectors appear first and the consumption sectors appear last. Our aim in this paper is to first represent this matrix as a linear ordering problem and to find the optimal permutation, which ensures that funds are input into sectors that generate the highest economic output. The novelty of this paper is that we achieve this objective by using a Tabu search algorithm and we show that by maximising the sum of the upper triangular part of the input-output matrix, that is the value of total intermediate consumption, we can effectively find the right permutation vector. Such study has never been carried out on the Mauritian economy before.

The national input output table shows how the different sectors of the economy interact with each other. Each row accounts for the sales by the industry named in the first column to the industries and final consumers listed in the first row. An illustration is given in Fig. 1. Quadrant 1 represents consumer behaviour and describes the consumption pattern of households, private investors, government and any other final users. Quadrant 2 illustrates the production relationships in the economy. It shows how raw materials and intermediate goods are combined to produce outputs for sale to other industries and to final consumers. Quadrant 3 describes the income of primary units of the economy, which consist of incomes of households, the depreciation and retained earnings of industries, and the taxes paid to government. It is also viewed as where the production cycle starts, where the contributions of different stakeholders to the economy are highlighted. Finally, quadrant 4 considers the non-market transfers between sectors of the economy which consists of gifts, savings, and taxes of households.

	Production	Final Demands	
Distribution	2 Inter-industry structure	1 Consumption patterns	Total Outputs
Final payments	3 Incomes	4 Non-market transfers	
	Total inputs		

Fig.1. A brief view of the input-output table

In this paper, we first describe the linear ordering problem and the tabu search method. Then we show that the input output table can be viewed as a linear ordering problem and how it can effectively be solved by our proposed method. Finally, we use some real data set to prove the efficiency of

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the tabu search algorithm.

II. The Linear Ordering Problem

A measure of the degree to which an economic structure agrees with a hierarchy of the sectors is the linearity coefficient. If the economy is perfectly linear, the flow of goods runs from the upper sectors to the lower sectors in the hierarchic ordering, that is the sectors which produce more will appear first. The input output matrix will then have an upper triangular structure. On the other hand, if there is a flow of funds back to the upper sectors, then there will be positive values on the entries in the lower triangular part too. This triangulation problem is same as finding a permutation of the rows and columns, such that the sum of the elements above the diagonal has the maximum possible value. We thus clearly see that this is equivalent to the well known linear ordering problem.

The linear ordering problem (LOP) can be stated as follows. Suppose we have a set M of m objects and a permutation p: M $\rightarrow$ M. Each permutation $p = (p(1), \ldots, p(\mathrm{m}))$ corresponds one-to-one to a linear ordering of the objects. Let e_{ij}, where $i,j = 1, 2, \ldots, m$, be the cost associated with the preference of having i before j in the ordering, and E be the m-square matrix of costs. Then the linear ordering problem is to find an optimal permutation p that maximises the total cost

$$C_E(p) = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m} e_{p(i)p(j)} \qquad (1)$$

Clearly, (1) is the sum of the elements above the main diagonal of the matrix E resulting from a simultaneous permutation of its rows and columns. The LOP is classified as an NP-hard combinatorial optimisation problem.

Triangulation of an input-output table is the process of determining a hierarchical ordering of the different industries of production such that the flow of monetary value between them is maximised. Such a hierarchical ordering allows the determination of a quantitative measure called *linearity*, which is the extent to which an economic structure agrees with the hierarchy of the industries. The linearity of an economy is given by

$$\lambda = \frac{\sum_{i=1}^{m-1} \sum_{j=i+1}^{m} e_{p(i)p(j)}}{\sum_{i=1}^{m} \sum_{j=1, i\neq j}^{m} e_{p(i)p(j)}}. \qquad (2)$$

It follows that highly developed economies tend to have a low degree of linearity, representing a high circulation of goods and services among industries, while underdeveloped economies tend to have larger values of linearity, typical ones of about 90%.

Since the denominator of (2), representing the sum of all the entries of the matrix E less those elements on the main diagonal is always constant, the triangulation problem is reduced to finding a permutation of the rows and columns such that the sum of the elements above the main diagonal is maximum. Clearly, this is equivalent to the linear ordering problem.

III. Tabu Search

The proposed Tabu search (TS) algorithm in this paper is mainly based on intensification and diversification search strategies [4]. Tabu search is a heuristic method originally presented by [3]. It provides solutions close to optimality and is effective in tackling difficult problems. TS is a procedure to explore the solution space beyond local optimality and it is based on schemes, which exploits the past history of the search in order to influence its future steps. The adaptive memory feature of the Tabu search allows the implementation of procedures that are capable of searching the solution space economically and effectively. There are two basic elements in a tabu search, these are the search space and its neighbourhood structure. The search space is defined as the space of all possible solutions that can be considered during the search. Denoting S to be the set of all feasible solutions, for each $s \in S$ we associate a subset of S called the neighbourhood, $N(S)$. The latter contains all those solutions which can be obtained with a simple modification of s, called *move*. Given a starting feasible solution s, ordinary local search iterates the following step:

- Compute the $N(S)$
- Select within $N(S)$ the solution S_N with the best objective value.
- If such a value is better than of s, replace s with S_N and go to step 1, otherwise stop and return s.

The final solution is a local optimum with respect to the defined neighbourhood. To go beyond local optimality, tabu search continues the search by performing even non-improving moves and adopting a memory structure to avoid a cyclic behaviour. To identify each solution at each iteration, Tabu search records in a memory structure, called tabu list, some information, called attributes, of the selected solution. Such information is not the complete structure of that solution; it is instead those aspects that are modified by the application of the move, that is, the aspects by which the current solution differs from the selected one. To stop the search the commonly adopted criterion is based on the total elapsed time or on the total number of iterations. A pseudo-code of the algorithm is given below:

Algorithm 1: Procedure Tabu Search(x)

```
begin
x_op:t := x;
f_opt := f(x);
while (termination criteria not satisfied) do
   begin
   evaluate f(.) for all x ∈ N(x)
   if ∃ f' | f(x') > (<) f_opt then
       x_opt := x'
     select x' best point in N(x)
   update tabu list
   x:= x'
```

end
end

A. Insert Move and Neighborhood and Measure of influence

Insertions are used as the primary mechanism to move from one solution to another. We define *Insert-Move* (p_j,i) function adapted from Laguna et al (1998) and it consists of deleting p_j from its current position j to be inserted in position i, in other words placing p_j between p_{i-1} and p_i. Thus we get the following ordering p' as follows:

$$p' = \begin{cases} (p_1, \dots p_{i-1}, p_j, p_i, \dots, p_{j-1}, p_{j+1}, \dots, p_n) & \text{for } i < j, \\ (p_1, \dots p_{j-1}, p_{j+1}, \dots, p_i, p_j, p_{i+1}, \dots, p_n) & \text{for } i > j. \end{cases}$$

The neighbourhood N consists of all permutations resulting from executing insertion moves and it is given as

$$N = \{p' : Insert - Move(p_j, i), \text{for } j = 1, \dots, n \text{ and } i = 1, 2, \dots j-1, j+1, \dots n\}$$

The *first* strategy, scans the list of sectors in the order given by the current permutation in search for the first sector (p_f) whose movement results in a strictly positive move value. The move selected by the first strategy is then *Insert-Move*(p_f, i^*), where i^* is the position that gives the best value.

For each sector, there are at most m-1 elements that may contribute to the objective value. The elements in the main diagonal are excluded because their sum does not depend on the ordering of the sectors. During search intensification, sectors should not be chosen with equal probability; we thus define weight values that will be used to bias the selection of sectors during the tabu search intensification phase. The weight of sector j w_j, is defined as $w_j = \sum_{i \neq j} (e_{ij} + e_{ji})$.

B. Intensification and Diversification Phase

The tabu search procedure starts from a randomly generated permutation p and alternates between an intensification and diversification phase. The intensification phase starts by randomly choosing a sector j with probability proportional to its weight w_j. The *Insert-Move* $(p_j,i) \in N$ with the largest move value is selected. The moved sector becomes tabu-active for *TabuTenure* iterations, and therefore it cannot be selected for insertions during this time.

The number of times that sector j has been chosen to be moved is recorded in the value *freq(j)*. This frequency will be used in the diversification phase. The intensification phase is terminated after a number of predefined number of iterations, *maxint*, without improvement. Before switching to the diversification phase, *first(N)* procedure is applied to the best solution found. This solution is denoted by $p^{\#}$ which represents at least a local optimum solution. In this paper we proceed the intensification phase with path relinking. The solution $p^{\#}$ is subjected to a relinking process. The process consists of making moves from $p^{\#}$ in the direction of a set of elite solutions or guiding solutions. This can be done as follows:

1. Find the position i for which the absolute value of (j-i) is minimized, where i is the position that p occupies in at least one of the guiding solutions.
2. Perform *Insert-Move*(p_j,i).

During the path relinking phase, a number of intermediate solutions are generated. These intermediate solutions are good candidates for additional exploration by applying a local search procedure. We apply *first(N)* to intermediate solutions once every four path relinking iterations. The path relinking process is terminated when all the sectors have been considered.

In the diversification phase, each sector is chosen at random with a probability which is inversely proportional to the *freq(j)*. The chosen sector is placed in the best position, as determined by the move values associated with the insert moves N^j. A long term diversification phase is implemented to complement the diversification phase. It is applied after a pre-defined number of iterations have elapsed without any improvements in the move values. The long term diversification phase is proceeded as follows: For each sector p_j, a rounded average position $\alpha(p_j)$ is calculated using the positions occupied by this sector in the set of elite solutions and the solutions visited during the last intensification phase. Then, m diversification steps are performed which insert each sector p_j in its complementary position m- $\alpha(p_j)$, i.e., *Insert-Move*$(p_j$, m- $\alpha(p_j))$ is executed for j = 1, …,m.

IV. SIMULATIONS AND RESULTS

The economic data considered in this study was obtained from the Central Statistical Office in Mauritius. Since the I/O tables are prepared on an interval of around 5 years, we considered the only available data for the years 1981, 1987, 1992, 1997 and 2002. When we compared the tables for the different years, we noted that there was a lack of similarity in the classification of the economic sectors. This can be explained by the addition of emerging sectors in this economy especially during the period 1997 to 2002. The latest update on the classifications is given in Table II, where each economic sector is assigned to a node as shown in Table I. For example, the construction sector was given the node 10 in 1981 and 1987 and assigned to the node 12 in the 1997 and 2002. This numbering process is an essential step for carrying out the Tabu search.

After representing the I/O tables as a linear ordering problem, we have applied the Tabu search algorithm to maximize the sum in the upper triangular part of the resulting input output matrix. In this process, a permutation vector is generated and the order in which its elements appear indicates the importance of the corresponding economic sector. In Table III, we see that for the year 1987, Sector 9 (Electricity distribution service; gas and water distribution services through mains) is ranked as the sector which generated most funds. Also, we see that for the year 1997, Sector 13 (Wholesale and retail trade, maintenance and repair services) is ranked first instead. This process of ranking sectors allows us to see the evolution in terms of importance of each sector in the economy. We can also observe the decline in the flow of funds for the sugar industry from the year 1981 to 2002.

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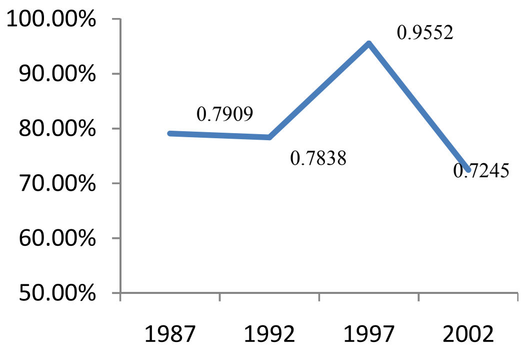

Fig. 2. Plot of linearity coefficient for the period 1987-2002.

In Fig. 2, we show the linearity coefficient of the Mauritius economy as calculated in Eq. (2). Our results show the strengthening of the economy in 2002 and actually Mauritius is no more considered as an under developed country but as a developing country.

TABLE I
CLASSIFICATION OF THE ECONOMIC SECTORS IN MAURITIUS

Product description based on Central product Classification	Number
Products of agriculture, horticulture and market gardening, forestry and logging except sugar cane	1
Sugar cane	2
Live animals and animal products	3
Fish and other fishing products	4
Coal and lignite, crude petroleum and natural gas, metal ores, stone, sand and clay	5
Meat, fish, fruit, vegetables, oils and fats, dairy products and grain mill products	6
Sugar	7
Beverages and tobacco products	8
Yarn and thread; woven and tufted textile fabrics and textile articles other than apparel	9
Knitted or crocheted fabrics; wearing apparel and leather products	10
Other manufactured goods	11
Constructions	12
Wholesale and retail trade, maintenance and repair services	13
Lodging; food and beverages serving services	14
Land, water, air, supporting and auxilliary transport services	15
Telecommunications, postal and courier services	16
Electricity distribution service; gas and water distribution services through mains	17
Financial intermediation, insurance, leasing and auxiliary services	18
Real estate, research and development, professional, scientific and technical services	19
Public administration and other services to the community as a whole; compulsory social security services	20
Education services	21
Health and social services	22
Sewage and refuse disposal, sanitation and other environmental protection services	23
Services of membership organisations	24
Recreational, cultural, sporting and other services	25
Other Services	26

TABLE II
CLASSIFICATION OF THE ECONOMIC SECTORS IN MAURITIUS(1981-2002)

Product description based on Central product Classification	1981	1987	1992	1997	2002
1	4	4	4	1	1
2	1	1	1	2	2
3	3	3	3	3	3
4				4	4
5	5	5	5	5	5
6	2	2	2	6	6
7	6	6	6	7	7
8				8	8
9			8	9	9
10	7	7	7	10	10
11	8	8	9	11	11
12	10	10	11	12	12
13	11	11	12	13	13
14	12	12	13	14	14
15	13	13	14	15	15
16				16	16
17	9	9	10	17	17
18	14	14	15	18	18
19				19	19
20			16	20	20
21				21	21
22			17	22	22
23				23	23
24				24	24
25				25	25
26	15	15			26

TABLE III
RANKING OF ECONOMIC SECTORS BY USING TABU SEARCH

Rank	1981	1987	1992	1997	2002
1	9	9	10	13	17
2	14	14	15	1	19
3	15	11	17	17	4
4	13	4	14	9	15
5	11	5	12	3	18
6	4	8	5	11	16
7	2	13	3	25	21
8	5	10	4	19	12
9	8	2	8	18	14
10	3	15	7	15	5
11	10	1	9	4	1
12	12	6	11	6	3
13	7	3	1	5	23
14	1	12	6	16	10
15	6	7	2	23	24
16			13	24	2
17			16	2	7
18				7	6
19				8	8
20				10	9
21				21	11
22				14	13
23				12	25
24				22	26
25				20	22
26					20

V. CONCLUSIONS

In this paper we have seen how the input-output table can be posed as a linear ordering problem and showed it can be used to determine the importance of each economic sector in an economy. This tool can be used when preparing budgets and it provides an idea on the amount of funds to be allocated in the different sectors of the economy so that the overall return on investment is maximized. We have also calculated the coefficient of linearity in an attempt to reflect the status of the economy.

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