Power System Security Constrained Economic Dispatch Using Real Coded Quantum Inspired Evolution Algorithm

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Abstract—This paper presents a new optimization technique based on quantum computing principles to solve a security constrained power system economic dispatch problem (SCED). The proposed technique is a population-based algorithm, which uses some quantum computing elements in coding and evolving groups of potential solutions to reach the optimum following a partially directed random approach. The SCED problem is formulated as a constrained optimization problem in a way that insures a secure-economic system operation. Real Coded Quantum-Inspired Evolution Algorithm (RQIEA) is then applied to solve the constrained optimization formulation. Simulation results of the proposed approach are compared with those reported in literature. The outcome is very encouraging and proves that RQIEA is very applicable for solving security constrained power system economic dispatch problem (SCED).

Keywords—State Estimation, Fuzzy Linear Regression, Fuzzy Linear State Estimator (FLSE) and Measurements Uncertainty.

I. INTRODUCTION

Optimization is the act of achieving the best possible solution to a problem in which there may be number of competing or conflicting parameters. The cost or the benefit can usually be expressed as a function of a set of design variables. Hence, optimization is a process of finding the settings that give the maximum or the minimum value of a function. Complex optimization problems arise all the time in such fields as science, engineering, business, industry, mathematical and computer science. Specialized methods such as gradient descent methods and other calculus based methods apply very well to certain non-complex problems. For many problems these specialized methods will not work and more robust techniques are required. Typically the objective function can be considered as a "black box" and may be non-convex with many local optima, non-differentiable, and possibly discontinuous. Both continuous and discrete variables may be involved in the problem.

In this paper, a new optimization method is presented. The developed method is novel in the concept of searching mechanism to find the optimum. The proposed Real Coded Quantum-Inspired Evolution Algorithm (RQIEA) is based on quantum computing principles which incorporate the ideas stemmed from the behavior of quantum computing elements encapsulated in a structured evolution process. Quantum computing is based on several phenomena of the quantum world that are fundamentally different from those encountered in classical computing. Such phenomena are complex probability amplitudes, quantum interference, quantum parallelism, quantum entanglement and the unitary nature of quantum evolution.

The main objective of this study is to introduce the proposed Real Coded Quantum-Inspired Evolution Algorithm to the subject of power system dynamic security with the most economical operating conditions. In this study RQIEA is employed to solve the security constrained economic dispatch problem (SCED). This new approach will be used to minimize the system objective cost function satisfying a set of constraints.

II. FUNDAMENTALS OF QUANTUM COMPUTING

Quantum computing is the result of merging quantum mechanics principles with computing. Quantum computing is shown to perform computation in polynomial time for problems, which would exhibit exponential computational complexity on a classical computer. Quantum computers are theoretically exponentially more powerful than classical computers [1]. The speed in processing is achieved by exploiting the quantum parallelism. Shor and Grover’s quantum algorithms are clear examples. The power of quantum computing relies basically on the laws of quantum mechanics, and they are fundamentally different from those in classical computing. Quantum computing requires an understanding of quantum parallelism, quantum entanglement, quantum evolution, quantum interference, and complex probability amplitudes. Computations based on quantum mechanics phenomena, processes and laws offer radically new and powerful possibilities, and lead to different constraints than those achieved on classical computers.
III. REAL CODED QUANTUM INSPIRED EVOLUTION ALGORITHM

The proposed Real Coded Quantum-Inspired Evolution Algorithm (RQIEA) consists of a register that holds a number of qubits and a quantum gate. Qubits hold the information in a superposition of all the states, while the quantum gates evolve the information to reach the desired objective, which in optimization is the maximum or the minimum. The work of the register and the quantum gate are guided through a set of rules. The Grover’s search algorithm [2] is designed to separate the required solution from the other data that are simultaneously presented in the quantum register. This separation is achieved by reinforcement of the amplitude of the desired state through interference between different states. This actually represents the basic idea of the proposed algorithm. The proposed algorithm uses the interference between qubit amplitudes for each problem variable guided by the cost (objective, fitness) function and a set of rules for updating qubits. In the following a description of the RQIEA, including representation of state variables (qubits) and the quantum gate is presented.

The binary coded QIEA proposed by [3] lacks fine local tuning capabilities. Therefore a modification is required to overcome such problems, and this leads to this new type of QIEA. Finding a global optimum in the continuous domain is a challenge to QIEA. The binary representation in QIEA has been used for solution vectors, which even discretizes real design space. Although such binary coded QIEAs have been successfully applied to benchmark optimization problems [3]. Binary coded QIEAs do not perform well when applied to more complex real problems involving large numbers of real valued design variables. This is because binary subvectors representing each parameter with the desired precision are concatenated to form a solution vector for the QIEA, and the resulting vector encoding a large number of design variables results in a huge vector length. To get around this problem one could sacrifice the precision or try to narrow down the search regions prior to the optimization. Another drawback of the binary representation applied to optimization problems in continuous domains comes from discrepancy between the binary representation space and the actual problem space. For example, two points close to each other in the continuous space might be far in the binary represented problem space. Also the QIEA search mechanism depends on a partially directed random movement but with partial directive forces to cover most of the search space. But with binary representation it may avoid or pass some regions. This happens when some bits stay unaltered based on their qubits and therefore their variables will only move in neighboring regions. Two opposite behaviors should be properties of QIEA, discovering neighboring points in the region of the best values and covering the entire search domain (exploring and exploiting). This is seen in the behavior of binary coded QIEA as it fine-tunes its search in the promising regions of the search space.

The objective of this paper is to develop robust and efficient quantum-inspired evolution algorithm applicable to complex optimization problems. To achieve this goal, the modification rule for changing the qubit vectors is incorporated with the use of the real value number representation. The real coded QIEA are expected to possess the advantage of the real value number representation to overcome the problem of having large search space that requires continuous sampling.

A. Representation of Qubits

The new QIEA uses a quantum representation of the solutions, which is similar to the variable representation in genetic algorithms except that each bit is associated with two arbitrary numbers namely probability amplitudes \((\alpha, \beta)\) taken as a real value between \([0,1]\). These two arbitrary numbers are the corner stone of the quantum evolution mechanism in the algorithm as it represents the probability amplitudes of the bit. The solution is represented as a combination of two vectors, solution vector \((x)\) and quantum bit vector \((qubit)\). The first vector encodes the solution, which consists of the problem variables, as real value number with a length of \((m)\). The length of the vector \((m)\) is actually corresponding to the number of problem variables. The second vector has the same length and holds a pair of real values \((\alpha, \beta)\) for each bit. These values are the bit probability amplitudes and represent arbitrary values between \([0,1]\) where \(|\alpha|^2 + |\beta|^2 = 1\). This means that \(|\alpha|^2\) represents, for real coded QIEA, the probability of changing the variable state and \(|\beta|^2\) represents the probability of keeping the variable in its current state. Upon measurement, the algorithm erases all the information in the qubit except for the single bit that the measurement reveals. The two vectors, which represent the problem solution, are associated together as shown:

\[
qubit = \begin{bmatrix} \alpha_1 & \alpha_2 & \ldots & \alpha_n \\ \beta_1 & \beta_2 & \ldots & \beta_n \end{bmatrix}
\]

\[
x = \begin{bmatrix} x_1 & x_2 & \ldots & x_n \end{bmatrix}
\]

where \(x = \text{variable value } \in [\text{Upper bound, Lower bound of variable domain}]\).

The qubit representation has the advantage that it can represent linear superposition of all the states (solutions). A classical computer has bits that exist in a state of one or zero, but quantum bits "qubits", exist in a state of one and zero simultaneously. This means that a large, even amount of data could be encoded in amplitudes of a single qubit by means of \(\alpha\) and \(\beta\) as the probabilities (amplitudes) of existing the state in all of the values in the variable domain. This representation has its power when applied to a quantum computer that can perform certain operation to all solutions...
encoded in qubits.

The proposed algorithm is population-based, as it maintains a number of solutions \((n)\) affiliated with their qubits in a register.

\[
\text{Register} = \begin{bmatrix}
\alpha_1 & \beta_1 \\
\alpha_2 & \beta_2 \\
\vdots & \vdots \\
\alpha_n & \beta_n
\end{bmatrix}
\]

(2)

B. Quantum Gates

The quantum gate used in the new version of QIEA is the same as in the binary coded QIA. However, for the purpose of this study, only the Hadamard gate is used with RQIEA (Figure 1b). Based on the results in the previous work in [3], only the Hadamard gate has a stable performance in all test functions used to study the performance of QIEA. For the RQIEA, Hadamard gate is the best selection of quantum gates as it has the average period length compared to other quantum gates. Also the selection is based on investigation by trial and error method for all quantum gates, which showed that the HADAMARD gate performed better than any other quantum gate for RQIEA.

\[
\begin{bmatrix}
1 & -1 \\
\sqrt{2} & \sqrt{2} \\
1 & -1 \\
\sqrt{2} & \sqrt{2}
\end{bmatrix}
\begin{bmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{bmatrix}
\]

Fig. 1 Matrix form of gates: (a) Square Root of NOT, (b) HADAMARD

(c) Quantum NOT, (d) Z, and (c) Rotation.

The qubit matrix multiplies the quantum gate matrix to alter the values of \(\alpha\) and \(\beta\). After a certain number of operations the probability amplitudes of a qubit returns to its first state (Fig. 2). This is considered as a periodic behavior, which is similar to movement of an electron in a quantum system. The number of operations (iterations) is dependent on the quantum gate type.

\[
\begin{bmatrix}
0 & 1 \\
-1 & 0
\end{bmatrix} \times \begin{bmatrix}
\alpha \\
\beta
\end{bmatrix} = \begin{bmatrix}
\beta \\
-\alpha
\end{bmatrix}
\]

Fig. 2 The effect of quantum gate on single qubit

C. Real Coded Quantum-Inspired Evolution Algorithm Structure

The proposed algorithm consists of two phases, the initialization phase and the evolution phase. In the initialization phase the qubits \((\alpha s \ and \ \beta s)\) are given a value of \((1/\sqrt{2})\). This means that qubits \(|\alpha|^2 = 0.5 \ and \ |\beta|^2 = 0.5\) have equal probability amplitudes. Also in the initialization phase an array of random numbers \(R = r_j\) is generated, where \(i = 1, 2, ..., m\) and \(j = 1, 2, ..., n\). The value of the element \(x\) in \(x\) is changed if the value of \(qubit_i\) is less than \(r_j\). Notice that the value of \(r_j\) is different for each bit and acts like the “water level mark” at which it changes the value of the bit (subvector) if it is above this mark line and maintains its value if it is below this line (Fig. 3). After that each vector \(x\) in the register is built based on the qubit values and substituted in the objective function. The best value in the register is stored as \(f_{best}^1 \ and \ x_{best}^1\).

Fig. 3 The probability amplitudes values in qubits are changed by the Hadamard quantum gate. \(r_j\) is a random number placed at the beginning of the algorithm at which it decides whether to change the value of the variable or not

In the next phase, the register is involved for a certain number of generations (iterations) MaxGen. The algorithm then updates the values of qubits based on the rule governing the relationship between each vector \(x_j\) in the register and the best vector \(x_{best}^1\). This rule states that the updating takes place if the parameter \((r)\), which is the distance between the solution variable and the variable in the current best vector, is less of a predefined arbitrary number \((pr)\). \(pr\) is a real value number assigned in the initialization phase of the program which represents the solution precision required.

IV. ED AND SECURITY ASSESSMENT

The dynamic security problem implies evaluating the system performance for all possible postulated contingencies. This means, for actual large systems, thousand of cases to be considered. The use of this approach in economic study of power system, as in our case, adds more computational
difficulties which make it impossible to be practically applicable. Therefore, it is simpler and desirable to have an indicator for different modes of security. Such indicator may be presented in the form of a simple mathematical function (classifier). The security assessment is then considered as a two class classification problem, namely secure or insecure. The classifier function is given as \[ \phi(x) = \omega^T x + \phi_0 \] (3)

where \( x \) is the vector of the system chosen features, \( \omega \) is the coefficient vector, \( \phi_0 \) is an arbitrary constant. Now, the system is considered secure if \( \phi(x) \) is greater or equal to zero. On the other hand if the function value is negative, then the system is insecure.

Power generations, in the power system, can be considered as features in deriving the classifier due to the great effect of it on the dynamic security [6]. In addition, this choice has the additional advantage of reducing the number of optimization variables, since the cost function is also formulated in terms of such powers.

V. SECURITY CONSTRAINT ECONOMIC DISPATCH

Power system security analysis is the process of detecting whether the power system is in a secure state or alert state. Secure state implies that the load is satisfied and no limit violations will occur under present operating conditions and in the presence of unforeseen contingencies. The alert state implies that particular limits are violated and/or the load demand cannot be met and corrective actions must be taken in order to bring the power system back to the secure state. The power system security problems are classified as static and dynamic. The static security problem implies evaluating the system steady state performance for all possible postulated contingencies. This means neglecting the transient behavior and any other time-dependent variations due to load-generation conditions. The dynamic analysis evaluates the time-dependent transition from the pre-contingent state to the post-contingent state. Dynamic security has been analyzed either by deriving dynamic security functions only, or along with the development of some preventive action techniques [7-11].

Often, security analysis is introduced to the economic study of an electric power system in the form of security constraint. A wide variety of optimization techniques have been applied in solving economic load dispatch problems (ELD). Some of these techniques are based on classical optimization methods while others are based on artificial intelligence methods or heuristic algorithms. Many references present the application of classical optimization methods, such as linear programming, quadratic programming to solve the ELD problem [7, 8]. Classical optimization methods are highly sensitive to starting points and some times converge to local optimum solution or diverge altogether. Linear programming methods are fast and reliable but the main disadvantage associated with the piecewise linear cost approximation. Non-linear programming methods have a problem of convergence and algorithmic complexity. Newton based algorithms have a problem in handling a large number of inequality constraints [11]. Methods based on artificial intelligence techniques, such as artificial neural networks, were also presented in many references [9, 10]. Recently, many heuristic search techniques such as particle swarm optimization [11] and genetic algorithms [12] were applied successfully to the ELD problem. Hybrid methods were also presented in some references such as reference [13]. In this reference, the conventional Lagrangian relaxation approach, first order gradient method and multi-pass dynamic programming are combined together.

The optimization problem is formulated as minimization of summation of the fuel costs of the individual generators, as in the economic dispatch. The objective function is then minimized subject to limits on generators outputs, as well as to the linear dynamic security constraints, set by pattern recognition technique [6]. A dynamic security constraint is derived for each contingency to be considered in the system. In mathematical form the problem can be stated as Minimization of:

\[ F = \sum_{i=1}^{N} F_i(P_i) = a_i P_i^2 + b_i P_i + c_i \] (2)

Subject to

\[ \sum_{i=1}^{N} P_{gi} = P_D + P_L \] (3)

\[ P_{g(i)}^{(\min)} < P_{gi} < P_{g(i)}^{(\max)}, i \in N_s \] (4)

\[ \phi_j(P_{gi}) > 0, j = 1, 2, \ldots, N_f \] (5)

where:

- \( F \) is the system overall cost function
- \( N \) is the number of generators in the system
- \( a_i, b_i, c_i \) are constants of fuel function of generator number \( i \)
- \( P_{gi} \) is the active power generation of generator number \( i \)
- \( P_D \) is the total power system demand
- \( P_L \) is the total system transmission losses
- \( P_{g(i)}^{(\min)} \) is the minimum limit on active power generation
- \( P_{g(i)}^{(\max)} \) is the maximum limit on active power generation
- \( N_s \) is the set of generators in the system
- \( P_{gi} \) is the array of active power generation in the system
- \( \phi_j \) is the classifier function related to contingency \( j \)
- \( N_f \) is the total number of contingency under consideration.
VI. APPLICATIONS AND RESULT ANALYSIS

To demonstrate the accuracy of the proposed method, RQIEA has been applied to solve two standard test systems are used [4, 5, 12]. A set of Matlab files implementing RQIEA algorithm has been built in order to solve the security constrained economic dispatch problem. The pseudo-code proposed of RQIEA is shown in the appendix.

The security functions used with the two systems for different three phase faults at different locations were derived before in reference [4]. For simplicity, transmission losses are ignored in the test.

A. Test System 1 [4, 5, 12]

Fig. 4 shows the 2-machine, 5-bus, 6-line test-system considered to test the proposed method. The system data is given in references [4, 5]. The system dynamic security has been studied under six different three-phase faults listed in Table I. The system classifiers which are given and driven in reference [4] are also shown in this Table I and the total demand = 7 p.u.

<table>
<thead>
<tr>
<th>Classifier#</th>
<th>Faultline</th>
<th>Near Bus</th>
<th>Classifier parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3-5</td>
<td>3</td>
<td>4.0 42000 -190000</td>
</tr>
<tr>
<td>2</td>
<td>4-5</td>
<td>5</td>
<td>2.0 36250 -381250</td>
</tr>
<tr>
<td>3</td>
<td>4-5</td>
<td>4</td>
<td>209 69875 -111875</td>
</tr>
<tr>
<td>4</td>
<td>3-5</td>
<td>5</td>
<td>4.0 42000 -190000</td>
</tr>
<tr>
<td>5</td>
<td>3-4</td>
<td>4</td>
<td>025 87812 -115371</td>
</tr>
<tr>
<td>6</td>
<td>3-4</td>
<td>4</td>
<td>320 112487 -141125</td>
</tr>
</tbody>
</table>

**TABLE I**

DYNAMIC SECURITY CLASSIFIERS FOR DIFFERENT CONTINGENCIES (SYSTEM 1)

The overall cost function $F$ will be,

$$F = F_1 + F_2$$  \hspace{1cm} (6)

where $F_1, F_2$ are the cost functions of the two generators given as:

$$F_1 = (155.5 + 0.489P_{g1} + 0.00393P_{g1}^2) \hspace{1cm} \text{$/hr$}$$  \hspace{1cm} (7)

$$F_2 = (59.4 + 0.703P_{g2} + 0.00745P_{g2}^2) \hspace{1cm} \text{$/hr$}$$  \hspace{1cm} (8)

Now, the problem can be stated as:

**Minimize $F$**

**Subject to:**

- Power balance equality constraint
  $$P_{g1} + P_{g2} = 7$$  \hspace{1cm} (9)

- Inequality constraints presenting the generation limits
  $$7 \geq P_{g1} \geq 3$$  \hspace{1cm} (10)
  $$4 \geq P_{g2} \geq 1$$  \hspace{1cm} (11)

- System security constraint using system classifiers given in table 1 and equation (3), the security constraints can be written as:

  $$1 \geq 4 \gamma_1 + 19 \gamma_2$$  
  $$-2 \geq 9.625 \gamma_1 + 38.125 \gamma_2$$  
  $$209 \geq 68.875 \gamma_1 + 111.875 \gamma_2$$  
  $$325 \geq 87.812 \gamma_1 + 115.371 \gamma_2$$  
  $$320 \geq 112.687 \gamma_1 + 141.125 \gamma_2$$  \hspace{1cm} (12)

**Optimal solution of test system 1**

The problem formulated above is solved using RQIEA optimization technique. The population size is 20. The constraints were augmented with the objective using penalty equal to $10^3$. Extensive runs, about 100 runs, show that the best generation states are:

$$P_{g1} = 4.6690 \hspace{1cm} \text{p.u.} \hspace{1cm} P_{g2} = 2.330 \hspace{1cm} \text{p.u.}$$

This state of generation is the absolute secure operating point for this system under any of the considered faults. The most economic generation cost for this situation is

$$F_{min} = 1868.61 \text{$/hr$}$$

Fig. 5 illustrates the convergence of RQIEA, where it clearly shows that the minimum has been obtained after 180 iterations. In addition the elapsed time is 1.25 seconds. It is important to mention that the worst solution obtained in 100 runs was 1869.983 $/hr$. 

![Fig. 4 Single line diagram of system 1](image_url)

![Fig. 5 Reduction cost function](image_url)
Table II shows a comparison of the optimal generators output obtained by RQIEA and those obtained in literature, [5, 12], by genetic algorithms and quadratic programming for test-system 1. Note that the cost obtained by RQIEA is almost the same as that obtained by QP (higher by less than 0.01%). On the other hand, optimal result obtained by GA is about 3% higher.

**TABLE II**

GENERATORS OUTPUT AND COST FOR TEST-SYSTEM 1

<table>
<thead>
<tr>
<th>Method</th>
<th>$P_{el1}$ (p.u.)</th>
<th>$P_{el2}$ (p.u.)</th>
<th>Cost ($/hr$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQIEA</td>
<td>4.6690</td>
<td>2.3300</td>
<td>1868.16</td>
</tr>
<tr>
<td>GA [21]</td>
<td>4.7214</td>
<td>2.4144</td>
<td>1925.88</td>
</tr>
<tr>
<td>QP [14]</td>
<td>4.6700</td>
<td>2.3300</td>
<td>1868.00</td>
</tr>
</tbody>
</table>

B. Test System 2 [5, 14]

The 7-machine test-system 2, consisting of 10 buses and seven generators, shown in Fig. 6 is used to demonstrate the preventive action control based on RQIEA optimization technique in more details. The system data is given in references [5, 14]. Four faults are considered for this system and the system classifiers are given in Table III and [4].

**TABLE III**

DYNAMIC SECURITY CLASSIFIERS FOR DIFFERENT CONTINGENCIES (SYSTEM 2)

<table>
<thead>
<tr>
<th>Class</th>
<th>Fault</th>
<th>Bus</th>
<th>Current</th>
<th>Voltage</th>
<th>Power</th>
<th>Reactive</th>
<th>Frequency</th>
<th>Speed</th>
<th>Security</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4-5</td>
<td>6</td>
<td>11.1</td>
<td>12.82</td>
<td>50.32</td>
<td>-26</td>
<td>5.15</td>
<td>70.2</td>
<td>0.9</td>
</tr>
<tr>
<td>2</td>
<td>3-6</td>
<td>4</td>
<td>11.2</td>
<td>11.12</td>
<td>50.15</td>
<td>-22</td>
<td>4.92</td>
<td>45.1</td>
<td>1.5</td>
</tr>
<tr>
<td>3</td>
<td>1-4</td>
<td>1</td>
<td>12.5</td>
<td>5.6</td>
<td>18.70</td>
<td>-10</td>
<td>7.8</td>
<td>81.1</td>
<td>0.5</td>
</tr>
<tr>
<td>4</td>
<td>2-3</td>
<td>2</td>
<td>23.5</td>
<td>-9.37</td>
<td>75.10</td>
<td>-10</td>
<td>11.12</td>
<td>79.8</td>
<td>0.9</td>
</tr>
</tbody>
</table>

In the same manner as in test-system 1, the problem is formulated as minimization of the total fuel function, in this case the summation of the seven individual functions, subject to total of 12 equality and inequality constraints.

The security constraint can be obtained directly, as in the previous case, using Table III and equation 1. Before solving the dispatch problem, the population size is equal to 80. The most economic operating state obtained via RQIEA optimization technique, is described in Table IV.

**TABLE IV**

GENERATORS OUTPUT AND COST FOR TEST-SYSTEM 2

<table>
<thead>
<tr>
<th>Method</th>
<th>$P_{el1}$ (p.u.)</th>
<th>$P_{el2}$ (p.u.)</th>
<th>$P_{el3}$ (p.u.)</th>
<th>$P_{el4}$ (p.u.)</th>
<th>$P_{el5}$ (p.u.)</th>
<th>$P_{el6}$ (p.u.)</th>
<th>Cost ($/hr$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RQIEA</td>
<td>1.47490405</td>
<td>1.3392</td>
<td>1.40722067</td>
<td>3.46636402</td>
<td>0.76889027</td>
<td>0.75109552</td>
<td>2980.500</td>
</tr>
<tr>
<td>GA [21]</td>
<td>1.3392</td>
<td>1.40722067</td>
<td>3.46636402</td>
<td>0.76889027</td>
<td>0.75109552</td>
<td>0.71109888</td>
<td>3125.1</td>
</tr>
<tr>
<td>QP [14]</td>
<td>1.40722067</td>
<td>3.46636402</td>
<td>0.76889027</td>
<td>0.75109552</td>
<td>0.71109888</td>
<td>0.8153</td>
<td>3091.3</td>
</tr>
</tbody>
</table>

In this case, as the system size increases, the proposed method shows better performance than the GA and QP in terms of attaining lower cost. This can be seen from table 4, where the total generation cost using RQIEA is $2980.5$ ($/hr$), which is about 4.87% and 3.72% less than the cost obtained using GA and QP respectively. The cost obtained using RQIEA is even less than the cost obtained neglecting the security constraints as reported in ref [5] (using QP). This reported cost was $3077.299$ ($/hr$). This means that the proposed RQIEA method converges to a less cost even with out considering the security constraints.

**Fig. 7** Reduction cost function F of system 2

Fig. 7 shows the convergence of RQIEA. This solution is obtained after 675 iterations which took about 5.88 seconds of an elapsed time. The worst solution obtained in 100 runs was 2991.33 $/hr$.

The main criteria of the assessment of RQIEA are speed, accuracy, robustness, simplicity, and generality [15]. According to the reference these criteria are used to evaluate any new optimization algorithm. In both test examples, Hadamard gate was used as it has positive effect on the RQIEA performance.

From the two figures, 5 and 7, it clear that the method is fast and reach near to the optimum in few iterations.
behavior represents a powerful feature of the new method. Observing the accuracy of the new method, the two test problems were performed for over 100 times and the worst results obtained are very close to the best answer found. The method is robust as it does not require changing only the main elements of the method which are the quantum gate and the population size. Looking at the code, it is clear that the new method have few code lines. Therefore the method can be coded in any computer language easily. For generality the new method so far can be implemented to different types of problems [3]. The new method was tested on several mathematical benchmark optimization problems and also on mechanical engineering optimization problems. Considering the problems of this study will add more generality to RQIEA. Overall the new method even in its preliminary stages of development it performed better than well known highly developed optimization techniques.

VII. FURTHER ANALYSIS

The first test case will be subjected to further analysis to investigate the performance of the RQIEA. The contour plot of the problem is illustrated in Fig. 8. From the plot it is clear that the region of the minima resides in the left bottom corner. This plot does not account for the constraints.

![Contour Plot of the 2-machine, 5-bus, 6-line test-system problem without constraints](image)

Fig. 8 Contour Plot of the 2-machine, 5-bus, 6-line test-system problem without constraints

In Fig. 9 (a), all the solution vectors are traced which showed that the proposed algorithm covers most of the search area. From this plot it clear that some solution vectors are surrounding the best found in quest to find better answer in this sub-region. While the other solution vectors are searching away from the region of the best answer found and therefore avoid trapping in local optima. Also, the plot illustrates that most of the search resides in the left corner of the search region. This primarily indicates that the RQIEA directed to search in the direction of the promising regions that potentially contains the optimum solutions. In Fig. 9 (b), the high convergence towards near the optimum in the early stages of the run is the main character of this proposed algorithm. Looking at the history of the location of the best solution found during executing the algorithm (Fig. 9 (c)), shows that only few points are founded which gives an indication to the speed of the algorithm in finding the best optima in the search area. By tracing the movement of one solution vector, as in Fig. 9 (d), it is clear that the individual movement covers most of the sub-regions of the search area. The vectors are not attracted to the best solution found but each vector remains in a local region of the search space for a period of time (number of iterations) (Fig. 9 (a), (c) and (d)). The search mechanism of the proposed algorithm keeps one or more variables unchanged while changing the others causes the behavior shown in Fig. 9 (a), which assures the search to cover most of the problem domain and not restrict itself in a local optimum. Nevertheless, results are promising and even at this early stage of development; the algorithm competes well with some matured methods.

![Convergence plot of the best results found, c) History of the position of the best found, and d) History of the Position of one individual during the search](image)

It is also important to point out that the evolution mechanism of the proposed algorithm is based on the interference between the qubits amplitudes motion to guide each solution in a partially random movement towards the optimum. This is done by placing rule that controls the action of changing the variable values. The rule maintains in a solution vector x some variable values unchanged while changing the remaining variable values. This mechanism assures that the solutions cover most of the search area in pursuing the best answer and this eliminates the possibility of falling in a local optimum region.

VIII. CONCLUSION

This paper introduces a new solution approach based on quantum computing principles to solve the problem of power system economic dispatch with security constraints. The proposed Real Coded Quantum-Inspired Evolution Algorithm RQIEA has been tested on a two and a seven unit systems. When compared with GA and QP, the results have
demonstrated that RQIEA outperforms them in terms of reaching a better optimal solution. Based on the outcome of this study, it is worth mentioning that RQIEA can be applied to a wide range of optimization problems in the area of power system.

REFERENCES


