Adaptive Total Variation Based on Feature Scale

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Abstract— The widely used Total Variation de-noising algorithm can preserve sharp edge, while removing noise. However, since fixed regularization parameter over entire image, small details and textures are often lost in the process. In this paper, we propose a modified Total Variation algorithm to better preserve smaller-scaled features. This is done by allowing an adaptive regularization parameter to control the amount of de-noising in any region of image, according to relative information of local feature scale. Experimental results demonstrate the efficient of the proposed algorithm. Compared with standard Total Variation, our algorithm can better preserve smaller-scaled features and show better performance.

Keywords— Adaptive, de-noising, feature scale, regularization parameter, Total Variation.

I. INTRODUCTION

Image de-nosing is to filter out the noise. The challenge is to preserve and enhance important features during the de-noising process. For example, edge, small details, texture are the most universal and crucial features. De-noising via the conventional filters (including linear filters and nonlinear filters) normally does not perform satisfactorily since both noise and features contain high frequencies. This led to a search for more effective alternatives. In recent years, wavelets [1], mathematical morphology [2] and partial differential equation (PDE) [3], [4] have been widely used in image de-noising. One of the de-noising methods that has draw a lot of attention is the Total Variation model proposed by Rudin, Osher and Fatemi[4]. Much of the appeal of Total Variation method lies in achieving numerical accuracy as well as stability and overcoming the basic limitations of all smooth regularization algorithms.

Though Total Variation method has shown impressive performance, recently the shortcomings began to raise attention [5], [6]. In Total Variation model, regularization parameter exhibits a critical behavior [7], [8]. When regularization parameter is very large small features are lost and when it is very small noise can not be removed.

Total Variation with the fixed regularization parameter over entire image is effective for large scale feature [8]. When it is used to process nature image with features at multiple scales, smaller-scaled features such as texture and small details will be partial or entirely removed.

The major aim of this paper is to provide a solution to preserve small features in nature images while removing noise. This is primarily obtained by allowing the regularization parameter to adapt automatically to the local feature scale.

II. TOTAL VARIATION REGULARIZATION

A. Total Variation Model [4]

Total Variation model is a classical PDEs image de-noising algorithm. This algorithm seeks for the solution to the minimization of the Total Variation norm $u(x, y)$ and the fidelity of this image to the noisy image $u_0(x, y)$:

$$\min_u \left\{ \alpha \iint_{D_u} |\nabla u| dxdy + \frac{1}{2} \iint_{D_u} (u - u_0)^2 dxdy \right\} \quad (1)$$

The Euler-Lagrange (E-L) equation is

$$\alpha \nabla \bullet \left(\frac{\nabla u}{|\nabla u|} \right) + (u_0 - u) = 0 \quad (2)$$

Where α is the regularization parameter that determines the trade-off between goodness of fit to the measured image, and the amount of regularization done to the measured image $u_0(x, y)$ in order to recover the desired image $u(x, y)$. The regularization parameter is positive constant and relative to the noise $\eta(x, y)$.

In real world, it is often supposed that noise $\eta(x, y)$ is additive white Gaussian with zero mean and σ^2 deviation. So image de-noising can be formulated as finding

$$\min_u \iint_{D_u} |\nabla u| dxdy$$

$$subject\ to\ \frac{1}{2|D_u|} \iint_{D_u} (u - u_0)^2 dxdy = \sigma^2 \quad (3)$$

This formulation is another common form of Total Variation model with noise-constrained. We note that solving (1) is equivalent to solving (3) when $\alpha = 1/\lambda$, where λ is the Lagrange multiplier computed by

$$\lambda = \frac{1}{2\sigma^2 |D_u|} \iint_{D_u} \nabla \bullet \left(\frac{\nabla u}{|\nabla u|} \right)(u - u_0) dxdy \quad (4)$$

We then have

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:2, 2007

$$\alpha = \frac{2\sigma^2 |D_u|}{\iint_{D_u} \nabla \bullet \left(\frac{\nabla u}{|\nabla u|}\right)(u-u_0)\,dxdy} \qquad (5)$$

This gives us a dynamic value α when noise level σ^2 is assumed to be known.

B. Regularization Properties [8]

The regularization parameter α in Total Variation model determines the properties of the image $u(x,y)$ approximating $u_0(x,y)$. When α is very large, small features are lost, and when it is very small, noise can not be removed. In addition, the change in image intensity δ due to Total Variation regularization is directly proportional to the regularization parameter α and inversely proportional to the scale of the image feature s. It is clear that

$$\delta = \frac{\alpha}{s} \qquad (6)$$

In general, features of very small scale relative to α are essentially removed while other features of larger scale preserved. So this can explain why Total Variation is well suited for images with large-scaled features.

III. REGULARIZATION WITH ADAPTIVE FEATURE SCALE

In real world, scenes are observed at more or less arbitrary scales, thus nature images should remain the same for image features at multiple scales. In order to better preserve small features in nature images, the regularization parameter α should not have a fixed value over the entire image, but locally adapt to feature scale. And the desired information about scale of various image features can be estimated by examining the change in the image due to Total Variation regularization.

A. Automatic Feature Scale Recognition

If we know the change in intensity level due to Total Variation regularization and regularization parameter α, we can find the scale of various image features, by re-writing (6) as

$$s = \frac{\alpha}{\delta} \qquad (7)$$

Here, the scale of a piecewise constant image feature s is defined as the ratio of the area of the feature $|\Omega|$ to its boundary length $|\partial\Omega|$; that is $s = |\Omega|/|\partial\Omega|$. Although this definition does not extend exactly to non-piecewise constant images features, this has heuristic effect on the results for the general cases. In general, each position with scale $s(x,y)$ can be considered as the pixel belonging to certain feature. The bigger $s(x,y)$ is, the larger the certain feature scale is.

Roughly speaking, δ is the change between the regularized image $u(x,y)$ and the original image $u_0(x,y)$, but not considering necessarily the direction (i.e. the sign) of the change. How to compute the intensity change at each position

in image is different from different cases. We first give a scheme for noise-free images, and then discuss how to extend it to noisy images.

For piecewise constant features of noise-free images, $\delta(x,y)$ can be computed on a pixel-by-pixel basis between the image before regularization $u_0(x,y)$, and the image after regularization $u(x,y)$. That is

$$\delta(x,y) = |u(x,y) - u_0(x,y)| \qquad (8)$$

For smooth features of noise-free images, since there is non-uniform change in intensity over the region of the feature of interest, we can take the average change as $\delta(x,y)$, which is given by

$$\delta(x,y) = \frac{1}{|\Omega|} \iint_{\Omega} |u(x,y) - u_0(x,y)|\,dxdy \qquad (9)$$

In order to extract the information about feature scale of noisy image, the above scheme would need to be modified to be effective, as much of change from the original image to regularized image is due to the de-noising that takes place. Moreover, the boundaries of different features are unknown for us and the change of boundary is actually realized by the intensity at the pixels nearer the boundary being reduced more than the intensity change for the pixel locations nearer the center of the feature. Therefore we can compute $\delta(x,y)$ for the noisy image as following formula:

$$\delta(x,y) = \frac{1}{|\Omega|} \iint_{\Omega} |u_0(\tilde{x},\tilde{y}) - u(\tilde{x},\tilde{y})| \omega_{x,y}(\tilde{x},\tilde{y})\,d\tilde{x}d\tilde{y}$$

$$(10)$$

Where $\omega_{x,y}(\tilde{x},\tilde{y}) = \omega(|\tilde{x}-x|,|\tilde{y}-y|)$ is the smoothing function, and the function is satisfied with $\int_{\Omega} \omega_{x,y}(\tilde{x},\tilde{y})\,d\tilde{x}d\tilde{y} = 1$.

As the noise is independent on signal mentioned before, the total intensity change $\delta(x,y)$ can be approximated as $\delta_{signal}(x,y) + \delta_{noise}(x,y)$, the sum of intensity change of signal and intensity change of the noise, respectively. Due to the smoothing role of $\omega_{x,y}(\tilde{x},\tilde{y})$ in intensity change of the noise, $\delta_{noise}(x,y)$ becomes small. Moreover, when the regularization parameter α gets bigger, the proportion of $\delta_{signal}(x,y)$ to $\delta_{noise}(x,y)$ becomes larger. Therefore, when the regularization parameter is sufficiently large, we can attain

$$\delta_{signal}(x,y) \gg \delta_{noise}(x,y) \qquad (11)$$

Here, intensity changes of the noise $\delta_{noise}(x,y)$ in different regions are approximately equal.

In fact, we need not know the actual values of $\delta_{signal}(x,y)$, but rather their relative values, that is, relative to each other. Considering the intensity changes of two regions, which are

World Academy of Science, Engineering and Technology
International Journal of Electrical and Computer Engineering
Vol:1, No:2, 2007

respectively denoted by $\delta'(x,y)$ and $\delta''(x,y)$, we easily get the ratio of the different signal changes, $\delta'_{signal}(x,y)$ and $\delta''_{signal}(x,y)$, as follows:

$$\frac{\delta'_{signal}(x,y)}{\delta''_{signal}(x,y)} \approx \frac{\delta'_{signal}(x,y)+\delta_{noise}(x,y)}{\delta''_{signal}(x,y)+\delta_{noise}(x,y)}$$

$$= \frac{\delta'(x,y)\Big/\delta_{\min}(x,y)}{\delta''(x,y)\Big/\delta_{\min}(x,y)} \qquad (12)$$

Here $\delta_{\min}(x,y)$ is the minimum of intensity change in entire image which is corresponding to the maximum feature scale $s_{\max}$, according to (7). Then the relative value to different scales of image features is given by

$$\frac{s(x,y)}{s_{\max}} = \frac{\delta_{\min}(x,y)}{\delta(x,y)} \qquad (13)$$

B. Choosing Adaptive Regularization Parameter

We know, from formula (6), that more regularization of the image is desired in regions of larger-scaled feature, while less regularization is appropriate in regions of smaller-scaled feature. We suggest the following adaptive regularization parameter

$$\alpha(x,y) = \alpha_0 \bullet \left(\frac{s(x,y)}{s_{\max}}\right)^p \qquad (14)$$

Where p is constant ($1 \le p \le 2$), α_0 is obtained by (5). Therefore the adaptive Total Variation based on feature scale is to solve the following Euler-Lagrange (E-L) equation:

$$\alpha_0 \bullet \left(\frac{s(x,y)}{s_{\max}}\right)^p \nabla \bullet \left(\frac{\nabla u}{|\nabla u|}\right) + (u_0 - u) = 0 \qquad (15)$$

IV. EXPERIMENTS AND DISCUSSION

In this section, toys image [9] from real world is illustrated to show the performance of the proposed algorithm.

Figure 1 is the original clean image and figure 2 is the noisy image. In order to show that the adaptive regularization parameter is appropriate, we normalize the regularization parameter computed by (14) to 8-bit gray image, as given by figure 3. The brightest region in figure 3 is corresponding to the biggest regularization parameter, and the darkest region to the smallest regularization parameter. Comparing figure 3 with figure 1, we find the darkest region in figure 3 is almost corresponding to the region of the smallest feature in figure 1, namely the middle-bottom part of image, and in other parts the change of bright is corresponding to the change of different scales of image features. Therefore, the adaptive regularization parameter is suitable for weighing the change of feature scale in original image.

Then, we explain the efficient for nature image by comparing the standard Total Variation method and the proposed algorithm. Figure 4 is de-noised by standard Total Variation and figure 5 is de-noised by the proposed algorithm. From the two images, it is obvious that the noise in left-up region is removed by both algorithms. However, in middle-bottom of image, textures are better preserved by the proposed method. In addition, our proposed algorithm shows better performance than standard Total Variation in terms of signal noise ratio (SNR) (and also visually).

V. CONCLUSION

An adaptive Total Variation based on feature scale is presented. The proposed algorithm better preserves smaller-scaled features and improve de-noising performance, in comparison with standard Total Variation. However, our scheme is rough in estimating the feature scale of original image. Further improvement may be gained by using more elaborated schemes.

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Figure 1 Original clean image

Figure 4 De-noised by standard Total Variation
(SNR＝16.24)

Figure 2 Noisy image (SNR＝12.53)

Figure 5 De-noised by adaptive Total Variation
(SNR=17.50)

Figure 3 Normalized image corresponding to
adaptive regularization parameter