Development of a Comprehensive Electricity Generation Simulation Model Using a Mixed Integer Programming Approach

Erik Delarue, David Bekaert, Ronnie Belmans, and William D'haeseleer

Abstract—This paper presents the development of an electricity simulation model taking into account electrical network constraints, applied on the Belgian power system. The base of the model is optimizing an extensive Unit Commitment (UC) problem through the use of Mixed Integer Linear Programming (MILP). Electrical constraints are incorporated through the implementation of a DC load flow. The model encloses the Belgian power system in a 220 – 380 kV high voltage network (i.e., 93 power plants and 106 nodes). The model features the use of pumping storage facilities as well as the inclusion of spinning reserves in a single optimization process. Solution times of the model stay below reasonable values.

Keywords—Electricity generation modeling, Unit Commitment (UC), Mixed Integer Linear Programming (MILP), DC load flow.

I. SIMULATION MODEL DESCRIPTION

A. Unit Commitment

THIS paper will introduce and illustrate the development of a simulation model, applied on the Belgian electricity generation system. The model has the characteristics of an advanced Unit Commitment (UC) model with the inclusion of electric network constraints. UC optimization is the determination of the activation of power plants, in order to meet a certain demand for electricity at lowest cost. Amongst the most well described methods to solve UC problems, we find priority listing, Lagrangian relaxation, dynamic programming, mixed integer programming, genetic algorithms etc. For an overview of methods to solve UC problems, we refer to [1]. In the work presented here, a Mixed Integer Linear Programming (MILP) model is developed.

UC models are currently used by several ISO's to perform the day ahead scheduling of the power plants in their system. These optimizations can be referred to as 'security constraint' UC [2]. A single player performs the UC for an overall

system. An example is the PJM market, where recently a MILP model has been put into use to perform the UC optimization [3]. On the other hand, when a single player is responsible for its own UC, this player will try to maximize its profit. The UC he performs is often called a 'price-based' UC [2].

B. Mixed Integer Linear Programming

The MILP approach has been chosen because of the recently improved capabilities of commercial solvers and increasing computational force of modern computers [4]. The MILP approach makes the model very readable and easily adaptable. Furthermore, when the UC problem is written as a MILP, it is possible to include network constraints (DC load flow) in the overall problem formulation.

A MILP problem is a Linear Programming (LP) problem, in which some of the variables can be limited to integer values, or, in a special case, to binary values (0 or 1). The problem is optimized towards a single objective function (i.e., a cost minimization function in the model presented in this work). Several constraints are being put on the variables.

C. Model Outline

In order to solve the UC problem, the time dimension is discretized in hours ('time periods'). The total cost (objective function) is written as the sum of fuel costs and startup costs, over all power plants i of all nodes n over all time periods j:

$$Cost = \sum_{i,j,n} Fuelcost(i,j,n) + \sum_{i,j,n} Startupcost(i,j,n) \quad (1)$$

with:

Cost the overall system cost (variable);

Fuelcost the fuel cost of plant i at node n in period j (variable);

Startupcost the startup cost of plant i at node n in period j (variable).

The fuel cost curves of the power plants are modeled as a step-wise linear function, in order to approximate the typically quadratic shaped cost curve of a power plant [5]. The equations required to define these fuel costs correctly are not presented. Technical constraints (e.g. minimum operating

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:1, No:4, 2007

points, ramp rates) of all power plants are taken into account (equations are not presented).

The constraint that enforces total supply must equal total demand at all time periods is presented in (2) (the use of pumping units is already reflected in this equation):

$$\forall j:$$
$$\sum_{i,n} g(i,j,n) + \sum_{n} (pueff_n \cdot pumpdown(j,n)) = \sum_{n} d_{j,n} + \sum_{n} pumpup(j,n) \tag{2}$$

with:

g presenting the electricity generation of certain unit i at node n in period j (variable);

$pueff_n$ the efficiency of the pumping unit (if any) at node n (parameter);

pumpdown the amount of energy released from storage at node n in period j (variable);

$d_{j,n}$ the demand for electricity at node n in period j (parameter);

pumpup the amount of energy pumped to the reservoir of node n in period j (variable).

The model is able to incorporate the use of a pumping storage unit in the overall optimization (see (2)). A storage unit uses electricity when demand is low to pump up water to a reservoir. In reversed mode, these pumps can also function as turbines to produce electricity by releasing water from the reservoir when demand peaks. The pumps have a certain power limit [MW], while the maximum amount of water that can be stored in the reservoir is expressed in terms of energy [MWh]. The whole pumping process has certain efficiency, typically around 70-80%. As an example, (3) presents the conservation of energy in the reservoir. In this formulation, the number of storage units is limited to 1 per node.

$$\forall n, \forall j, j \neq 1:$$
$$puct(j,n) = pumpup(j,n) - pumpdown(j,n) + puct(j-1,n) \tag{3}$$

with:

puct the total amount of energy stored by the pumping unit of node n at the end of period j (variable).

Besides actual electricity generation, a certain amount of spinning reserves is enforced in the system as well. When a power plant faces an unexpected outage, these reserves can be drawn on quickly and are used to overcome the loss of the failed unit.

Several methods exist to determine the required amount of spinning reserves (e.g., largest unit reserve, percentage reserve etc.). In this work, the size of the largest unit in the system is enforced as spinning reserve. The model is able to determine the optimal contribution of power plants to spinning reserves. Each power plant has a certain maximum contribution [MW] to these reserves and must be activated (generate at least its minimum output) in order to be able to contribute to spinning reserves. If a storage unit's reservoir is filled, it can also

contribute to spinning reserves.

The output of the model consists of hourly values for electricity generation for each power plant, together with pump storage outcomes, contribution to spinning reserves, flows on the network etc. Greenhouse gas emission values due to electricity generation can also easily be obtained.

Only the objective function of the model (1) and two constraints (2-3) are shown, the other equations (definitions of costs, technical constraints of power plants and pumping storage units, spinning reserve requirements) are not shown for the sake of simplicity. For the implementation of part of the constraints, we refer to [6]. The implementation of the electric network will be discussed further on.

The model is implemented partly in Matlab and partly in GAMS (using the Matlab/GAMS link) and is solved using the Cplex 10.0 solver.

II. ELECTRIC NETWORK

A. The Choice for DC Load Flow

A grid principally exists of nodes and branches. Each node is characterized by four variables, although only two are initially given. By using Ohm's and Kirchoff's laws it is possible to calculate the other variables. By doing so all power flows through the branches are known. This method, called power flow or load flow, is well known and extensively described in the literature (see, for instance, [7]).

Load flow describes both active and reactive power flows. A simplified variant of a full load flow is called DC load flow. In a DC load line, resistances are neglected and voltage angle differences are supposed to be small. Furthermore, bus voltages are considered equal to 1 p.u.

The following assumptions have to be made for a DC load flow:

- Voltage angle differences are small:
  $$\sin(\delta_i - \delta_j) \approx \delta_i - \delta_j, \quad \cos(\delta_i - \delta_j) \approx 1$$
- Line resistances are negligible (compared to reactances):
  $$R << X$$
- Flat voltage profile

When taking into account these assumptions the power flow through a line can be described by:

$$P_{line} = \frac{\delta_{ij}}{X_{line}} = \delta_{ij} \cdot B_{line} \tag{4}$$

with $\delta_{ij} = \delta_i - \delta_j$. Equation 4, together with the active power balances, yields a linear relationship between flows over lines and nodal injections. The proportionality factor is the well known PTDF-matrix.

As an example, (5) presents this relationship in a triangular network. $PTDF_{r,s-t}$ is flow on line s-t caused by a unit of injection in node r and withdrawal of this same injection at the reference (swing) node. In (5), node 1 is the reference node.

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
Vol:1, No:4, 2007

$$\begin{bmatrix} PTDF_{2,1-2} & PTDF_{3,1-2} \\ PTDF_{2,1-3} & PTDF_{3,1-3} \\ PTDF_{2,2-3} & PTDF_{3,2-3} \end{bmatrix} \cdot \begin{bmatrix} inj_2 \\ inj_3 \end{bmatrix} = \begin{bmatrix} flow_{1-2} \\ flow_{1-3} \\ flow_{2-3} \end{bmatrix} \qquad (5)$$

B. Justification of Assumptions DC Load Flow

It is not always certain that the assumptions stated above are met. Therefore, Purchala et al. [7] have verified the accuracy of the assumptions and checked the likelihood of the assumptions with the Belgian HV grid as a case study. First, the difference in voltage angles has been plotted for the Belgian grid with rated voltages from 70kV to 380kV and a winter peak scenario of 13 GW. The highest angle differences amount to 6°-7°, but 94% of the voltage angle differences are, however, lower than 2°. These voltage angles are presented in Fig. 1.

Second, the ratio X/R has to be bigger than 4, otherwise the resistance cannot be neglected. This is typically true for overhead lines. The Belgian 220kV and 380kV grid consists solely out of overhead lines. Since in this work, only the 220 kV and 380 kV lines of the Belgian grid are considered (see further), the assumption is permitted. R and X values for different voltage levels in the Belgian grid are presented in Table I.

Third, a flat voltage profile is needed. This means that voltage deviations should be avoided. Again, it is not obvious that this requirement is met. By taking only the 220 kV and 380 kV into consideration, deviations are kept as small as possible. To conclude, the DC load flow, taking into account the Belgian 220 – 380 kV HV grid, should provide reliable results.

Besides the fact that technical constraints must be met in order to use the DC load flow method, there are advantages in using this technique. Most important, the equations for a DC load flow are linear and therefore make this method highly suited to be used in the MILP approach.

Moreover, in techno-economic issues, like the work presented here, DC load flow is well suited because only active power flows are considered.
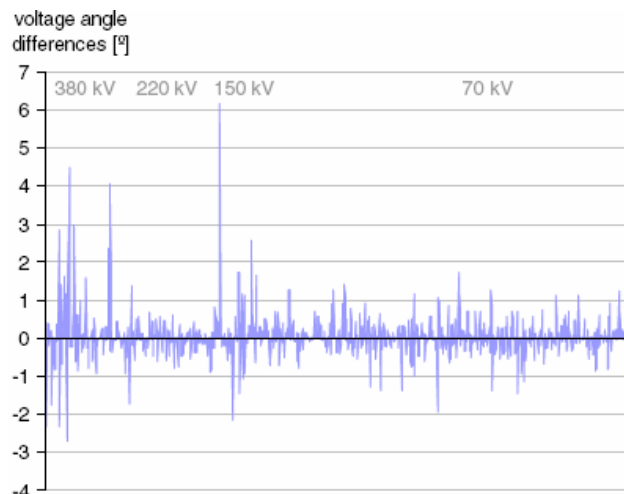

Fig. 1 Voltage angle differences in the Belgian HV grid. Figure taken from [8]

C. Representation of the Electric Network

The Belgian High Voltage (HV) grid consists of several voltage levels, i.e., from 30 kV to 380 kV. The 380 kV lines are the backbone of the Belgian grid. International supplies are carried over these lines. The 150 kV and 220 kV lines carry the electric power from generation to the big load centers and ensure the domestic power supply. The lower voltage levels connect the transmission grid to the distribution grid. Large industrial consumers are directly connected to the HV grid. Although the impact of the 150 kV grid is significant for the Belgian grid as it is meshed and forms a large part of the grid (2356 km 150 kV-lines against 1187 km 220 kV and 380 kV lines on a total of 8344 km HV lines) [9], for pragmatic modeling reasons, only the 220 kV and 380 kV lines are considered in this work.

The electric load distribution throughout the country is based on a solved load flow of the entire grid. Out of this solved load flow, all nodes on the 220 kV and 380 kV level and the lines between them are selected. If there is a power flow from one of these nodes to a node on a lower voltage level through a transformer, the flow is added to the load of the node with the higher voltage, as is illustrated in Fig. 2.

TABLE I
TYPICAL X AND R VALUES IN THE BELGIAN HV GRID

Voltage [kV]	min R [Ω]	max R [Ω]	avg R [Ω]	min X [Ω]	max X [Ω]	avg X [Ω]	min X/R [-]	max X/R [-]	avg X/R [-]
380	0.025	0.038	0.031	0.278	0.353	0.325	8.4	12.5	10.5
220	0.038	0.088	0.067	0.184	0.429	0.364	3.5	8.0	5.5
150	0.018	0.292	0.090	0.071	1.458	0.374	1.0	12.0	4.2
70	0.034	0.425	0.174	0.034	0.756	0.360	0.8	9.0	2.1

World Academy of Science, Engineering and Technology
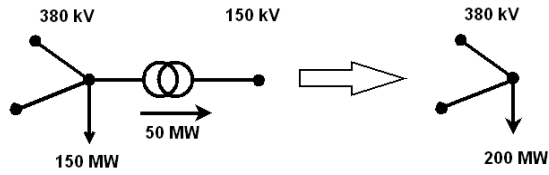International Journal of Energy and Power Engineering
Vol:1, No:4, 2007

Fig. 2 Load redistribution in case of lower voltage level

All internal and cross-border 220 kV and 380 kV lines are implemented. Across the border with the Netherlands, additional lines are also used, i.e., Zandvliet – Geertruidenberg – Eindhoven – Maasbracht. In the south, at the French border the same is done with Avelin – Mastaing – Lonny – Chooz – Moulaine. Both in France and the Netherlands, one extra virtual node is constructed. Via these two nodes a certain cross border flow is forced. In this way the Belgian HV grid can be placed in an international context. The overall model consists of 96 Belgian nodes, 8 foreign nodes, 2 virtual nodes, and 149 HV lines.

III. SIMULATION RESULTS

A. Basic Simulation

This section will demonstrate applications and results of the developed model. In a first a case, a straightforward optimization of the Belgian system is performed. A 24 hour problem is looked at, with a typical Belgian load pattern (winter season) [10], peaking at 13 GW. Fuel prices are taken from [11]. The model contains 106 nodes and 149 lines. 93 Belgian power plants are modeled.

As an example of one of the outcomes of the model, Figure 3 presents the electricity generation in the actual geographical location at the 8th hour, together with the electricity network. As can be seen from this figure, the Belgian electricity generation relies heavily on nuclear based power. The coal fired power plants are also found in base load for the fuel prices assumed. While cogeneration, hydro sources and electricity generation from waste and furnace gases are considered as must-runs, the gas fired power plants are used for modulation.

A similar figure as Fig. 3 could be made for spinning reserves. This is presented in Fig. 4. In this example, spinning reserves consist mainly of pumping storage and gas fired power plants.

In the solution, the storage facilities are used for pumping during the night (when demand is low), while water is released from the reservoir during daytime (when demand peaks).

B. Forced Outage Simulation

This section will demonstrate the simulation of a forced outage of a power plant. The methodology can be described as follows. In a first step, a regular UC optimization is performed. The provided solution is retained up to the hour previous to the hour of an unexpected outage. Then, a new optimization is performed, starting at the hour of the outage, respecting the scheduled on/off states of power plants of the regular overall solution of this hour. If an additional power plant is brought online after a certain loss, this will require a certain startup time. This process is illustrated in Fig. 5.
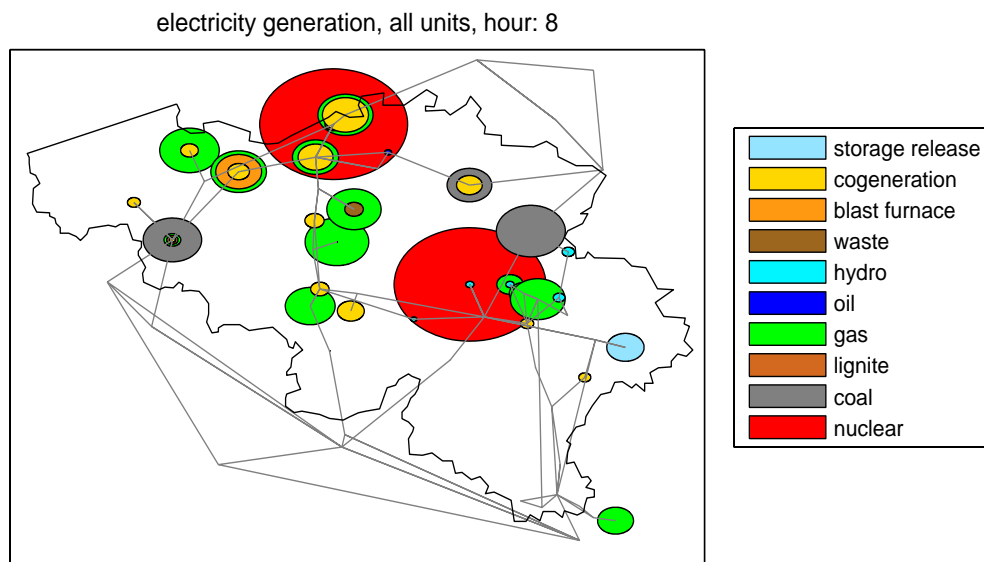

Fig. 3 Geographical representation of electricity generation by fuel type. The surface of the colored ellipses is proportional to the magnitude of the electricity generation

World Academy of Science, Engineering and Technology
International Journal of Energy and Power Engineering
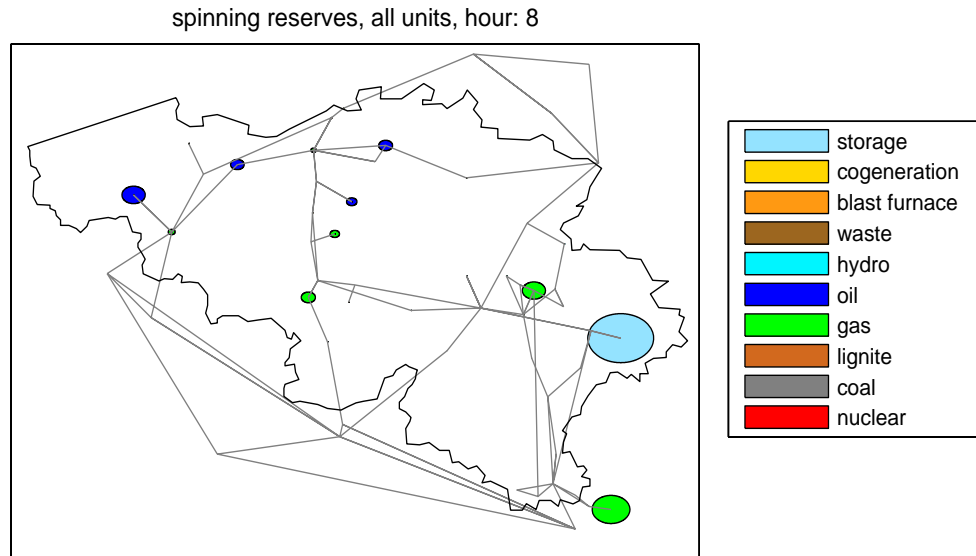Vol:1, No:4, 2007

Fig. 4 Geographical representation of spinning reserves by fuel type. The surface of the colored ellipses is proportional to the magnitude of the spinning reserves
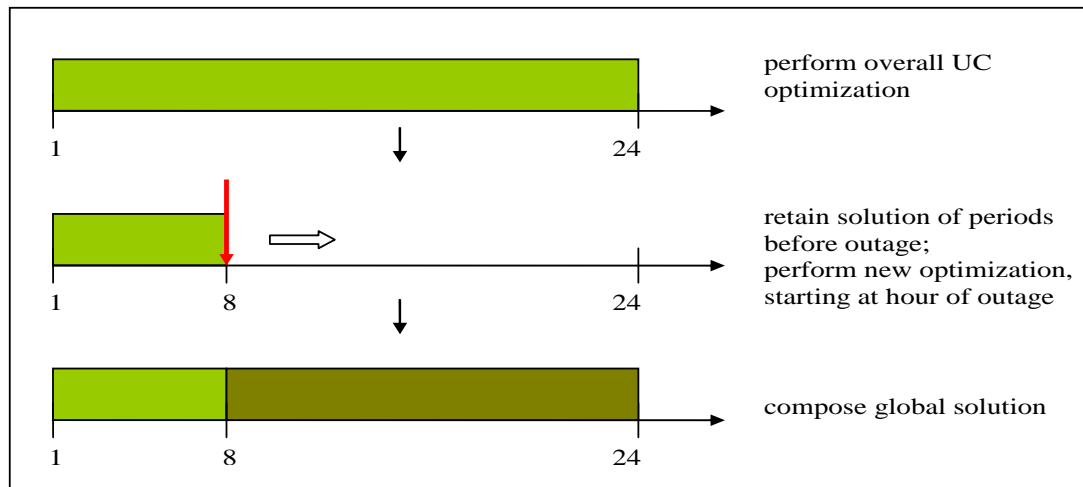

Fig. 5 Methodology used to simulate forced outages of power plants and HV lines. An outage is assumed to occur in hour 8

As a demonstration, the forced outage of a power plant is illustrated. In this case, the Pressurized Water Reactor (PWR) "Doel 3" with a full power capacity of 1006 MW faces an unexpected outage in the 8th hour. In the simulation, the main response of the system is releasing water from the storage reservoir in order to provide enough electricity generation.

Additional (more expensive) electricity is also imported from France. Because of the nuclear loss in hour 8, additional power is to be brought online in order to optimize the electricity production for the rest of the day. Additional gas fired plants are therefore started up in hour 8. These plants become operational in hour 12 (having a startup time of 4

hours). Fig. 6 presents the generation per fuel type aggregated over all nodes, for the 24 hours, with the nuclear outage in hour 8 clearly visible.

Serious shifts in flows on the HV grid will also occur due to the outage. However, the power system is capable to shift generation in such a way congestion on all HV lines is avoided. In this case, the cost of this perturbation (deviation from normal solution) amounts to €410,500 for the considered day.

C. Calculation Time of the Model

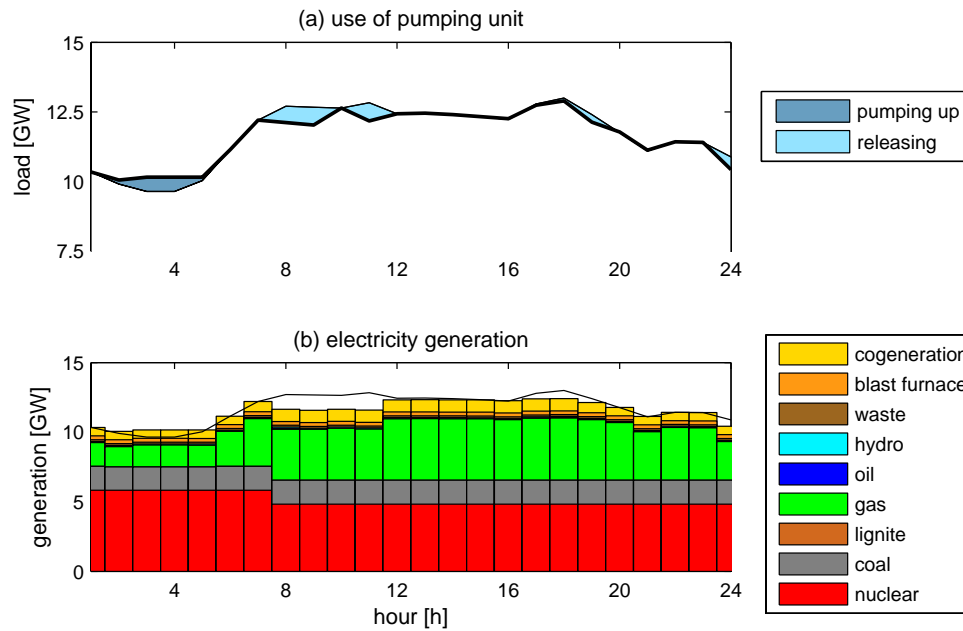All simulations in this paper are run on an Intel® Pentium®

Fig. 6 (a) presents the use of the pumping units (aggregated) and (b) presents the electricity generation by fuel type. The nuclear outage in the 6th hour is clearly shown

Dual Core 3.2 GHz processor with 3.5 GB of RAM. A 24 hour run of the Belgian system takes around 23 seconds to solve (MILP), with 29,150 binary variables. This calculation time increases when solving for larger timescales, for instance, up to 180 seconds for a 6 day problem, having 169,070 binary variables. The MILP solution time depends mainly on the problem structure, size and formulation.

IV. CONCLUSION

This paper has presented the development of an electricity simulation model using MILP, focusing on the incorporation of a representative electricity network, of the Belgian power system. Power plants are modeled in great detail, spinning reserves constraints are respected and pumping storage units are considered. The use of the model has been demonstrated in a 24 hour case.

Solution times of the model stay within reasonable limits (23 seconds for 24 hour case), which make the model very suitable for scenario analysis and reliability studies. An example has demonstrated the simulation of a forced outage of a nuclear power plant.

REFERENCES

[1] Padhy N. Unit commitment – a bibliographical survey. IEEE Transactions on power systems 2004; 19:1196-1205.
[2] Shahidehpour M, Yamin H, Li Z. Market operations in electric power systems. IEEE, Wiley Interscience, New-York, 2002.
[3] Streiffert D, Philbrick R, Ott A. A mixed integer programming solution for market clearing and reliability analysis. Power Engineering Society General Meeting, 2005, vol. 3. IEEE, 2005. p. 2724-2731.
[4] Hobbs B, Rothkopf M, O'Neill R, Chao H. The next generation of electric power unit commitment models. Kluwer Academic Publishers, Boston, 2001.
[5] Wood A, Wollenberg B. Power generation, operation and control – 2nd ed. Wiley, New York, 1996.
[6] Delarue E, D'haeseleer W. Advanced Priority Listing versus Mixed Integer Programming in solving the Unit Commitment problem. Submitted to Energy, February 2007.
[7] Purchala K. Modeling and analysis of techno-economic interactions in meshed high voltage grids exhibiting congestion, doctoral thesis, K.U.Leuven, Electrical Energy Research Group (ELECTA), Leuven, 2005.
[8] Purchala K, Meeus L, Van Dommelen D, Belmans R. Usefulness of DC Power Flow for Active Power Flow Analysis, IEEE PES General Meeting 2005.
[9] Elia. Information on the Belgian grid. Available at http://www.elia.be
[10] Beroepsfederatie van de Elektriciteitssector (BFE). Statistisch jaarboek 2002, 2002. Available (in Dutch) at: http://www.synergrid.be/Media/StatistiekenBFE2002.pdf
[11] IEA. World Energy Outlook 2005, Middle East and North Africa Insights. International Energy agency/OECD, Paris, 2005.