

Control Technology for a Daily Load-following Operation in a Nuclear Power Plant

Keuk Jong Yu, Sang Hee Kang and Sung Chang You

Abstract—In Korea, the technology of a load follow operation in a nuclear power plant has been being developed. In this study, an automatic controller which is able to control reactor average temperature and axial power distribution was developed. The automatic controller is functionally divided into a system identification algorithm and a model predictive control algorithm. The former transforms the nuclear reactor status into model parameters numerically. And the latter uses them and generates optimized manipulated values such as two kinds of control rod positions. Using this automatic controller, the performance of a daily load follow operation was evaluated. As a result, the automatic controller properly generated model parameters of a nuclear reactor and enabled the nuclear reactor average temperature and axial power distribution to track the desired targets during a daily load follow operation.

Keywords—axial power distribution, model predictive control, reactor temperature, system identification

I. INTRODUCTION

MODEL Predictive Control (MPC) [1], which is sketched in Figure 1, is an optimal control method. The basic concept of MPC is to solve an optimization problem for a finite future at current time and to implement the first optimal control input as the current control input. That is, the manipulated variables are sequentially selected such that the predicted output has a certain desirable target and only the first computed value in the manipulated variables is implemented. This procedure is repeated. So, MPC is a suitable control strategy for nonlinear time varying systems such as nuclear reactor because of its concept. Proper system identification is needed to have a good performance using MPC method. In general, when system dynamics are varying over time, recursive system identification is used. It is the name for estimation algorithms where the estimated parameters are updated for each new observation. It relies on fast algorithms where the computation burden and require memory do not increase with time. In this study, for a daily load-following operation, a model predictive control method is developed to optimize the manipulated variables such as control rod positions and the recursive system identification method is applied to generate model parameters of nuclear reactor which is a nonlinear time varying system.

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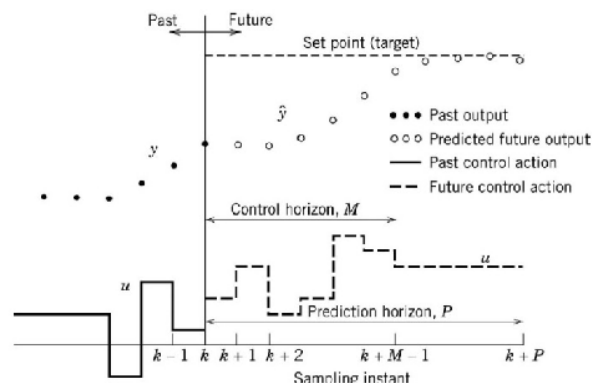


Fig. 1 Basic Concept for Model Predictive Control

II. METHODOLOGY

A. Method of Model Predictive Control

The Generalized Predictive Control (GPC) [2-3] method proposed by Clarke et al. has become one of the most popular MPC methods in both industry and academia. GPC uses a Controller Auto-Regressive Integrated Moving-Average (CARIMA) model for a multivariable process. A CARIMA model can be expressed as

$$A(z^{-1})y(t) = B(z^{-1})u(t-1) + \frac{1}{\Delta} C(z^{-1})e(t) \quad (1)$$

where $A(z^{-1})$ and $C(z^{-1})$ are $n \times n$ monic polynomial matrices and $B(z^{-1})$ is an $n \times m$ polynomial matrix defined as

$$\begin{aligned} A(z^{-1}) &= I_{n \times n} + A_1 z^{-1} + A_2 z^{-2} + \dots + A_{n_a} z^{-n_a} \\ B(z^{-1}) &= B_0 + B_1 z^{-1} + B_2 z^{-2} + \dots + B_{n_b} z^{-n_b} \\ C(z^{-1}) &= I_{n \times n} + C_1 z^{-1} + C_2 z^{-2} + \dots + C_{n_c} z^{-n_c} \end{aligned}$$

The operator Δ is defined as $\Delta = 1 - z^{-1}$. The variables $y(t)$, $u(t)$ and $e(t)$ are the $n \times 1$ output vector, the $m \times 1$ input vector and the $n \times 1$ noise vector at time t . The noise vector is supposed to be a white noise with zero mean. And matrix $C(z^{-1})$ is considered as $I_{n \times n}$. The reason for this is that the coloring polynomials are very difficult to estimate with sufficient accuracy in practice, especially in the multivariable case. The cost function of GPC is shown below. $\hat{y}(t+j|t)$ is an optimum j step ahead prediction of the system output on data up to time t ; that is, the expected value of the output vector at time t if the past input and output vectors and the future control sequence are known. N_1 and N_2 are the minimum and maximum prediction horizons and $w(t+j)$ is a future setpoint or reference sequence for the output vector. R and Q are positive definite weighting matrices.

$$J(N_1, N_2, N_3) = \sum_{j=N_1}^{N_2} \|\hat{y}(t+j|t) - w(t+j)\|_R^2 + \sum_{j=1}^{N_3} \|\Delta u(t+j-1)\|_Q^2 \quad (2)$$

The optimal prediction of the output vector can be estimated using the Diophantine equation as follows

$$I_{n \times n} = E_j(z^{-1})\tilde{A}(z^{-1}) + z^{-j}F_j(z^{-1}) \quad (3)$$

With

$$\begin{aligned} \tilde{A}(z^{-1}) &= A(z^{-1}) = I_{n \times n} + \tilde{A}_1 z^{-1} + \tilde{A}_2 z^{-2} + \dots + \tilde{A}_{n_a} z^{-n_a} \\ &\quad + \tilde{A}_{n_a+1} z^{-(n_a+1)} \\ &= I_{n \times n}(A_1 - I_{n \times n})z^{-1} + (A_2 - A_1)z^{-2} \dots \\ &\quad \cdot + (A_{n_a} - A_{n_a-1})z^{-n_a} - A_{n_a} z^{-(n_a+1)} \\ E_j(z^{-1}) &= E_{j,0} + E_{j,1}z^{-1} + E_{j,2}z^{-2} + \dots + E_{j,j-1}z^{j-1} \\ F_j(z^{-1}) &= F_{j,0} + F_{j,1}z^{-1} + F_{j,2}z^{-2} + \dots + F_{j,n_a}z^{-(n_a)} \end{aligned}$$

where $E_j(z^{-1})$ and $F_j(z^{-1})$ are unique polynomial matrices of order $j-1$ and n_a respectively. If Eq.(1) is multiplied by $\Delta E_j(z^{-1})z^j$:

$$E_j(z^{-1})\tilde{A}(z^{-1})y(t+j) = E_j(z^{-1})B(z^{-1})\Delta u(t+j-1)$$

By using Eq.(3) and after some manipulation we get

$$y(t+j) = F_j(z^{-1})y(t) + E_j(z^{-1})B(z^{-1})\Delta u(t+j-1) \quad (4)$$

The Diophantine equation corresponding to the prediction for $\hat{y}(t+j+1|t)$ is

$$I_{n \times n}E_{j+1}(z^{-1})\tilde{A}(z^{-1})z^{-(j+1)}F_{j+1}(z^{-1}) \quad (5)$$

Subtract Eq.(3) from Eq.(5)

$$0_{n \times n} (E_{j+1}(z^{-1}) - E_j(z^{-1}))\tilde{A}(z^{-1})z^{-j} (z^{-1}F_{j+1}(z^{-1}) - F_j(z^{-1})) \quad (6)$$

The matrix $(E_{j+1}(z^{-1}) - E_j(z^{-1}))$ is of degree j . Let us make

$$(E_{j+1}(z^{-1}) - E_j(z^{-1})) = \tilde{R}(z^{-1}) + R_j z^{-j}$$

where $\tilde{R}(z^{-1})$ is an $n \times n$ polynomial matrix of degree smaller or equal to $j-1$ and R_j is an $n \times n$ real matrix. By substituting in Eq.(6)

$$0_{n \times n} \tilde{R}(z^{-1})\tilde{A}(z^{-1}) + z^{-j} (R_j \tilde{A}(z^{-1}) + z^{-1}F_{j+1}(z^{-1}) - F_j(z^{-1})) \quad (7)$$

As $\tilde{A}(z^{-1})$ is monic, it is easy to see that $\tilde{R}(z^{-1}) = 0_{n \times n}$. That is, matrix $E_{j+1}(z^{-1})$ can be computed recursively by

$$E_{j+1}(z^{-1}) = E_j(z^{-1}) + R_j z^{-j}$$

The following expressions can easily be obtained from Eq.(7)

$$R_j = F_{j,0} \\ F_{j+1,i} = F_{j,i+1} - R_j \tilde{A}_{i+1} \text{ for } i = 0 \dots \delta(F_{j+1})$$

The initial conditions for the recursion equation are given by

$$E_i = I \\ F_1 = z(I - \tilde{A})$$

By making the polynomial matrix $E_j(z^{-1})B(z^{-1}) = G_j(z^{-1}) + z^{-1}G_{j,p}(z^{-1})$ the prediction equation can now be written as

$$\hat{y}(t+j|t) = G_j(z^{-1})\Delta u(t+j-1) + G_{j,p}(z^{-1})\Delta u(t-1) + F_j(z^{-1})y(t) \quad (8)$$

The last two terms of the right-hand side of Eq.(8) depend on past values of the process output and input variables and correspond to the free response of the process considered if the control signals are kept constant, while the first term depends only on future values of the control signal and can be interpreted as the forced response. That is, the response obtained when the initial conditions are zero $y(t-j) = 0, \Delta u(t-j) = 0$ for $j = 0, 1 \dots$. Eq.(8) can be rewritten as

$$\hat{y}(t+j|t) = G_j(z^{-1})\Delta u(t+j-1) + f_j$$

with $f_j = G_{j,p}(z^{-1})\Delta u(t-1) + F_j(z^{-1})y(t)$. Let us now consider a set of N_j ahead predictions

$$\begin{aligned} \hat{y}(t+1|t) &= G_1(z^{-1})\Delta u(t) + f_1 \\ \hat{y}(t+2|t) &= G_2(z^{-1})\Delta u(t+1) + f_2 \\ &\vdots \\ \hat{y}(t+N|t) &= G_N(z^{-1})\Delta u(t+N-1) + f_N \end{aligned} \quad (9)$$

Because of the recursive properties of the E_j polynomial matrix described earlier, Eq.(9) can be rewritten as; The predictions can be expressed in condensed form as

$$y = Gu + f$$

The free response term(f) can be calculated recursively by

$$f_{j+1} = z(I - \tilde{A}(z^{-1}))f_j + B(z^{-1})\Delta u(t+j)$$

with $f_0 = y(t)$ and $\Delta u(t+j) = 0$ for $j \geq 0$. The computation of $G_j(z^{-1})$ and f_j is also considerably simplified. If the control signal is kept constant after the first N_3 control moves, the set of predictions affecting the cost function, Eq.(2), $y_{N_{12}} = [\hat{y}(t+N_1|t)^T \dots \hat{y}(t+N_2|t)^T]$ can be expressed as

$$y_{N_{12}} = G_{N_{123}}U_{N_3} + f_{N_{12}}$$

where $U_{N_3} = [\Delta u(t)^T \dots \Delta u(t+N_3-1)^T]^T, f_{N_{12}} = [f_{N_1}^T \dots f_{N_2}^T]^T$ and $G_{N_{123}}$ is the following submatrix of G

$$G_{N_{123}} = \begin{bmatrix} G_{N_1-1} & G_{N_1-2} & \dots & G_{N_1-N_3} \\ G_{N_1} & G_{N_1-1} & \dots & G_{N_1+1-N_3} \\ \vdots & \ddots & \ddots & \vdots \\ G_{N_2-1} & G_{N_2-2} & \dots & G_{N_2-N_3} \end{bmatrix}$$

with $G_i = 0$ for $i < 0$. Eq.(2) can be rewritten as

$$J = (G_{N_{123}}U_{N_3} + f_{N_{12}} - w)^T \bar{R} (G_{N_{123}}U_{N_3} + f_{N_{12}} - w) + U_{N_3}^T \bar{Q} U_{N_3}$$

where $\bar{R} = \text{diag}(R, \dots, R)$ and $\bar{Q} = \text{diag}(Q, \dots, Q)$. If there are no constraints, the optimum can be expressed as

$$U = (G_{N_{123}}^T \bar{R} G_{N_{123}} + \bar{Q})^{-1} G_{N_{123}}^T \bar{R} (w - f_{N_{12}})$$

Because of the receding control strategy, only $\Delta u(t)$ is needed at instant t . Thus only the first m rows of

$(G_{N_{123}}^T \bar{R} G_{N_{123}} + \bar{Q})^{-1} G_{N_{123}}^T \bar{R}$, say K , have to be computed. The control law can then be expressed as $\Delta u(t) = K(w - f)$. That is a linear gain matrix that multiplies the predicted errors between the predicted references and the predicted free response of the plant.

B. Method of System Identification

The proposed generalized predictive control method needs appropriate parameters of a plant model. The parameters are usually obtained by optimizing a function that measures how well the model, with a particular set of parameters, fits the existing input-output data. When process variables are perturbed by a transient nature, such as a nuclear reactor, the model identification problem is interpreted as a parameter estimation problem. The multivariable CARIMA model described by Eq. (1) can easily be expressed as Eq.(10). That is, the parameter estimation equation using CARIMA model is

$$\hat{y}(t + j) = F_j(q^{-1})y(t) + G_j(q^{-1})\Delta u(t) = \theta^T(t) \cdot \varphi(t) \quad (10)$$

where θ^T is the vector of the parameters to be estimated, $\varphi(t)$ is a vector of the past input and output measures, and $\hat{y}(t + j)$ is a vector of the latest output measures. θ^T , $\varphi(t)$ can be expressed as below.

$$\theta^T(t) = [A_1(t)A_2(t) \cdots A_{na}(t)B_0(t)B_1(t) \cdots B_{nb}(t)]$$

$$\varphi^T(t) = [-y(t) - y(t-1) \cdots - y(t-na+1)\Delta u(t)\Delta u(t-1) \cdots \Delta u(t-nb)]$$

The parameter vector $\theta(t)$ is estimated with the aid of a recursive least-squares method as follows:

$$\theta(t) = \theta(t-1) + \Sigma(t)\varphi(t-1)[y(t) - \theta^T(t-1)\varphi(t-1)]$$

$$\Sigma(t) = \Sigma(t-1) - \frac{\Sigma(t-1)\varphi(t-1)\varphi^T(t-1)\Sigma(t-1)}{\lambda(t) + \varphi^T(t-1)\Sigma(t-1)\varphi(t-1)} \quad (11)$$

$\lambda(t)$ is a forgetting factor and usually used to account for the exponential decay of the past data. The parameters estimated by Eq.(11) are used to predict the future outputs over prediction horizon N .

III. NUMERICAL SIMULATION ON A DAILY LOAD-FOLLOWING OPERATION

The key of a load-following operation is to control the axial power distribution within the operating limits while the reactor power follows the target power. In APR1400, control rods and soluble boron are used as the means for a daily load-following operation. During a daily load-following operation, the power change is controlled mainly by control rods. Boron is used to compensate for power defects when the reactor returns to full power. However, boron is usually provided as a scenario in advance because boron has a slow response characteristic and it is difficult to control its concentration automatically. Therefore, in this paper, control rods and a boron scenario are used as the means for a daily load-following operation of APR1400.

In order to apply a MPC method to a daily load-following operation of APR1400, average reactor temperature (Tavg)

which is proportional to the power and axial power distribution are selected as control variables and two kinds of control rods are selected as manipulated variables. That is, in Eq.(1), $y(t)$ is composed of Tavg and axial power distribution and $u(t)$ consists of the Part Strength Control Element Assembly (PSCEA) and the Full Strength Control Element Assembly (FSCEA). Then, the predictive $y(t)$ is calculated by Eq.(9) and the optimized $u(t)$ is also calculated. However, as above mentioned, the GPC method needs appropriate parameters of a nuclear reactor. Thus, Eq.(9) can be expressed as Eq.(10). And the polynomial parameters (θ^T) of the NPP model, which are optimized to fit the measured values of Tavg and axial power distributions, are calculated by Eq.(11) and these parameters are provided with those of the GPC CARIMA model.

For simulation of a daily load-following operation, KISPAC-1D code [4] is used as a reactor system. The reason why KISPAC-1D code is selected is that it generates one-dimensional power distribution as well as Tavg. In KISPAC-1D code, one-dimensional power distribution called Axial Shape Index (ASI), is calculated by Eq.(12). A ASI is the rate of the upper and lower thermal reactor power. A positive ASI means core power leans toward the bottom and a negative ASI leans toward the top of the core.

$$ASI = \frac{FZBOT - FZTOP}{FZBOT + FZTOP} \quad (12)$$

$$FZBOT = \sum_{i=1}^{13} FZ_i, FZTOP = \sum_{i=14}^{26} FZ_i$$

where FZ_i = relative axial power in the i^{th} node
 FZBOT = power in the bottom half of the core
 FZTOP = power in the top half of the core

In order to connect two methods and the KISPAC-1D code, algorithms of GPC and model identification are coded using standard C programming language, respectively. These codes are coupled as an automatic controller that is capable of receiving and processing control rod positions, Tavg and ASI from the KISPAC-1D code. However, the programming language of KISPAC-1D code is different from that of an automatic controller. So, for the interface between the automatic controller and the KISPAC-1D code, the method of a Dynamic Link Library (DLL) is used. Using the DLL, the KISPAC-1D code calls the automatic controller as a library file. The detailed procedure of the numerical simulation is illustrated in Figure 2. The automatic controller receives Tavg, ASI and positions of the PSCEA and FSCEA as inputs. Using them, the system identification algorithm calculates the proper parameters of the GPC CARIMA model. And the GPC algorithm generates optimized control rod positions in control horizons. Then, the first positions of PSCEA and FSCEA are used for current control inputs. And the KISPAC-1D code receives optimized positions and recalculates a new Tavg and ASI. These procedures are repeated every 4th second.

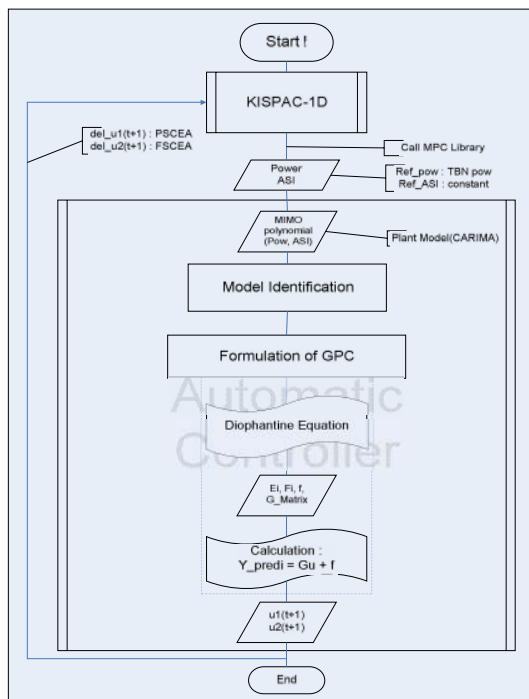


Fig. 2 A procedure for a daily load-following operation using an automatic controller

The target pattern of power change for a daily load-following operation consists of a 25%/h power decrease and increase, respectively. The power decreases from 100% to 50% in two hours after a daily load-following operation starts. Then, the power is maintained at 50% for six hours. After that, the power increases to 100% for another two hours. The boron concentration is slightly changed during simulation and the control rods move only two velocities of 0.127cm/s and 1.27cm/s. For a daily load-following operation, especially, constant average temperature program is used. It means reactor average temperature (Tavg) is not proportional to the reactor power except specific power region. And it has an advantage of reducing reactor temperature feedback effect while reactor temperature is changed. In my study, it is assumed that Tavg is 585°F between 100% power and 75% power. Detailed simulation conditions [5] are given in Table I.

TABLE I
 SIMULATION CONDITION FOR THE DAILY LOAD-FOLLOWING OPERATION

Reactor thermal power	3983 MWth
Fuel burn-up	500 MWD/MTU
Interface time	Every 4 th second
A change rate of Tavg	±10 °F/h
Boron concentration	Simplified(Figure 3)
# of PSCEA and FSCEA	17/76
Reactivity of CEAs	PSCEA : -0.38 %Δp FSCEA(G5) : -0.25 %Δp

	FSCEA(G4) : -0.36 %Δp FSCEA(G3) : -0.80 %Δp
A type of Tavg program	Constant Tavg (75%~100%)

IV. RESULTS

Numerical simulation, using the KISPAC-1D code coupled with an automatic controller, is conducted to verify the performance of a daily load-following operation.

Usually, in a nuclear reactor, reactor temperature is proportional to the reactor power linearly. When reactor power is changed, additional reactivity worth related with the temperature reduction is added. So, for a power change operation such as a daily load-following operation, additional reactivity worth should be controlled and compensated. In this simulation, simplified boron concentration is used in order to compensate the temperature reactivity effect. The boron concentration change is presented in Figure 3.

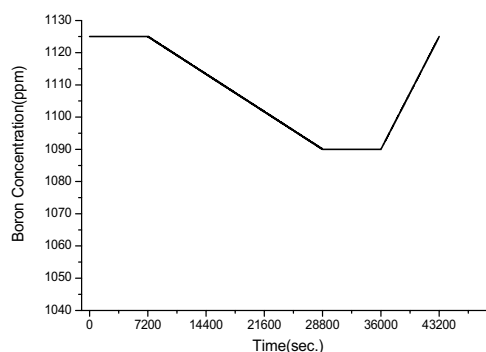


Fig. 3 Boron concentration change for a daily load-following operation

The change of control rod positions and reactor power ratio from 10s to 43,210s are shown in Figure 4. The power(target) is the target power trajectories for the daily load-following operation. And power(MPC) is indirectly controlled reactor power related with Tavg which is one of the controlled variables of GPC algorithm.

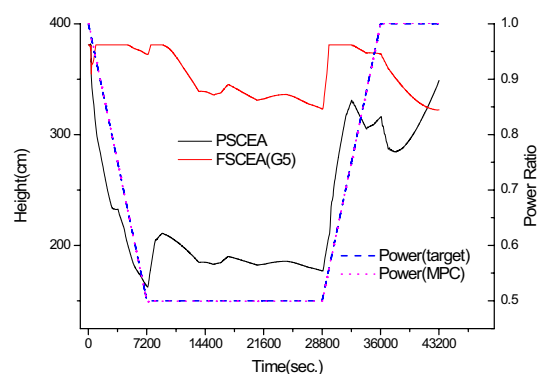


Fig. 4 A change of control rod positions and reactor power ratio every 4th second

The values of the power(target) and power(MPC) are nearly the same and the average deviation of the two powers is less than about 0.2%. As shown in Figure 4, for a daily

load-following operation, the only two types of control rods are used among four control rods. It means they have sufficient reactivity worth for a daily load-following operation in reactor core. Control rods are inserted and withdrawn in order to decrease and increase reactor power related with reactor temperature. However, even though reactor power remains unchanged between 7,200s and 28,800s control rods positions are changed. This is because ASI value which is another controlled variable is changed around 10,000s and xenon concentration which affects reactor reactivity is also changed.

The temperature and ASI from 10s to 43,210s are shown in Figure 5. Tavg.(target) and ASI(target) are the target trajectory of reactor average temperature and ASI. Especially, ASI target value is selected as zero. It means axial power shape is ideally the same as sin function one. And Tavg.(MPC) and ASI(MPC) are the controlled variables of MPC algorithm. In figure 5, Tavg.(target) and Tavg.(MPC) have a good agreement except for beginning of simulation. This is because reactor model parameters estimated by model parameter estimation method are not stabilized.

And at the beginning of the simulation in Figure 5, ASI increases positively because PSCEA and FSCEA are inserted and located at the top half of the core. As control rods are inserted deeply, however, ASI decreases until Tavg reach 575°F around 7,200s. After that, ASI approaches zero value.

And ASI decreases again when Tavg increases from 575°F to 585°F. Considering that ASI operating band of APR1400 is between negative 0.27 and positive 0.27, an automatic controller also control axial power shape properly. Meanwhile, it is confirmed that controlled Tavg by automatic controller is more good agreement than controlled ASI. This is because the automatic controller prioritizes to control reactor average temperature rather than ASI value using reactor average temperature weighting factor in GPC algorithm. Generally, a developed automatic controller properly controls reactor average temperature and axial power shape.

initial core of APR1400 was used. And a simplified boron scenario and constant Tavg program was used to reduce the movement of the control rods. Simulation results confirmed that reactor average temperature and axial power distribution were properly controlled and the developed automatic controller was suitable for the daily load-following operation of APR1400.

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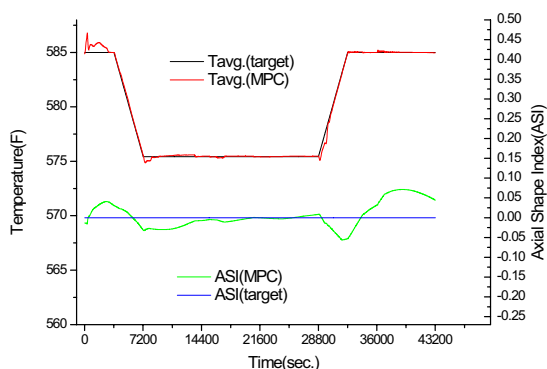


Fig. 5 Comparison of target and controlled results every 4th second

V. CONCLUSION

In this study, an automatic controller has been developed to control the nuclear reactor average temperature and axial power distribution for a daily load-following operation of APR1400. For simulation using a developed automatic controller, the