

# Light Confinement in Low Index Nanometer Areas

N. Aravantinos-Zafiris, and M. M. Sigalas

**Abstract**—In this work we numerically examine structures which could confine light in nanometer areas. A system consisting of two silicon disks with in plane separation of a few tens of nanometers has been studied first. The normalized unitless effective mode volume,  $V_{\text{eff}}$ , has been calculated for the two lowest whispering gallery mode resonances. The effective mode volume is reduced significantly as the gap between the disks decreases. In addition, the effect of the substrate is also studied. In that case,  $V_{\text{eff}}$  of approximately the same value as the non-substrate case for a similar two disk system can be obtained by using disks almost twice as thick. We also numerically examine a structure consisting of a circular slot waveguide which is formed into a silicon disk resonator. We show that the proposed structure could have high Q resonances thus raising the belief that it is a very promising candidate for optical interconnects applications. The study includes several numerical calculations for all the geometric parameters of the structure. It also includes numerical simulations of the coupling between a waveguide and the proposed disk resonator leading to a very promising conclusion about its applicability.

**Keywords**—Disk resonators, field enhancement, optical interconnect, slot waveguides.

## I. INTRODUCTION

THE localization of light in nanometer size areas can have important applications such as nanometer size sensors, surface enhanced Raman scattering [1]-[3] (SERS), nanometer size sources [4], [5]. Most of the research has been focused on metallic nanoparticles such as spheres, rods or triangles. In that case, the incident light excites surface plasmons in the metallic nanoparticles. When these particles are brought close enough with separations of few tens of nanometers or smaller the field enhanced by several hundred times in the gap between the metallic nanoparticles. The enhancement increases as the gap gets smaller [3].

More recently, similar high field enhancements have been found in dielectric (such as silicon) nanoparticles that are in close proximity [6]. There are two important factors for the appearance of these high field enhancements. Resonant modes should exist in the dielectric nanoparticles such as the whispering gallery modes [6]. In addition, the localized modes should be predominantly polarized with their electric field perpendicular to the surface of the dielectric nanoparticles. When two dielectric nanoparticles are brought in close proximity of a few tens of nanometers, the electric field in the

gap between the dielectric nanoparticles increases. As in their metallic counterpart, the electric field increases as the gap decreases. The dielectric nanoparticles can combine very high Q factors (provided that the absorption of the dielectric nanoparticles is negligible) with potentially ultra small mode volumes [7] making ideal candidates for sources and sensor applications.

The last two decades there has also been a lot of academic interest in creating photon based structures that could be used as information carriers within a personal computer. Silicon based structures have been the leading candidate to this sector, a fact resulting from its low fabrication cost, its performance enhancements and its compatibility with the CMOS technology. An optical resonator based on the circulation of light in dielectric volumes enables storage of optical power and is the basic idea for a wide range of fields such as quantum electrodynamics [8], photonics [9]-[11], biosensing [12], non-linear optics [13] and filters [14]. Additionally, studies of optical resonators in glass microspheres [15], microrings [16] and microtoroids [17] gave rise to the applications afforded by the extremely long lifetime Whispering Gallery Modes (WGM) supported by these structures.

Slot waveguides consist of a nanometer-sized low index slot region embedded between two strips of a high-index material. Those structures have recently attracted much academic attention because of a large number of applications that they could perform. Those applications rely on the tight confinement of the optical field in the low index slot region. Microring resonators [18], photonic crystal cavities [19], optical switches [20], signal processing [21] and sensing applications [22] are some of the uses that slot waveguides could support.

Here, two types of structures have been numerically studied. Both, they are able to confine light in low refractive index nanometer size areas close to disk resonators. The first is a system of two disk resonators separated by a few tens of nanometers air gap. In this case, the light is localized in the air gap. In the second structure the light is confined in an in-plane low refractive index slot. The light is confined in the few tens of nanometers thick slot area.

## II. RESULTS AND DISCUSSION

### A. System of Two Closely Spaced Si Disks

In the first part of this work the mode integral is calculated using the finite difference time domain (FDTD) method. The dependence of normalized unitless effective mode volume [7],

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$V_{\text{eff}}$ , on the disk separation and the effect of the substrate are considered.  $V_{\text{eff}}$  is given by the formula:

$$V_{\text{eff}} = \frac{\int \varepsilon(\vec{r}) |\vec{E}(\vec{r})|^2 d^3r}{\varepsilon(\vec{r}_{\text{max}}) \max [|\vec{E}(\vec{r})|^2]} \left( \frac{2n(\vec{r}_{\text{max}})}{\lambda} \right)^3 \quad (1)$$

where  $\vec{r}_{\text{max}}$  is the location of the maximum squared field,  $\vec{E}$  is the electric field,  $\varepsilon$  is the dielectric constant and  $n$  is the refractive index. The disks (shown in Fig. 1) are made of silicon and they have a diameter of 2 micrometers. The silicon is assumed to have a frequency independent refractive index of 3.4 with no-absorption. This is a good assumption for the near infrared frequency region.

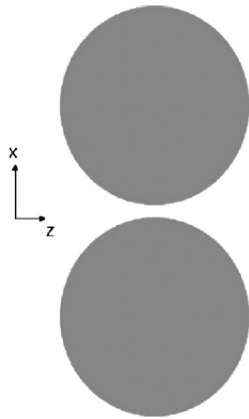


Fig. 1 x-z cross section of the two disks

Fig. 2 shows the  $V_{\text{eff}}$  as a function of the separation between the disks for the case where the disks are surrounded by air and their thickness is 200nm. As it is shown in Fig. 2 of Ref. 6, the electric field in the middle of the gap between the disks as a function of wavelength for a fixed separation shows several peaks. These peaks correspond to the whispering gallery modes resonances of the individual disks. For the lowest resonance,  $V_{\text{eff}}$  has a minimum at 20 nm separation with a value of 0.0274 (black line in Fig. 2). This is the smallest calculated separation. The integral in the numerator of Eq. 1 is reduced by 12% changing the separation from 40 nm to 20 nm while the maximum of the electric field in the denominator increases by a factor of 2. Since the maximum of the electric field increases even more for smaller separations [6], it is expected that the mode volume will get even smaller as the separation decreases.  $V_{\text{eff}}$  for the second lowest resonance shows similar behavior as a function of separation with a value of 0.0545 at 20 nm separation (grey line in Fig. 2). For comparison,  $V_{\text{eff}}$  of the single disk is 0.55 and 0.63 for the lowest and second lowest resonances, respectively.

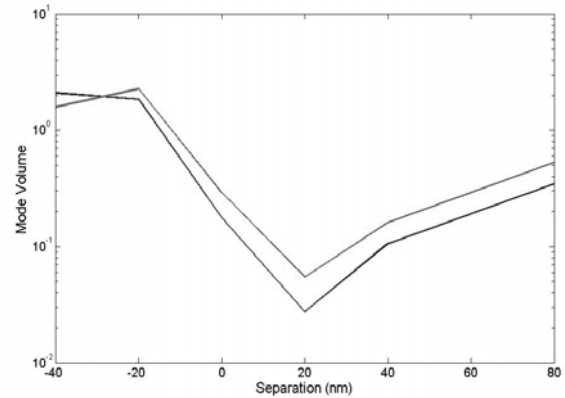


Fig. 2 The change of the normalized unitless effective mode volume is shown as a function of the separation between the disks. Black and grey lines correspond to the longest and the second longest resonance, respectively

By increasing the thickness of the disks and keeping the distance between them at 20 nm (black line in Fig. 3),  $V_{\text{eff}}$  becomes smaller reaching a saturation value of 0.021 for thicknesses higher than 360nm. This is due to the fact that the mode is better confined in the disks resulting in higher Q values and higher values of the field in the area between the disks.

Placing the disks on a  $\text{SiO}_2$  ( $n=1.414$ ) substrate, the enhancement of the field is reduced. This is due to the increased leakage of the resonant modes to the substrate and the corresponding lower values of Q. However increasing the thickness of the disks the enhancement increases reaching a maximum value of 38 for a thickness of 680nm and for the longest wavelength resonance. The grey line in Fig. 3 shows  $V_{\text{eff}}$  as a function of the disk thickness. The behavior is similar with the no-substrate case (compare black and grey lines in Fig. 3). By increasing the thickness,  $V_{\text{eff}}$  decreases reaching saturation value of 0.0272 for thicknesses higher than 680nm. The saturation value of  $V_{\text{eff}}$  is almost the same for both the air and the substrate cases but the thickness of the disks for the substrate case is almost twice as much for the substrate case.

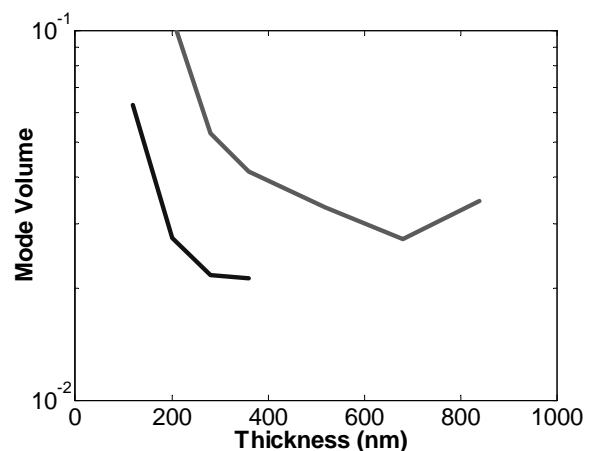


Fig. 3 The change of the normalized unitless effective mode volume is shown as a function of the thickness of the disks. Black and grey lines correspond to the non-substrate and substrate cases, respectively

*B. In Plane Slot Waveguide into a Disk Resonator*

In this work we also numerically investigate a structure which could be considered as a combination of a slot waveguide into a disk resonator and could be used for optical interconnects applications. The structure (yz profile is shown in Fig. 4) is constructed from two silicon layers. The upper layer is a silicon disk on top of a SiO<sub>2</sub> substrate. The lower layer is a silicon film into the SiO<sub>2</sub> substrate and is consisted of a SiO<sub>2</sub> disk created on it. The axis of the upper layer silicon disk and the lower layer SiO<sub>2</sub> disk are coinciding. The lower film is into the SiO<sub>2</sub> substrate whereas the upper silicon film is standing on top of the substrate.

As mentioned previously, two homocentric disks are considered in each film. The upper disk is from silicon and the lower from SiO<sub>2</sub>. The upper disk's radius (symbolized as  $r_u$ ) is larger than the lower disk's radius (symbolized as  $r_d$ ) thus creating an overlap between the two disks. The radius of the upper silicon disk is 2 $\mu$ m and retains this value for all the calculations in this work. The radius of the lower SiO<sub>2</sub> disk is growing from 1.265 $\mu$ m to 1.789 $\mu$ m. The two disks are separated by a thick SiO<sub>2</sub> film which is considered 20nm in our calculations. So even though the structure is based on the well known Silicon on Insulator (SOI) fabrication technique, we could say that it is a different consideration of it, since, as we will show in this work, we take advantage of SiO<sub>2</sub> as the in plane medium.

The numerical investigations were for all the different parameters of the structure. Those parameters are the thickness of the two disks (indicated as  $w$  in Table I), the overlap between them and the thickness of the film that separates them. The excitation is a Gaussian pulse created in the space among the disks and more specifically in the center of the overlap. The well known Finite Difference Time Domain (FDTD) numerical method was used for the simulations [23]. The response results were collected and Fourier transformed in order to find the resonant frequencies for the different parameters of the structure.

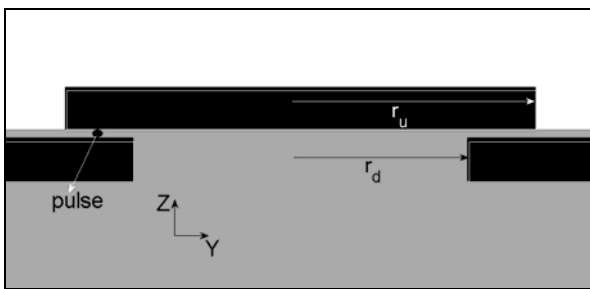


Fig. 4 The y-z plane of the structure which is taken at the center of the x-axis of the simulation cell showing a profile of the two disks. The black (gray) area is silicon (SiO<sub>2</sub>). White color represents air

The results show that this structure could have high Q resonances thus raising the belief that could be used as a candidate for optical interconnects applications. For the highest Q resonance frequencies the numerical investigations took place by considering a continuous sinusoidal pulse. The

well known Whispering Gallery Modes (WGM) appeared in this part of the calculations and the field, for certain values of the structure parameters, is localized in the low index material (SiO<sub>2</sub>) that separates the two films (Fig. 5 and Fig. 6). Fig. 5 shows the field profile of a sinusoidal pulse generated in the middle of the thin SiO<sub>2</sub> film between the two silicon layers. The frequency of the pulse is 276THz, the thickness of the two layers is 0.12 $\mu$ m, the radius of the upper silicon disk is 2 $\mu$ m and the radius of the lower SiO<sub>2</sub> disk is 1.414 $\mu$ m. The thin SiO<sub>2</sub> layer between the two disks is 20nm. Fig. 6 shows the x-z field profile where we can clearly see that the field is focused in the plane between the two silicon layers and especially in the overlap of them.

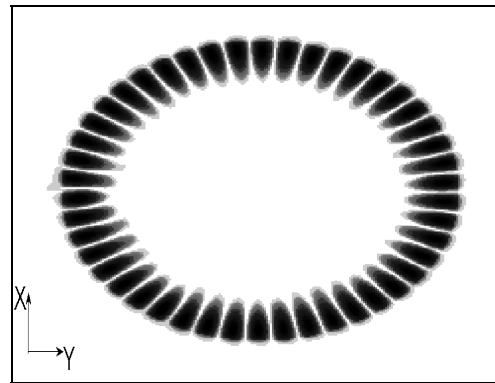


Fig. 5 WGM created in the SiO<sub>2</sub> material in the overlap of the two disks of the structure studied. The frequency of the pulse is 276THz and the thickness of the two layers is 0.12 $\mu$ m

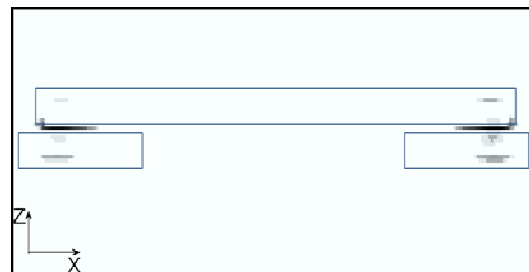


Fig. 6 Field profile showing the localization of the field in the SiO<sub>2</sub> material in the overlap of the two disks of the structure studied. The frequency of the pulse is 276THz and the thickness of the two layers is 0.12 $\mu$ m

In order to have a countable measure of the ability of the structure to hold resonant frequencies we considered an average Q factor ( $Q_{av}$ ). This average Q factor is the mean value of the three highest first order resonance frequencies Q of this part of the work are collected in Table I.

TABLE I  
 AVERAGE Q FACTORS FOR DIFFERENT THICKNESS OF THE TWO DISKS AND  
 DIFFERENT RADIUS OF THE LOWER DISK (OVERLAP CHANGE)

w(μm) \ r <sub>d</sub> (μm)	0.10	0.12	0.14	0.16	0.18	0.20
1.265	99	487	186	107	168	160
1.414	524	496	125	193	197	183
1.550	198	379	365	327	248	147
1.673	257	392	399	246	134	70
1.789	70	91	89	70	48	22

Table I shows that there are certain parameters of the structure which makes it work better, thus giving high Q resonances, reaching the value of 524. This value appears for radius of the lower SiO<sub>2</sub> disk r<sub>d</sub>=1.414μm and thickness of the two layers 0.1μm. The thin SiO<sub>2</sub> layer between the two disks is 20nm and the upper silicon disk has radius 0.2μm. It is obvious that for thickness of the disks equal 0.12μm the structure gives on average higher Q factors for all the values of the lower disk radius.

For thickness of the disks w=0.12μm and radius of the upper silicon disk equal 2μm, we have numerically examined how the Q<sub>av</sub> factor is affected when the separation between the two films becomes 40nm and the radius of the lower SiO<sub>2</sub> disk varies, as previously, from 1.265μm to 1.673μm. Table II contains the results for those calculations. It is clear that for higher separation among the disks the Q<sub>av</sub> reduces dramatically.

TABLE II  
 AVERAGE Q FACTORS FOR DIFFERENT RADIUS OF THE LOWER DISK

w(μm) \ r <sub>d</sub> (μm)	0.12
1.265	40
1.414	100
1.550	83
1.673	71

In order to check the accuracy of our numerical results we made some simulations for the double simulation time. The simulation for thickness of the two films equal to 0.12μm and the radius of the down SiO<sub>2</sub> disk equal to 1.265μm showed

that the Q<sub>av</sub> rises to 1074 which is more than the double of the respective value indicated in Table I. Similar longer time simulations for other low Q cases did not have any significant change in their Q values. So, it becomes clear that Q values higher than about 400 should be significantly higher and their low values are just due to the limiting computational time.

It is obvious that the proposed structure is strongly affected by the thickness of the two silicon layers, the thickness of the middle SiO<sub>2</sub> layer and the radius of the lower SiO<sub>2</sub> disk. The later parameter is in fact the parameter that affects the overlap of the two silicon layers since the difference r<sub>u</sub>-r<sub>d</sub> is the width of the overlap. The proposed structure seems to prefer small thickness of the two films, with the value w=0.12μm to be the optimum, very small width of the thin SiO<sub>2</sub> layer and large overlap of the two silicon layers.

Additionally, numerical calculations performed in order to check the stability of the resonances of the structure when the two disks are not homocentric. In the fabrication process, making a thin SiO<sub>2</sub> layer between the two silicon layers can be more easily control than the precise alignment of the upper silicon disk with the lower SiO<sub>2</sub> disk. The calculations performed in this part of the work were made by moving the upper disk towards y direction, thus achieving the change of the structure. The displacement of the centre of the upper disk (without changing its radius) is symbolized as d. We examined two cases in which r<sub>u</sub>=2μm, thickness of the two disks equal 0.12μm and thickness of the thin SiO<sub>2</sub> layer 20nm. The first case includes radius of the lower disk equal 1.673μm and the second case includes radius of the lower disk equal 1.414μm. It is clear from both cases that the Q<sub>av</sub> of the structure's resonances reduces as the upper disks displacement grows. Tables III and IV contain the results of this part of the work and Fig. 7 contains the diagram of Q<sub>av</sub> versus d. It is clear that Q decreases rapidly as the displacement increases. The reduction is more pronounced when r<sub>d</sub> increases. In that case, the overlap of the upper layer silicon disk with the lower silicon layer is reduced (see Fig. 4) making the concentration of the field in the thin SiO<sub>2</sub> layer between them more difficult.

TABLE III  
 AVERAGE Q FACTORS FOR DISPLACEMENT OF THE UPPER DISK

r <sub>d</sub> (μm) \ d(nm)	1.673
3	486
5	468
8	414
10	294

TABLE IV  
 AVERAGE Q FACTORS FOR DISPLACEMENT OF THE UPPER DISK

$r_d(\mu\text{m})$	1.414
$d(\text{nm})$	
5	486
10	468
15	414
20	294
22	137

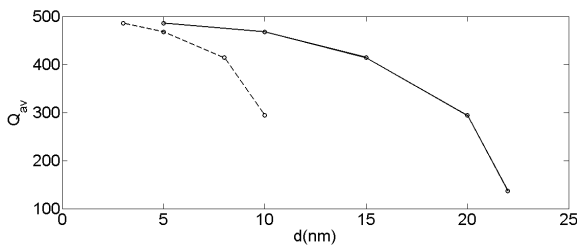


Fig. 7  $Q_{av}$  versus  $d$  of the structure for two different values of the lower disk radius. Solid line is for  $r_d=1.414\mu\text{m}$  and dashed line is for  $r_d=1.673\mu\text{m}$

Similar to the previous structure is the next one. The structure that we numerically examined is almost the same as the first one but we have replaced the  $\text{SiO}_2$  material in the overlap of the two disks by air. The excitation takes place again at the same point as before and we examined how this structure could perform as a disk resonator. After several numerical calculations, we concluded that resonance frequencies with high  $Q_{av}$  appear for higher thicknesses of the two disks. This result is expected since the effective refractive index rises thus leading the field modes to extend in the high index material. The calculations took place for radius of the upper silicon disk  $2\mu\text{m}$  and the most interesting thickness of the silicon film was  $0.2\mu\text{m}$ . The radius of the lower  $\text{SiO}_2$  disk is growing from  $1.265\mu\text{m}$  to  $1.673\mu\text{m}$ . The two disks are separated by a thick film which is considered  $20\text{nm}$  in our calculations and consists of air in the overlap of the two disks and  $\text{SiO}_2$  for a radius the same as the lower disk. We again consider the  $Q_{av}$  factor as a measure of the structures capability to hold resonance frequencies. Although there are very interesting results giving  $Q_{av}=525$  for  $r_d=1.265\mu\text{m}$  and  $Q_{av}=460$  for  $r_d=1.550\mu\text{m}$ , the field is not as much localized in the low index materials (as in the previous cases). Instead, there is significant amount of the field in the silicon.

The next part of our work contains a proposal for coupling of the above two layer structure with a waveguide. So considering an extension of the first proposed structure we

added another part that consists of a straight waveguide which consists of two silicon layers. The upper is above the  $\text{SiO}_2$  substrate and the other into the substrate. This waveguide is in fact an extension towards  $y$  axis of the previous structure. The pulse is created in the beginning of the waveguide into the thin  $\text{SiO}_2$  film between the two silicon layers and we numerically examined how it is transmitted through the  $\text{SiO}_2$  layer (along the  $x$  direction of the simulation supercell) with and without the existence of the disks on the right side of it. The results showed that it is indeed a strong coupling between the two parts as shown by the relative transmission spectrum of Fig. 9.

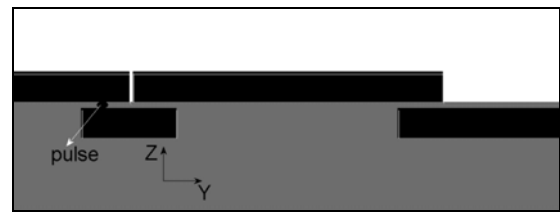


Fig. 8 Disk structure coupled to a waveguide. The pulse is created in the beginning of the waveguide, as shown in figure and transmitted through the thin  $\text{SiO}_2$  film. The figure shows the  $y$ - $z$  plane in the middle of the  $x$  axis of the simulation supercell. White color represents air, grey represents  $\text{SiO}_2$  and black color represents silicon

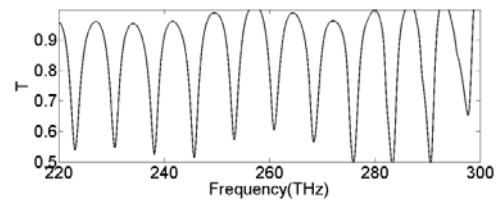


Fig. 9 Mode coupling between a waveguide and the proposed disk structure

### III. CONCLUSION

In conclusion, the normalized unitless effective mode volume ( $V_{eff}$ ) is calculated for the whispering gallery modes propagating in a system of two silicon disks when the separation between the disks is a few tens of nanometers.  $V_{eff}$  is reduced significantly when the disks are brought in very close proximity with a small gap between them. The combination of very high  $Q$  values with ultra small mode volumes in these systems can have significant applications in nanometer size sensors, SERS, and sources.

Furthermore, we studied a structure consisting of a circular slot waveguide, formed in a circular silicon resonator. Several numerical calculations were performed and showed the in plane concentration of the field at the low index material, raising the belief that this structure could be a very promising candidate for optical interconnects applications. For radius of the upper silicon disk equal  $2\mu\text{m}$ , thickness of the two Si films equal  $1.2\mu\text{m}$ , separation of the two films by a thin  $\text{SiO}_2$  layer of thickness  $20\text{nm}$  we calculated  $Q$  values up to 496 when the radius of the lower  $\text{SiO}_2$  disk is  $1.414\mu\text{m}$ . Generally the proposed structure gives better results for small thickness of

the two layers and it is negatively affected when the overlap of the two disks reduces, thus raising the radius of the lower SiO<sub>2</sub> disk. It also gives lower Q values as the thin SiO<sub>2</sub> film between the two layers rises or by moving the centre of the upper silicon disk, making the two disks of the structure non homocentric.

Additionally, a proposed structure for coupling between the circular in plane resonator and a simple waveguide was investigated further supporting its possible use in optical interconnects applications.

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