Nearfield UWB Pulse Array Beamformer based on Multirate Filter Bank

Min Wang, Shuyuan Yang

Abstract—The paper presents a method of designing ultrawide band (UWB) pulse array beamformer in the case of nearfield. Firstly the principle of space-time processing of UWB pulse array is discussed. The radical beampattern transform based on spherical coordinates is employed to solve the nearfield beamforming of UWB pulse array. The frequency invariant technology is considered for the frequency dependent beampattern of UWB pulse array. We use a multirate bank scheme of to implement the FI beamformer of UWB pulse array. By using multirate filters in each element channel, it can make the response of the UWB array to avoid distortion in the whole band. The simulation resultes are given to prove the efficiency and feasibility of this method.

Keywords—UWB pulse array, frequency invariant, multirate bank, nearfield beamformer, radical transform

I. INTRODUCTION

In recent years, the array antenna technology has found numerous applications in the military radar as well as the. wireless communication. In the past, the array antenna technology had mainly used in the narrowband and wideband system. Now with the development of emerging ultrawide-band (UWB) pulse technology in impulse radar and impulse-radio communication^[11], the principle of space-time processing can be also applied in the design and research of the UWB system^[2-7].

The commonly used UWB signal model is Gaussian impulse waveform, which is of high resolution and penetration for its very short duration. Because phase weighting cannot be used, the obvious method is to compensate time delay using a time delay-sum beamformer in UWB system. In order to achieve electronic beam steering for enhancing the received signal from a desired look direction, firstly the time-delay of elements should be estimated correctly and then be compensated. An optical realization is true time delay beamforming in UWB pulse array. However, it is of a complex system. Now the rapid development of sampling technology has made the digital realization of UWB impulse arrays possible, thus some traditional beamforming method in broadband can be used for reference. However, a UWB array has so wide range of frequency band, and its array response varies with the frequency obviously. Consequently a large number of sensors may be needed under general broadband solution. With the increase of the range of frequency, the number of sensors also increases. Moreover, many application occasions of the UWB pulse array cannot satisfy an ordinary farfield condition, such as the diagnose way in some medical treatment^[3]. The goal of paper is to consider the realization of nearfield beamforming in UWB pulse array, which can produce an undistorted response in the whole frequency band.

The frequency invariant (FI) technology can be used in broadband array to get a undistorted array response in the interest frequency scope^[8]. For nearfiled beamforming a time delay compensation^[9] or radical transformation method^[10] can be a solution. In this paper, we applied a method of multirate filter bank based on the FI principle to nearfield UWB pulse array and obtained the same response in whole frequency and angle. Firstly, we convert a desired nearfield beampattern to a corresponding farfield beampattern. Then the multirate filter bank based on FI technology is designed for each element channel aiming at the transformed farfield beampattern. Firstly, the principle of space-time processing of UWB pulse array is discussed. Then the radical beampattern transform is introduced to solve the nearfield beamforming and an improved design steps are given. Subsequently the method constructing multirate band is presented after the description of FI technology. Finally, the simulation results are given.

II. PRINCIPLE OF UWB PULSE ARRAY SPACE-TIME PROCESSING

A. UWB signal model and the principle of space-time processing of UWB pulse array

The commonly used form of UWB pulse is waveform based on Gaussian model. We give a representation of generalized Gaussian pulse (GGP) which has been tested in the transmission and receiving experiment in the following:

$$\Omega(t) = \frac{E_0}{1-\alpha} \left\{ e^{-4\pi \left(\frac{t-t_0}{\Delta T}\right)^2} - \alpha e^{-4\pi \alpha^2 \left(\frac{t-t_0}{\Delta T}\right)^2} \right\}$$
(1)

Here E_0 is the peak amplitude at the time $t=t_0$ (usually $E_0=1$), ΔT is a nominal duration, and α is a scaling parameter.

Paper [2] gives a structure of UWB (carrier free) pulse array beamformer, as shown in figure 1. The beamforming system consists of a linear array of omnidirectional of M=2a+1 sensors uniformly spaced with interelement distance d which are grouped into two subarrays with same size. Each channel of

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beamformer firstly carries out temporal correlation processing equivalent to matched filter for received samples (it yields maximum SNR output and fine temporal resolution). An adjustable digital delay line or a transverse digital filter is employed to get an accurate compensation of time delay. Thus a beam can be formed in the direction of desired signal.

Take m=0 as a referenced point, the time delays of each array unit is a function of the incident angle ϕ and d:

$$\tau_m(\phi) = \frac{md}{c}\sin\phi = \frac{m}{2a}\rho\sin\phi \cdot \Delta T$$
(2)

where $\rho = 2md/(c\Delta T) = L\Delta f/c$, which represents the ratio of the transmitted time of signal along the array to ΔT , we call it the bandwidth of space frequency.

To increase the signal came from the angle ϕ_0 , a time delay $\tilde{\tau}_i$ is exert upon the received signals $x_m(t,\phi) = \gamma(t - \tau_m(\phi))$. $\gamma(t)$ is the correlation function of $\Omega(t)$. When $\tilde{\tau}_i$ and ϕ_0 has

such relations, then a main beam is formed in the angle ϕ_0 :

$$\tilde{\tau}_i / \Delta T = (i/2m) \rho \sin \phi_0 \tag{3}$$

Then we can get the maximum energy of array output. The array response is the sum which can be depicted using the error function:

$$y(t,\phi) = \sum_{i=-m}^{i=m} \gamma(t + \tilde{\tau}_i - \tau_i(\phi))$$
(4)

The directive patterns of UWB pulse array is defined as the ratio of the output energy of array form angle ϕ and ϕ_0 :

$$\tilde{W}(\phi) = \tilde{U}(\phi) / \tilde{U}(\phi_0) \quad (\text{where } \tilde{U}(\phi) = \int_{-\infty}^{\infty} [y(t,\phi)]^2 dt) \quad (5)$$

The width of the main beam is a function of the size L of array and nominal bandwidth $\triangle f$:

$$\Delta \theta = Kc \,/\,\Delta f \cdot L \tag{6}$$

B. The frequency response of the UWB pulse array

For the UWB pulse array in Fig.2, we study its principle changed into frequency domain. Let $s(t) = \gamma(t)$, then we get the following formula:

$$x_m(t) = s(t) \otimes \delta(t - \frac{d}{c}\sin\theta)$$
(7)

$$y(t) = \sum_{m=1}^{M} x_m(t) \otimes \delta(t - \Delta + \tau_m)$$
(8)

$$x_m(\omega) = s(\omega) \exp(-j\frac{\omega}{c}d_m\sin\theta)$$
(9)

$$Y(\omega) = \sum_{m=1}^{M} x_m(\omega) \exp(-j\omega\tau_m)$$
(10)

Here we weight both the amplitude and phase of the weights:

$$Y(\omega) = \sum_{m=1}^{M} g_m^*(\omega) \exp(-j\frac{\omega}{c} d_m \sin\theta) \cdot s(\omega)$$
(11)

$$g_m^*(\omega) = a_m \exp(-j\omega\tau_m) \tag{12}$$

Accordingly, we get the frequency response function of UWB impulse array:

$$B(\omega,\theta) = \sum_{m=1}^{M} g_m^*(\omega) \exp(-j\frac{\omega}{c} d_m \sin\theta) = G^H(\omega) A(\omega,\theta)$$
(13)

$$y(t,\theta) = s(t) * b(t,\theta)$$
(14)

III. UWB PULSE NEARFIELD BEAMFORMER

Given the radical distance r, azimuth ϕ , and elevation angle θ in a spherical coordinate system. A general response of beamformer should satisfy the following wave equation:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial b}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial y}{\partial \theta}\left(\sin\theta\frac{\partial b}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 b}{\partial \phi^2} = \frac{1}{c^2}\frac{\partial^2 b}{\partial t^2}$$
(15)

The classical solution of the wave equation can be written as the beampattern form:

$$b_r(\theta,\phi;k) = r^{-1/2} \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \alpha_n^m \cdot H_{n+1/2}^{(1)}(kr) \cdot P_n^{[m]}(\cos\theta) \cdot e^{jm\theta}$$
(16)

where *m* and *n* are integers, $k = 2\pi f/c$ is the wavenumber. The function $P_n(.)$ are associated Legendra functions and $H_{n+1/2}^{(1)}(\cdot)$ is half odd integer order spherical Hankel function of the first kind, which is defined by:

$$H_{n+1/2}^{(1)}(\cdot) = J_{n+1/2}(\cdot) + jY_{n+1/2}(\cdot)$$
(17)

where $J_{n+1/2}(\cdot)$ is a half integer order Bessel function of the first kind, and $Y_{n+1/2}(\cdot)$ is a half integer order Neumann function. The Fourier-like complex constant α_n^m can be expressed as:

$$=\frac{\zeta_{n}^{m}}{r^{1/2}H_{n+1/2}^{(1)}(kr)}\int_{n=0}^{2\pi}\int_{0}^{\pi}b_{r}(\theta,\phi)\cdot P_{n}^{|m|}(\cos\theta)\cdot\sin\phi\cdot e^{-jm\phi}d\theta d\phi^{(18)}$$
$$\zeta_{*}^{*}=\sqrt{\frac{2n+1}{4\pi}\frac{(n-|m|)!}{(n+|m|)!}}$$
(19)

Applying the equations described above we can convert a beampattern in short distance to a beampattern in infinite distance, then a farfield method can be used to design a beamformer. As we mentioned above, this method is suitable for arbitrary nearfield beamformer with arbitrary geometry. In comparison with nearfield delay compensation, the desired response is achieved exactly over all angles, not just the primary look direction. However, this radical transformation method involves the multidimensional integral and is too complex in computation even in a linear array. A further research found that the beampatterns in two different distances have the reciprocity relationship. So a new way can be used to reduce the computational complexity. Assume $r_1 = r$, $r_2 = \infty$, the farfiled response which is corresponding to the desired nearfield beampattern satisfies such an approximately equal equation:

$$\tilde{b}_{\infty}(\theta,\phi) = \tilde{b}_{r_1}(\theta,\phi)$$
, when $r_1 \to \infty$ (20)

So the design procedure of a nearfield beampattern can be summed as:

Step 1: Specify a desired nearfiled response $b_{rl}(\theta, \phi) = b$

located at r;

 α_n^m

Step 2: Synthesize the beampattern $\tilde{b}_{\infty}(\theta, \phi) = b^*$ according to a farfield assumption;

Step 3: Compute the beampattern $\tilde{b}_{r}(\theta, \phi) = a$ located at r using the obtained weight;

Step 4: Synthesize the beampattern a^* in a nearfield model. The resulting weights obtained form the nearfield beampattern b located at r.

IV. THE FREQUENCY INVARIANT TECHNOLOGY

The frequency invariant technology can fulfill an invariant response over a wide range of bandwidth. This kind of design method is deduced based on an assumption of a continuous array, and in practical a discrete array is employed to approximate the continuous array. Commonly a FIR filter is used (or a group of IIR filter bank) in each sensor channel. Assume the frequency response of the linear continuous array for the farfield plane wave is:

$$r(f,\theta) = \int_{0}^{X_{\max}} g(x,f) e^{-j2\pi f x e^{-1} \sin \theta} dx$$
(21)

If let $g(x, f) = f \cdot \beta(fx)$, where $\beta(x)$ is any absolutely integrabel function. It can be seen easily that the response is invariant, as shown in the following formula:

$$r(f,\theta) = \int_0^{\xi_{\max}} \beta(\xi) e^{-j2\pi\xi c^{-1}\sin\theta} d\xi \qquad (\xi = fx)$$
(22)

Thus we know that the filters which are used in the frequency invariant beamformer (FIB) can be divided into two sections: the primary filter with response $H_{i}(f) = \beta(f \cdot x)$ and the second filter f with position irrelevant response. The most important characteristic of the primary filters in FIB array is dilation relationship, so all the primary filters can be originated from a reference filter.



If $u = c^{-1} \sin \theta$, we can seen from (22) that the aperture distribution and the desired array response has an inner Fourier transformation relationship. That is,

$$r_d(u) = \int_0^{\xi_{\text{max}}} \beta(\xi) e^{-j2\pi\xi u} d\xi$$
(23)

$$H_1(f) = \int_{1/c}^{1/c} r_d(u) e^{j2\pi f u} du$$
 (24)

The impulse response of the primary filter $H_1(f)$ at x=1 is very similar to the desired response of FIB. After a direct sampling we get the coefficients $h_1(k) = r_d(kT)$. Then for the primary filter of referenced sensor located at x_{ref} , its impulse response and tap coefficients are:

$$H_{\rm tref}(f) = H_1(x_{\rm ref}f) \tag{25}$$

$$h_{\rm ref}(k) = \frac{1}{x_{\rm ref}} r_d \left(\frac{kT}{x_{\rm ref}}\right)$$
(26)

Getting the coefficients $h_{ref}(k)$ of the filters with order L located at reference position, we can give the response of the primary filter for the *n*-th array unit:

$$H_n(f) = \sum_{k=-(L-1)/2}^{(L-1)/2} h_{\text{ref}}(k) \exp(-j2\pi f T_n k)$$
(27)

It can be proved that the frequency invariant relationship for different *n*-th sensor. Define T_n as the sampling period of the filter of *n*th sensor, $T_n = T x_n / x_{ref}$. Accordingly we get a group of multirate filters. A common method of multirate sampling is to sample every sensor at the highest rate required and then to use decimation to achieve the desired sampling rate. Thus, each of the primary filters would be implemented by downsampling by x_n/x_{ref} . Select x_n to make x_n/x_{ref} be an integer.

Taking each $H_n(f)$ as the row vector of $G(\omega, \theta)$, each column of matrix $G(\omega, \theta)$ is corresponding to the complex weight at some frequency of UWB pulse array. Select an appropriate window function to weight the amplitude of the response of each sensor. The decreasing of the width of mainbeam and the decreasing of the plus of sidelobe cannot be obtained simultaneously, so we can have a balance between them according to the practical requirement.

The sensor positions can be determined by the minimum of the number of sensors. To avoid the spatial aliasing, the positions of the sensors for linear array are given by:

$$x_n = \begin{cases} \lambda_v/2, & 0 \le n \le P \\ P \lambda_v/2 \left(\frac{P}{P-1}\right)^{n-P}, & p \le n \le N \end{cases}$$
(28)

where $P \in N$ is the aperture of array in a measurement of half $N = P + \left[\log \left(\frac{f_U}{f_L} \right) / \log \left(\frac{P}{P-1} \right) \right]$, λ_U is the wavelength,

maximum wavelength of signal.

V.SIMULATION AND ANALYSIS OF THE PERFORMANCE

We consider the case of an omnidirectional linear UWB pulse array. If the signal is located at $r = 10\lambda_U$ and produce impulse sequences with very short duration $\Delta T = 1$ ns, 3G~4Ghz approximate bandwidth. The waveform and frequency spectrum are shown in Figure 3. Take $[f_1, f_2] = [100Mhz, 4.5Ghz]$, the sensor number M=11, array size L=3.6m. Figure 4 gives the space-time output of the UWB pulse array. Beampatterns in nine observed frequency in [fl, fh] are shown in figure 5. It can be seen that they differ from each other, and the width increases with the frequency bins





decreases. So we must take the distortion of response in the whole bandwidth into consideration. The response of UWB pulse array using multirate filter bank based on FI technology is shown in figure 6. It can be seen that the beampattern of different frequency is almost same.

Suppose a Chebyshev windowed beampattern with -25dB sidelobe is the desired nearfield array response with source location at $r = 10\lambda u$. We design the UWB pulse beamformer with the desired beampattern focused on the center frequency 2.5GHz. First the corresponding farfield beampattern is

obtained by exerting the radical transformation on the given nearfiled beampattern. Then the multirate filter bank are constructed by design their coefficients using equation (27). The FIB response is the transformed farfield beampattern. The design results are depicted in Figure 5 and Figure 6. It can be seen in Fig.5 that farfield pattern transformed from the desired nearfield FI beampattern of UWB pulse array has distortion with frequency and does not maintain the FI property after transformation into farfield.

VI. CONCLUSION

The paper presents a method of multirate filter bank based on FI technology to design UWB pulse array beamformer in the case of nearfield. After discussion of UWB pulse array processing principle, we can see that the frequency domain processing of UWB array also can be done. The multirate filter bank designed for each element based on FI technology can be used to solve the frequency dependent problem in so wide band of UWB pulse array. At the same time, the radical beampattern transform based on spherical coordinates is employed to solve the nearfield beamforming of UWB pulse array. We show that the employment of these methods in UWB pulse array nearfield beamforming has effective results. The simulation results prove the feasibility of this method.

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