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Compromise Ratio Method for Decision Making under Fuzzy Environment using Fuzzy Distance Measure

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***Abstract*—**The aim of this paper is to adopt a compromise ratio (CR) methodology for fuzzy multi-attribute single-expert decision making proble. In this paper, the rating of each alternative has been described by linguistic terms, which can be expressed as triangular fuzzy numbers. The compromise ratio method for fuzzy multi-attribute single expert decision making has been considered here by taking the ranking index based on the concept that the chosen alternative should be as close as possible to the ideal solution and as far away as possible from the negative-ideal solution simultaneously. From logical point of view, the distance between two triangular fuzzy numbers also is a fuzzy number, not a crisp value. Therefore a fuzzy distance measure, which is itself a fuzzy number, has been used here to calculate the difference between two triangular fuzzy numbers. Now in this paper, with the help of this fuzzy distance measure, it has been shown that the compromise ratio is a fuzzy number and this eases the problem of the decision maker to take the decision. The computation principle and the procedure of the compromise ratio method have been described in detail in this paper. A comparative analysis of the compromise ratio method previously proposed [1] and the newly adopted method have been illustrated with two numerical examples.

***Keywords*—**Compromise ratio method, Fuzzy multi-attribute single-expert decision making, Fuzzy number, Linguistic variable

I. INTRODUCTION

MAKING decision is undoubtedly one of the most fundamental activities of human beings. Multi Attribute Decision Making (MADM) problems have an important part in real life situations. Since MADM has found acceptance in areas of operation research and management studies, different methodologies have been created for making decision. But the application of the different methods is complex and fuzzy in nature. In recent times, with the help of computers, the decision making methods have found great acceptance in all areas of decision making process. Especially, in the last few years, with computers becoming connected to every field of life, the applications of various methodologies for MADM have become easier for the decision maker. The main concept of the MADM problem is to find the best option among all feasible alternatives based on multiple attributes both qualititative and quantitative.

There are various methods that exist by which we can deal with MADM problem. Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) is one of the known classical MADM methods, developed by Hwang and Yoon [7]. The TOPSIS method is based upon the idea that the chosen alternative has shortest distance from the positive ideal solution and farthest distance from the negative ideal solution. But in the classical TOPSIS method the weights of the attributes and rating of the alternatives are given crisp values.

Under many conditions, in decision making problems, crisp data are insufficient to model real life situations [8]. Decision maker's response to the different alternatives and also preferences to the various attributes may be sometimes expressed in linguistic variables [9, 10, 11, 12, 13]. Therefore fuzzy set theory is used to deal with MADM problem. Many researchers have used fuzzy set theory in decision making when fuzziness present in human judgment. Based on the similarity measure proposed by Chen [15], in the literatures [3, 16, 18, 19], a ranking strategy is developed for the subjects. A web-based decision-support-system based on fuzzy set approach is implemented in [20] that integrates the subjective and objective information for the evaluating the grade of journals. Under these circumstances, the TOPSIS method was extended for group decision making problems under fuzzy environment [14].

Compromise ratio method for fuzzy Multi Attribute Decision Making method was introduced by Deng-Feng Li in 2006 [1]. The basic principle of the compromise ratio method is that the chosen alternative should have the closest distance from the positive ideal solution and the farthest distance from the negative ideal solution. But it is not possible in real life situation that one particular alternative satisfy both these conditions simultaneously. So, the question here arises, that how the decision is to be made, under such kind of circumstances. In the paper [1] relative importance has been given to both of these distances. A compromise ratio methodology to solve fuzzy multi attribute group decision making problems has been developed in the paper [1]. In the process of compromise ratio method, linguistic variables have been used to capture fuzziness in decision making information

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and decision making process by means of a fuzzy decision matrix.

Now, in this paper, Compromise Ratio method, introduced by Deng-Feng Li [1], has been modified. Distance measures have important role in Compromise ratio method. But in paper [1] the distance measure between two imprecise numbers has been used, which give us a crisp value. But a logical consequence in defining a fuzzy distance measure for generalized fuzzy numbers is that distance between two imprecise numbers should also be an imprecise (i.e. fuzzy) number. Also the compromise ratio, introduced in paper [1], give crisp value. This leads to a problem for the decision maker, as for two different alternatives the CR can give the same value. So in this paper, a new fuzzy distance measure, introduced by Chakraborty and Chakraborty [2], has been used. So the CR is a fuzzy number here. Also, for ranking the alternatives, the ranking method proposed in the paper [6] has been used.

In section II, notation of fuzzy number has been given. And also the fuzzy distance measure and the ranking method have been discussed in this section. The basic principle and procedure of the methodology has been given in section III. In section IV, the proposed method has been illustrated with two numerical examples. A comparative analysis between the two methods has also been given in this section. Finally a short conclusion has been given in section V.

II. PRELIMINARIES

A Notation of fuzzy number

In 1965 Zadeh [7] first introduced the fuzzy set for dealing with vagueness type of uncertainty. A fuzzy set $\tilde{A}$ defined on the universe X which is characterized by a membership function such that $\mu_{\tilde{A}} : X \to [0,1]$. The support of $\tilde{A}$, say $\text{supp}(\tilde{A})$ is defined by the set $\{x \in X / \mu_{\tilde{A}}(x) > 0\}$ and the α level set of $\tilde{A}$ leads to a set such that $\{x \in X / \mu_{\tilde{A}}(x) \ge \alpha\}$ for all $\alpha \in [0, 1]$.

B Generalized fuzzy number (GFN)

A generalized fuzzy number $\tilde{A}$, conventionally represented by $\tilde{A} = (a_1, a_2; \beta, \gamma)$ i.e. (left point., right point, left spread , right spread),is a normalized convex fuzzy subset on the real line R if

(i) $\text{Supp}(\tilde{A})$ is a closed and bounded interval i.e. $[a_1 - \beta, a_2 + \gamma]$;

(ii) $\mu_{\tilde{A}}$ is an upper semi continuous function.

(iii) $a_1 - \beta < a_1 \le a_2 + \gamma$; and

(iv) the membership function is of the following form:

$$\mu_{\tilde{A}}(x) = \begin{cases} f(x) \;\; for \;\; x \in [a_1 - \beta, a_1] \\ 1 \;\; for \;\; x \in [a_1, a_2] \\ h(x) \;\; for \;\; x \in [a_2, a_2 + \gamma] \end{cases}$$

Where $f(x)$ and $h(x)$ are the monotonic increasing and decreasing functions respectively.

C LR-type fuzzy number

A GFN $\tilde{A} = (a_1, a_2; \beta, \gamma)$ is said to be *LR*-type if there exists reference functions L (for left), R (for right) and sealers β >0, γ >0 with the membership function of the form

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\dfrac{a_1 - x}{\beta}\right) & \text{if } a_1 - \beta \le x \le a_1 \\ 1 & \text{if } a_1 \le x \le a_2 \\ R\left(\dfrac{x - a_2}{\gamma}\right) & \text{if } a_2 \le x \le a_2 + \gamma \end{cases}$$

Where, for $L(x)$ and R(x) different functions may be chosen. For example, $L(x) = max\ (0,1\text{-}x^p)$ with $p>0$ or $L(x)= e^{-x}$ (Zimmerman 1996). In particular, if $a_1 = a_2$ =m then $\tilde{A}$ is written as $(m, \beta, \gamma)_{LR}$. The formula of an opposite fuzzy number is $-(m, \beta, \gamma)_{LR} = (-m, \beta, \gamma)_{LR}$.

D Triangular fuzzy number (TFN)

A *LR*-type fuzzy number $\tilde{A}$ is said to be a triangular fuzzy number (TFN) denoted by $\tilde{A} = (m, \beta, \gamma)_{TFN}$ if its membership function is of the following form

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{m - x}{\beta} & \text{if } \; m - \beta \le x \le m \\ 1 - \dfrac{x - m}{\gamma} & \text{if } \; m \le x \le m + \gamma \end{cases}$$

E Distance between generalized fuzzy numbers

In this section we take the point of view that the distance between two fuzzy numbers should itself be fuzzy.

Let us consider two GFNs as $\tilde{A}_1 = (a_1, a_2; \beta_1, \gamma_1)$ *and* $\tilde{A}_2 = (a_3, a_4; \beta_2, \gamma_2)$.

Therefore α -cut of $\tilde{A}_1$ and $\tilde{A}_2$ represents following two intervals respectively $[\tilde{A}_1]_\alpha = [A_1^L(\alpha), A_1^R(\alpha)]$ and $[\tilde{A}_2]_\alpha = [A_2^L(\alpha), A_2^R(\alpha)]$ for all $\alpha \in [0,1]$.

It is clear that distance between two intervals can be measured by taking their difference. So here the interval-difference operation for the intervals $[A_1^L(\alpha), A_1^R(\alpha)]$ and $[A_2^L(\alpha), A_2^R(\alpha)]$ has been used to formulate the fuzzy distance between $\tilde{A}_1$ and $\tilde{A}_2$. Now, the distance between $[\tilde{A}_1]_\alpha$ and $[\tilde{A}_2]_\alpha$ for all $\alpha \in [0,1]$ is one of the following

either (a) $[\tilde{A}_1]_\alpha - [\tilde{A}_2]_\alpha$

if $\frac{A_1^L(1)+A_1^R(1)}{2} \geq \frac{A_2^L(1)+A_2^R(1)}{2}$

or $(b)\ [\tilde{A}_2]_\alpha - [\tilde{A}_1]_\alpha$

if $\frac{A_1^L(1)+A_1^R(1)}{2} < \frac{A_2^L(1)+A_2^R(1)}{2}$

In order to consider both the notations together an indicator variable η is introduced such that

$$\eta([\tilde{A}_1]_\alpha - [\tilde{A}_2]_\alpha) + (1-\eta)([\tilde{A}_2]_\alpha - [\tilde{A}_1]_\alpha) = [d_\alpha^L, d_\alpha^R] \quad (1)$$

for $\eta = \begin{cases} 1 \ \text{if} \ \frac{A_1^L(1)+A_1^R(1)}{2} \geq \frac{A_2^L(1)+A_2^R(1)}{2} \\ 0 \ \text{if} \ \frac{A_1^L(1)+A_1^R(1)}{2} < \frac{A_2^L(1)+A_2^R(1)}{2} \end{cases}$

after comparing from (1)

$$d_\alpha^L = \eta[A_1^L(\alpha) - A_2^L(\alpha) + A_1^R(\alpha) - A_2^R(\alpha)] + [A_2^L(\alpha) - A_1^R(\alpha)] \text{ and}$$

$$d_\alpha^R = \eta[A_1^L(\alpha) - A_2^L(\alpha) + A_1^R(\alpha) - A_2^R(\alpha)] + [A_2^R(\alpha) - A_1^L(\alpha)]$$

Therefore, the fuzzy distance between $\tilde{A}_1$ and $\tilde{A}_2$ is defined by

$$\tilde{d}(\tilde{A}_1, \tilde{A}_2) = (d_{\alpha=1}^L, d_{\alpha=1}^R; \theta, \sigma) \quad (2)$$

Where

$$\theta = d_{\alpha=1}^L - \max\left\{\int_0^1 d_\alpha^L d\alpha,\ 0\right\} \text{ and } \sigma = \int_0^1 d_\alpha^R d\alpha - d_{\alpha=1}^R$$

F Ranking Method

This subsection gives a short description of the ranking method proposed by [6].

Here centroid point of a fuzzy number has been denoted by $\overline{x}$ on the horizontal axis and $\overline{y}$ on the vertical axis. The centroid point ($\overline{x}, \overline{y}$) for a fuzzy number $\tilde{A}$ (subsection 2.2.1) has been defined as

$$\overline{x}(\tilde{A}) = \frac{\int_{a_1-\beta}^{a_1} xf(x)dx + \int_{a_1}^{a_2} xdx + \int_{a_2}^{a_2+\gamma} xh(x)dx}{\int_{a_1-\beta}^{a_1} f(x)dx + \int_{a_1}^{a_2} dx + \int_{a_2}^{a_2+\gamma} h(x)dx}$$

$$\overline{y}(\tilde{A}) = \frac{\int_0^1 y[h^-(y) - f^-(y)]dy}{\int_0^1 [h^-(y) - f^-(y)]dy}$$

where *f(x)* and *h(x)* are the left and right membership functions of fuzzy number $\tilde{A}$ respectively. $f^-(y)$ and $h^-(y)$ are the inverse functions of *f(x)* and *h(x)* respectively.

The area between the centroid point ($\overline{x}, \overline{y}$)and the original point (0, 0) of the fuzzy number $\tilde{A}$ is defined as area$(\tilde{A}) = \overline{x}.\overline{y}$.

The area $(\tilde{A})$ has been used for ranking the alternatives. For any two different fuzzy numbers $\tilde{A}_i$ and $\tilde{A}_j$ if $area(\tilde{A}_i)=area(\tilde{A}_j)$ then $\tilde{A}_i = \tilde{A}_j$, if $area(\tilde{A}_i) > area(\tilde{A}_j)$ then $\tilde{A}_i > \tilde{A}_j$. Finally, if $area(\tilde{A}_i) < area(\tilde{A}_j)$ then $\tilde{A}_i < \tilde{A}_j$.

III. FUZZY COMPROMISE RATIO METHOD (FCRM) FOR MADM (SINGLE EXPERT)

In this paper, the following MA (Single Expert) DM in fuzzy environment has been discussed. Suppose there exist n possible alternatives $s_1, s_2, ..., s_n$ from which the decision maker has to choose on the basis of m attributes $c_1, c_2, ..., c_m$, both qualitative and quantitative. Now here the attributes set C has been divided into two subsets C^1 and C^2 where C^* (k=1, 2) is the subset of benefit attributes & cost attributes respectively. Furthermore $C = C^1 \cup C^2$ and $C^1 \cap C^2 = \Phi$. Here it has been assumed that the m attributes have equal weights. Let us suppose that the rating of alternative s_j (j= 1, 2…n), on attribute (i= 1, 2…n), as given by the decision maker be $\tilde{f}_{ij} = (m_{ij}; \alpha_{ij}, \beta_{ij})$. Hence, a FMA Single expert DM problem has been concisely expressed in matrix format as follows:

$$\tilde{Y} = (\tilde{f}_{ij})_{m\times n} = \begin{pmatrix} \tilde{f}_{11} & \cdots & \tilde{f}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{f}_{m1} & \cdots & \tilde{f}_{mn} \end{pmatrix}$$

This is referred to as fuzzy decision matrices, which is usually used to represent the FMA Single expert DM problem. Since the m attributes may be measured in different ways, the decision matrix $\tilde{Y}$ needs to be normalized. The linear scale transformation has been used here to transform the various attribute scale into a comparable scale [5]. After normalization we get

$$\tilde{r}_{ij} = \left(\frac{m_{ij}}{d_i^{\max}}; \frac{\alpha_{ij}}{d_i^{\max}}, \frac{\beta_{ij}}{d_i^{\max}}\right) \text{ for } c_i \in C^1$$

and

$$\tilde{r}_{ij} = \begin{cases} \left(\frac{a_i^{\min}}{m_{ij}}; \frac{a_i^{\min}.\beta_{ij}}{m_{ij}.(m_{ij}+\beta_{ij})}, \frac{a_i^{\min}.\alpha_{ij}}{m_{ij}.(m_{ij}-\alpha_{ij})}\right) (a_i^{\min} \neq 0) \\ \left(1 - \frac{m_{ij}}{d_i^{\max}}; \frac{\beta_{ij}}{d_i^{\max}}, \frac{\alpha_{ij}}{d_i^{\max}}\right) (a_i^{\min} = 0) \end{cases} \text{ for } c_i \in C^2$$

Where

$$d_i^{\max} = \max_{1<j<n}\{m_{ij} + \beta_{ij} \mid \tilde{f}_{ij} = (m_{ij}; \alpha_{ij}, \beta_{ij})\} \text{and}$$

$$a_i^{\min} = \min_{1<j<n}\{m_{ij} - \alpha_{ij} \mid \tilde{f}_{ij} = (m_{ij}; \alpha_{ij}, \beta_{ij})\}$$

Denote $\tilde{r}_{ij} = \left(\sigma_{ij};\xi_{ij},\nu_{ij}\right)$

The normalization method mentioned above is to preserve the property that the range of a normalized triangular fuzzy number belongs to the closed interval [0, 1]. Then the fuzzy decision matrix $\tilde{Y} = \left(\tilde{f}_{ij}\right)_{m\times n}$ can be transformed into normalized fuzzy decision matrix:

$$\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \begin{pmatrix} \tilde{r}_{11} & \cdots & \tilde{r}_{1n} \\ \vdots & \ddots & \vdots \\ \tilde{r}_{m1} & \cdots & \tilde{r}_{mn} \end{pmatrix}$$

Obviously, all $r_{ij} = \left(\sigma_{ij};\xi_{ij},\nu_{ij}\right)$ are normalized positive triangular fuzzy numbers and their ranges belong to the closed interval [0, 1]. Then, the fuzzy positive ideal solution s^+ and the fuzzy negative ideal solution s^- have been defined, whose weighted normalized fuzzy vectors are $\tilde{a}^+ = \left(\tilde{a}_1^+,\tilde{a}_2^+\cdots\tilde{a}_m^+\right)$ and $\tilde{a}^- = \left(\tilde{a}_1^-,\tilde{a}_2^-\cdots\tilde{a}_m^-\right)$ respectively, where $\tilde{a}_i^+ = \left(1,0,0\right) = 1$ and $\tilde{a}_i^- = \left(0,0,0\right) = 0$.

Now in this paper difference between each alternative s_j (j=1, 2…n) and the positive ideal solution and the fuzzy negative ideal solution has been measured by using equation (1)as follows respectively:

$$\tilde{D}(s_j,s^+) = \sum_{i=1}^{m}\tilde{d}(\tilde{r}_{ij},\tilde{a}_i^+)$$
$$\tilde{D}(s_j,s^-) = \sum_{i=1}^{m}\tilde{d}(\tilde{r}_{ij},\tilde{a}_j^-) \tag{3}$$

Now the smaller $D(s_j,s^+)$, the better s_j. Therefore, rank the alternative s_j (*j*=1, 2…,n) by $\tilde{D}(s_j,s^+)$ in increasing order. An alternative s_{j^*} satisfying

$$\tilde{D}(s_{j^*},s^+) = \min_{1<j<n}\{\tilde{D}(s_j,s^+)\}$$

should be the best compromise solution which has the shortest distance from the positive ideal solution. However shortest distance from the positive ideal solution may not always automatically imply maximum distance from the negative ideal solution.

Similarly when the decision maker ranks the alternatives with respect to the negative ideal solution; then it is clear that bigger the value of $\tilde{D}(s_j,s^-)$, the better s_j. So rank the alternatives s_j (*j*=1,2….*n*) by $\tilde{D}(s_j,s^-)$ in decreasing order. An alternative $s_{j^{**}}$ satisfying $\tilde{D}(s_{j^{**}},s^-) = \max_{1<j<n}\{\tilde{D}(s_j,s^-)\}$ should be the best compromise solution, which has the farthest distance from the negative ideal solution.

But in every situation it may not happen that $s_{j^*} = s_{j^{**}}$. So here also a compromise ratio for every alternative $s_j \in S$ (*j*=1,2,…*n*) is calculated as

$$\varepsilon(S_j) = \varepsilon\frac{\tilde{D}_1(s^+)-\tilde{D}(s_j,s^+)}{\tilde{D}_1(s^+)-\tilde{D}_2(s^+)} + \left(1-\varepsilon\right)\frac{\tilde{D}(s_j,s^-)-\tilde{D}_2(s^-)}{\tilde{D}_1(s^-)-\tilde{D}_2(s^-)} \tag{4}$$

where,

$$\tilde{D}_1(s^+) = \max_{1<j<n}\{\tilde{D}(s_j,s^+)\}$$
$$\tilde{D}_2(s^+) = \min_{1<j<n}\{\tilde{D}(s_j,s^+)\}$$
$$\tilde{D}_1(s^-) = \max_{1<j<n}\{\tilde{D}(s_j,s^-)\}$$
$$\tilde{D}_2(s^-) = \min_{1<j<n}\{\tilde{D}(s_j,s^-)\}$$

Here, it is considered that $\varepsilon \in [0,1]$ indicates the attitudinal factor of the decision maker.

When $\varepsilon = 1$, the decision maker gives more importance to the distance from the positive ideal solution.

When $\varepsilon = 0$, the decision maker is then interested the distance from the negative ideal solution. And equally importance to both the distances $\tilde{D}(s_j,s^+)$ and $\tilde{D}(s_j,s^-)$ will be given when $\varepsilon = 1/2$. Also it is assumed that if $\varepsilon > 0.5$, then, the decision maker is biased above the fuzzy positive ideal solution. The index $\varepsilon(s_j)$ measures the extent of compromise of the proximity of the alternative s_j to the positive ideal solution s^+ and its distance from the negative ideal solution s^-. The bigger $\varepsilon(s_j)$ is the better s_j. The preference order of the alternative s_j (*j*=1, 2…*n*) is generated according to $\varepsilon(s_j)$. An alternative s_{j^0} which has the best compromise level between the distance from the positive ideal solution s^+ and the distance from the negative ideal solution s^-, satisfying $\varepsilon(s_{j^0}) = \max_{1<j<n}\{\varepsilon(s_j)\}$ should be the best compromise solution.

Now in this newly developed method we will get $\varepsilon(s_j)$ itself as a fuzzy number. So here how decision maker compare $\varepsilon(s_j)$ to each other. In this regards, we will apply the ranking method proposed in the paper [6] and using this method we choose the alternative s_j.

IV. NUMERICAL EXAMPLE

Example 1: Let us assume that a reputed management company wants to hire a person as the general manager. After the written test, the experts conduct an interview and GD. The expert will, then, take the final decision based upon the following criteria:

C1. M.B.A degree from a well-known management institute.

C2. Oral communication skill

C3. Presence of mind and the capacity to handle critical situations
C4. Group work and leading power.
C5. Personality

Now here it is assumed that based on the above five criteria the expert will take the interview of each candidate. The expert use linguistic terms in the making of his expert comments, which are expressed in terms of triangular fuzzy number (without loosing its impreciseness.). A PC-aided evaluation procedure [4] may be considered. It helps the decision maker to express their opinion. To design a PC-based information system it needs to have an initial knowledgebase (KB) that store the knowledge about a domain represented in machine-processable form. The PC-aided procedure should be user-friendly so that as and when DM feels initial knowledge base (KB) could be updated by incorporating more options having more terms. The options having different term differentials help the DM to express the responses comfortably.

Let us consider the domain as [0,100] on which DM's responses are to be explained; e.g., a sample of KB consisting of four options [3] is considered here given below:

Option 1 (2 terms) Not-satisfactory/satisfactory (NS/S),
Option 2 (3 terms) Not-satisfactory/satisfactory/good (NS/S/G)
Option 3 (4 terms) Notsatisfactory/satisfactory/good/excellent (NS/S/G/EX)
Option 4 (5 terms) No-merit/poor/satisfactory/good/excellent (NM/P/S/G/EX)

Actually here the domain [0,100] has been fuzzily partitioned into different number of options. As the numbers of options are increased, the domain will be partitioned into more number of intervals. Now the necessity of taking different options in knowledgebase is that, if the DM is not satisfied with the term from option1, the opportunity is to be given to him to choose further fruitful option.

An algorithm is designed here to transform the fuzzy terms of different options into corresponding triangular fuzzy numbers as follows:

Algorithm.

Step 1: Consider kth option that consists of (*k+1*) term-differentials.

Step 2: Suppose for the *j*th term where *j* = 1, 2… (*k+1*), the spread is equal to (*100/k*) and center say, m_j is computed as:

$$m_j = \begin{cases} 0 \quad \text{for } j = 1 \\ m_{j-1} \text{ for } j = 2, 3 \cdots k \\ 100 \quad \text{for } j = k+1 \end{cases}$$

Step 3: (boundary condition) If the left or/and right point of a fuzzy number is/are outside the domain [0,100], the left and right points would be automatically replaced by 0 and 100, respectively.

Therefore, the outputs of the algorithm for various options are represented as fuzzy numbers as follows:

:	*option 1*	*option 2*	*option 3*	*option 4*
No-merit	-	-	-	$(0, 0, 25)_{LR}$
Poor	-	-	-	$(0, 25, 50)_{LR}$
Non-satis.	$(0, 0, 100)_{LR}$	$(0, 0, 50)_{LR}$	$(0, 0, 33.3)_{LR}$	-
Satis	$(0,100,100)_{LR}$	$(0,50,100)_{LR}$	$(0,33.3,66.7)_{LR}$	$(25,50,75)_{LR}$
Good	-	$(50,100,100)_{LR}$	$(33.3, 66.7, 100)_{LR}$	$(50, 75,100)_{LR}$
Excellent	-	-	$(66.7, 100,100)_{LR}$	$(75,100,100)_{LR}$

A human expert's linguistic expression is an outcome of a reaction in his mind toward a particular query. In fact, an expert (let us assume that an expert is not at all aware of what is called 'fuzzy mathematics') may be more confident and spontaneous in expressing his opinion in one linguistic term or in one linguistic phrase (given in different options) with his own way of perception, reasoning and expression. Depending on the DM's satisfaction level, the same linguistic term in all of the sets of term differentials that carries multiplicity of meaning is placed into various positions. One interpretation can be drawn here: the term 'satisfactory' in option 4 is relatively more precise that that in option 3 where it is mostly imprecise in option 1. In view of this, the DM selects the same term from different options to make response more meaningful. Thus a response matrix is obtained as follows

TABLE I:
Expert's linguistic responses from KB having different options

Candidates > / Criteria ∨	s_1	s_2	s_3
C1	S from option2	S from option3	S from option4
C2	NS from option 3	S from option 4	NS from option1
C3	G from option1	EX from option2	G from option3
C4	G from option 4	NS from option 1	NS from option 4
C5	Ex from option3	Ex from option4	S from option2

Now from the above table 1 we can write the fuzzy decision matrix n the following table:

TABLE II:
The fuzzy decision matrix of three candidates:

	s_1	s_2	s_3
C1	$(50, 50, 50)_{LR}$	$(33.3, 33.3, 33.3)_{LR}$	$(50,25,25)_{LR}$
C2	$(0, 0, 33.3)_{LR}$	$(50, 25, 25)_{LR}$	$(0, 0,100)_{LR}$
C3	$(100, 100, 0)_{LR}$	$(100, 50, 0)_{LR}$	$(66.7, 33.3, 33.3)_{LR}$
C4	$(75, 25, 25)_{LR}$	$(0, 0,100)_{LR}$	$(25, 25, 25)_{LR}$
C5	$(100, 33.3, 0)_{LR}$	$(100, 25, 0)_{LR}$	$(50, 50, 50)_{LR}$

TABLE III:
The normalized fuzzy decision matrix of three candidates:

	s_1	s_2	s_3
C1	$(0.5, 0.5, 0.5)_{LR}$	$(0.33, 0.33, 0.34)_{LR}$	$(0.5,.25,.25)_{LR}$
C2	$(0, 0, .33)_{LR}$	$(0.5, 0.25, 0.25)_{LR}$	$(0, 0,1.00)_{LR}$
C3	$(1.0, 1.0, 0.0)_{LR}$	$(1.00,0.5,0.0)_{LR}$	$(0.67,0.34,0.33)_{LR}$
C4	$(0.75, 0.25, 0.25)_{LR}$	$(0,0,1.00)_{LR}$	$(0.5, 0.5,0.5)_{LR}$
C5	$(1.00, 0.33, 0)_{LR}$	$(1.00,0.25,0)_{LR}$	$(0.5, 0.5, 0.5)_{LR}$

TABLE IV:

Ranking results obtained by the modified compromise ratio method (using equation 3)

Candidates	s_1	s_2	s_3
$\tilde{D}(s_j, s^+)$	(1.75,.875,1.04)	(3.17,0.96,.665)	(2.83,.665,.96)
Ranking order	$s_1 > s_3 > s_2$		
$\tilde{D}(s_j, s^-)$	(3.25,.04,375)	(2.83,.665,0.96)	(2.17,.795,1.29)
Ranking order	$s_1 > s_2 > s_3$		

Hence here the decision maker will be in confusion. So here we calculate $\varepsilon(s_j)$ for j =1, 2, 3. by equation (4). And here the expert's attitude is specified by ε =0.6

Here the value of the fuzzy number $\varepsilon(s_j)$ is written (for ε =0.6)

	s_1	s_2	s_3
$\varepsilon(s_j)$	(1, 1, 1)	(-0.674, 0.244, 1.5)	(-0.979, .144, .948)
Ranking order	$s_1 > s_2 > s_3$		

The compromise solution obtained by the modified compromise ratio method is the alternative s_1.

Example 2**:** Our next aim is to show, with help of this example, the shortcomings of the Deng Feng Li's methodology and the advantages of proposed fuzzy compromise ratio method (FCRM).

Let us consider a problem with a given criterion and two alternatives s_1 and s_2 among which the expert have to choose the best option. The expert gives his expert's comments linguistically and without any loss of generality this responses are expressed in terms of triangular fuzzy numbers as follows:

$$s_1 = (0.6; 0.3, 0.165)_{TFN}$$

$$s_2 = (0.55; 0.55, 0.45)_{TFN}$$

Decision results obtained by the compromise ratio method introduced by Deng- Feng Li [1] and fuzzy compromise ratio method are given in the following table. Here, in the compromise ratio methodology (CRM), to calculate the distance between two triangular fuzzy numbers, the distance measure proposed by Chen [14] has been used.

Table V.

Ranking results obtained by Deng-Feng Li's compromise ratio method and the fuzzy compromise ratio method:

Candidates	s_1	s_2	Ranking order
CRM			
$\tilde{D}(s_j, s^+)$	0.5	0.6	$s_1 > s_2$
$\tilde{D}(s_j, s^-)$	0.6	0.7	$s_2 > s_1$
$\varepsilon(s_j)$	0.5	0.5	#
FCRM			
$\tilde{D}(s_j, s^+)$	$(0.4; 0.15, 0.08)_{TFN}$	$(0.45; 0.225, 0.275)_{TFN}$	$s_1 > s_2$
$\tilde{D}(s_j, s^-)$	$(0.4; 0.15, 0.08)_{TFN}$	$(0.45; 0.275, 0.225)_{TFN}$	$s_1 > s_2$
$\varepsilon(s_j)$	(1;0,0)	(0;0,0)	$s_1 > s_2$

The expert's attitude is specified by ε =0.5

\# means that with help of the Deng Feng Li's compromise ratio method, the expert can not come to a conclusion. Under this kind of circumstances, the proposed fuzzy compromise ratio method will give better result.

In this way, from the above example it is proved that in some cases the modified fuzzy compromise ratio method will give better result. Here the best alternative is s_1.

The compromise ratio method [1] introduces an aggregating function for ranking in equation (2), which reflects the extent that the alternative s_j (*j*=1, 2... *n*) closes to the positive ideal solution s^+ and is far away from the negative ideal solution s^-.

Now in the compromise ratio method [1] we choose the alternative s_j (j=1, 2,…, n) for which $\varepsilon(s_j)$ has the maximum value .But for any two alternatives sj (j=k, m) $\varepsilon(s_j)$ can give the same value, then this creates a very big problem to the decision maker (as shown in the example 2).

Now under this point of view, we use here the fuzzy distance measure introduced by Chakraborty and Chakraborty [2]. Using this distance measure in the newly developed method we get $\varepsilon(s_j)$ itself as fuzzy number and then easily decision maker using the ranking method [6] can come to a conclusion. In this way we can overcome the shortcomings of compromise ratio method [1].

Also in paper [1], to measure the distance from positive and negative ideal solution, Minkowski distance (or Lp metric) is used. But this distance method basically compute crisp distance values for particular fuzzy numbers, not for generalized fuzzy numbers. But it is very natural question that, if the numbers itself are not known exactly, how the distance between them can be an exact value. In this regard, the fuzzy distance measure proposed by Chakraborty and Chakraborty [2], is used here.

V. CONCLUSION

Since attitude of an expert has a necessary part in decision making activities and also the ranking or ordering of the candidates change due to the attitude of the decision maker. In this regard the modified fuzzy compromise ratio method with the attitudinal factor will play an important role in decision making activities. Fuzzy Multiple Attribute Group Decision Making is future work of this paper.

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