The Use of Dynamically Optimised High Frequency Moving Average Strategies for Intraday Trading

Abdalla Kablan, Joseph Falzon

Abstract—This paper is motivated by the aspect of uncertainty in financial decision making, and how artificial intelligence and soft computing, with its uncertainty reducing aspects can be used for algorithmic trading applications that trade in high frequency. This paper presents an optimized high frequency trading system that has been combined with various moving averages to produce a hybrid system that outperforms trading systems that rely solely on moving averages. The paper optimizes an adaptive neuro-fuzzy inference system that takes both the price and its moving average as input, learns to predict price movements from training data consisting of intraday data, dynamically switches between the best performing moving averages, and performs decision making of when to buy or sell a certain currency in high frequency.

Keywords—Financial decision making, High frequency trading, Adaprive neuro-fuzzy systems, moving average strategy.

I. INTRODUCTION

AS computational power and data base capacities increase, there will always be a parallel increase in the availability of more data and information for trading systems to process. High-frequency trading, despite being a new area, has proved to be a ripe field for future implementation of trading systems that can make use of this type of data and develop high frequency trading strategies. The fundamental problem that this thesis aspires to solve is to improve algorithmic trading systems by taking a common sense based approach reflected by fuzzy logic a decision making mechanism. Therefore, the decisions taken by an algorithmic trading system should be significant rather than just precise. When analysing the problem from this perspective, it appears that fuzzy logic, as a reasoning mechanism, is an obvious choice. However, fuzzy logic on its own is never sufficient. Fuzzy logic can provide very good results when used in designing trading systems, yet using a hybrid fuzzy system would provide a much better optimised trading system. Various artificial intelligence and signal processing mechanisms will have to be incorporated in the trading algorithms implemented to yield very good results. The used application will The second application to be utilised will be an Adaptive Neuro Fuzzy Inference System (ANFIS) for high-frequency trading.

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This is an expert system that combines fuzzy reasoning with the pattern recognition capability of neural networks. A new event based volatilitymodel will counter unnecessary input in the training phase and will therefore be proposed. The Intraday Seasonality Observation Model is greatly enhanced by taking factors such as the volatility and scaling laws of financial time series into account. Excess data is thus removed as this enhanced model makes it possible to observe specific events and seasonalities in the data. The overall performance of the Adaptive Neuro Fuzzy Inference System has been greatly improved due to the more accurate input/data provided by the new event based volatility model.

This paper extends on the concept of ANFIS that has been introduced in [1,5] to optimise moving average strategies that are very widely used in the industry. In this paper, ANFIS is utilised to learn from the input of high frequency data and moving averages applied on them to perform predictions on the next market movement.

Due to the random nature of financial times series, the prediction and forecast is extremely complicated, the predictability/accurate forecast of most financial time series such as stock prices or indices is a highly contentious issue as the efficient market hypothesis declares that the current price takes into account/uses/makes use of/contains all available market information/data. [14]. Moreover, a filtering process should be applied in order to separate the substantial noise created by/caused by financial time series from the signal [3]. Furthermore, the practice of performing calculations in real-world scenarios for the standard deviation in fixed time, employed by traditional volatility models, has revealed major drawbacks [11]. Measuring volatility should be done from an event-based perspective by analyzing observations made after the event. In addition, making the right decisions having analyzed all of the inputs from other blocks is key to producing an accurate system [6,7]. Artificial intelligence and soft computing can provide a major solution to the above. ANFIS combines two important aspects of artificial intelligence. Neuro-fuzzy systems successfully combine the human-like reasoning process of fuzzy logic with the ability of neural networking to identify data patterns [16,1]. This provides the reasoning to extend ANFIS further to become a major block in an overall trading system. Conventional mathematical applications such as differential equations or statistical analysis are not suited to dealing with uncertain systems and nonlinearity in the data series [9,24,33].

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Alternatively, fuzzy inference systems can express in a linguistic fashion (if-then statements) aspects of human knowledge and the reasoning process without employing extensive quantitative analysis.

The reason for using such a methodology lies in that Takagi and Sugeno [28] have used and expanded on fuzzy modelling for systems as well as fuzzy identifications in similar nonlinear simple control systems. However, when the problem becomes more complex and involves nonlinear prediction and estimation, the following problems arise: There is no standard method of directly transferring expert knowledge into a set of rules in the fuzzy inference system Fuzzy membership functions need to be tuned in order to minimise the output error or maximise the performance index.Therefore, in order to solve the above fuzzy logic drawbacks, ANFIS has been proposed. It serves as a basis for constructing a set of fuzzy if-then rules with appropriate membership functions to generate the stipulated input/output pairs. This will then be combined with three different moving average strategies in order to optimize the trading behavior by dynamically switching between the various strategies (modes).

II. LOW PASS FILTERS (LPFs):

The concept of signal filtering traces its roots to electrical engineering. To clean the Signal back to its original value we need to *filter* it, and since in the case of financial signals we are concerned only with the low frequency, thus we can use a *Low-Pass-Filter*
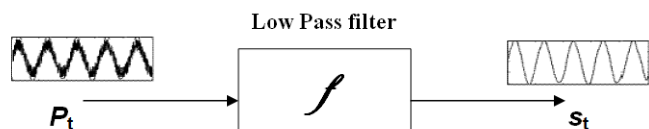
Low Pass filter

Fig. 1 Low pass filters filter high frequency noise

A noisy signal P_t can be filtered from noise to produce a pure signal S_t. Simple forecasting and smoothing methods model components in a series that are usually easy to see in a time series plot of the data. Such procedure decomposes data down to its basic structure of trend and seasonal components, and then extend the estimates of the components into the future to provide forecasts.

There are two different types of filtrations, static and dynamic.

Static methods have patterns that do not change over time; dynamic methods have patterns that do change over time and estimates are updated using neighboring values.

You may use two methods in combination. That is, you may choose a static method to model one component and a dynamic method to model another component.
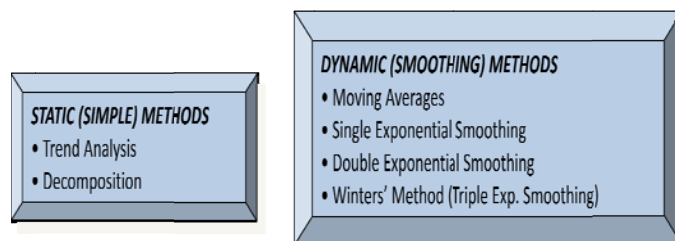
STATIC (SIMPLE) METHODS
• Trend Analysis
• Decomposition

DYNAMIC (SMOOTHING) METHODS
• Moving Averages
• Single Exponential Smoothing
• Double Exponential Smoothing
• Winters' Method (Triple Exp. Smoothing)

Fig. 3 Static and dynamic smoothing methods

This paper will use the *Simple Moving Average (*S.M.A) as the simplest for of a moving average, and can be defined as: the Simple 'arithmetical' moving average which is calculated by summing up the prices of instrument closure over a certain number of single periods (for instance, 12 hours or days ..etc). This value is then divided by the number of such periods

It is to be noted that if the average range is taken to be very big, the signal will tend to lose information! But what is of our concern at this point is that the Signal was filtered and each time it produced a cleaner Signal, however a *LAG* (delay) was introduced.

We tend to buy and sell at the crossovers, since the M.A system catches the trades however it introduces the problem of the whipsaw.
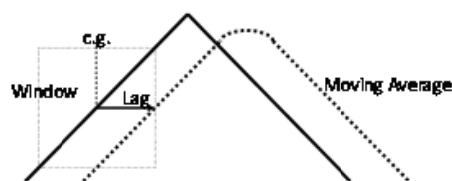
Fig 2 Lag with moving averages

Moving average (MA) models constitute another set of tools for forecasting future movements of a financial instrument. While autoregressive AR models estimate what proportion of past period data is likely to persist in future periods, MA models focus on how future data reacts to innovation in the past data. In other words, AR models estimate future responses to the expected component realized in the past persistence, whereas MA models measure future responses to the unexpected component realized in the past data.

This paper will compare five different trading systems

- A Buy and Hold trading system which places a buy order when the prices have deviated downwards from the mean.,
- Two momentum (trend following) trading signals that trade after the preset moving averages have performed a crossover.,
- A contrarian (trend reversal) trading signal that performs a trade when the moving average has crossed over with a delayed version of the signal.
- An optimised Neuro-Fuzzy system that learns from the behavior of the other trading signals and performs trades accordingly.

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III.   ARCHITECTURE

A.   Moving Average Systems

Empirically, traders modify their order placement as soon as market conditions change, that is why we need different moving average strategies [22,26,27]. The idea that this paper proposes is to create a system that not only can switch between various moving average strategies, but can also learn from their output. Of course, these strategies depend on the available information when submitting their orders. Monitoring the order book and using all available public information, they attempt to update their information set to optimise the order activities. In order to describe the buy-or-sell decision, *trend modes* and *cycle modes* are central to the process of selecting a trading strategy.

Buying and holding is the obvious strategy in an uptrend whilst in a downtrend the strategy is to sell and hold. Similarly, top-picking and bottom-fishing is the best strategy in a cycle mode. Trading the trend mode often involves the use of a variant of moving averages whilst an oscillator is commonly used to trade the cycle mode. This is attributed to prices in trend mode vary slowly with respect to time, high frequency components are ignored and only slowly varying low-frequency components are used to pass to their output. It is for this reason that trend modes are particularly effective for trend mode trading. However, oscillators are high-pass filters that almost completely disregard any low-frequency components, and are used in cycle oriented trading strategies. The proposed optimized ANFIS system will be able to select the right strategy afte learning from the various available input strategies.

This paper utilizes one of the simplest yet most effective ways to filter the signal, the Simple Moving Average S.M.A, which can be defined as the Simple 'arithmetical' moving average which is calculated by summing up the prices of instrument closure over a certain number of single periods (for instance, 15 minute samples, 30 minutes samples, 1 hour samples.. etc). This value is then divided by the number of such periods. For example, when dealing with high frequency 5-minute data one first has to establish the *window* size **n** of the moving average and then calculate the moving average at each point from the previous **n** prices **P**:

$$MA = \sum_{i=1}^{n} P_i / n$$

where **n** is window size (number of periods) for the moving average. The simple moving average can be defined as the sum of the last **n** periods divided by **n**. The bigger the window size of periods, the 'smoother' and less noisy the moving average line will be but the lag or delay will be higher. For illustration purposes of this paper, the various simple moving averages that were chosen for the two momentum based signals and the contrarian signal are described in the Table 1. Momentum 1 and Momentum 2 signals are trend following signals that open a buy position when the two moving averages cross over. The contrarian signal is a trend reversal signal that closes the buy position when the moving

averages crosses over with the price or a slightly delayed version of the price[1].

TABLE I
Types of Simple Moving Averages used

Signal	MA Periods	Data type
Momentum 1	5 & 10	5-min
Momentum 2	10 & 15	5-min
Contrarian	10 & $P-2$	5-min

B.   ANFIS

ANFIS is an adaptive network of nodes and directional links with associated learning rules. The approach learns the rules and membership functions from the data [29], and has been used for high frequency financial trading in [1,2,4,5,6,7]. It is called adaptive as some or all of the nodes have parameters that affect the output of the node. These networks identify and learn relationships between inputs and outputs; they have high learning capability and membership function definition properties. Although adaptive networks cover a number of different approaches, for this research, a detailed investigation of the method proposed by Jang et al. [24], and used successfully by [1,4,5,6,8,9] with the architecture shown in Fig 4 will be conducted. ANFIS can also be used to design forecasting systems [19,25,31].
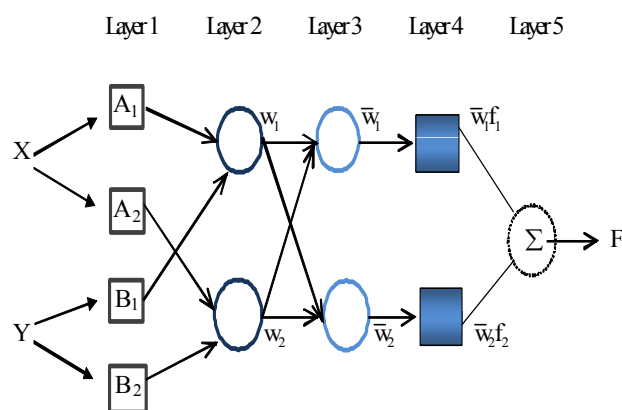

Fig. 4 ANFIS architecture for a two rule Sugeno system

The circular nodes have a fixed input-output relation, whereas the square nodes have parameters to be learnt. Typical fuzzy rules are defined as a conditional statement in the form:

$$If\ X\ i\ sA_1, \qquad then\ Y\ is\ B_1\ . \qquad (1)$$

$$If\ X\ i\ sA_2, \qquad then\ Y\ is\ B_2\ . \qquad (2)$$

X and Y are linguistic variables; A_i and B_i are linguistic values determined by fuzzy sets on the particular universes of discourse X and Y respectively. However, in ANFIS, the first-order Takagi-Sugeno system [28,29] is utilised; this is described in Equations (1) and (2).

X and Y represent the universes of discourse; A_i and B_i are linguistic terms defined by their membership functions, and p_i, q_i and r_i are the consequent parameters that are updated in the forward pass of the learning algorithm. The forward pass propagates the input vector through the network layer by

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[1] Future work will cater for optimizing the selection process of the moving average periods.

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layer. In the backward pass, the error is returned through the network, in a similar manner to back-propagation. The five layers can be described as the following:

The output of each node in Layer 1 is:

$$O_{1,i} = \mu_{A_i}(x) \qquad for \ i = 1,2$$
$$O_{1,i} = \mu_{B_{i-2}}(x) \qquad for \ i = 3,4$$

Hence, $O_{1,i}(x)$ is essentially the membership grade for x and y. Although the membership functions could be very flexible, experimental results lead to the conclusion that for the task of financial data training, the bell-shaped membership function is most appropriate (Abonyi et al. , 2001).

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x - c_i}{a_i}\right|^{2b_i}} \ , \tag{3}$$

Here a_i, b_i, c_i are parameters to be learnt. These are the premise parameters.

In Layer 2, every node is fixed. This is where the t-norm is used to 'AND' the membership grades, for example, the product:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \quad i = 1,2 \ . \tag{4}$$

Layer 3 contains fixed nodes that calculate the ratio of the firing strengths of the rules:

$$O_{3,i} = \overline{w_i} = \frac{w_i}{w_1 + w_2} \ . \tag{5}$$

The nodes in Layer 4 are adaptive and perform the consequent of the rules:

$$O_{4,i} = \overline{w_i}f_i = \overline{w_i}(p_i x + q_i y + r_i) \ . \tag{6}$$

The parameters (p_i, q_i, r_i) in this layer are to be determined and are referred to as the consequent parameters.

In the final layer, a single node computes the overall output:

$$O_{5,i} = \sum_i \overline{w_i} f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \ . \tag{7}$$

This is how the input vector is typically fed through the network layer by layer. Next a consideration needs to be justified in how the ANFIS learns the premise and consequent parameters for the membership functions and the rules.

The ANFIS system will be taking input from the three developed moving average strategies and dynamically switch to the best performing one.

IV. Data

All simulations in this paper have been implemented for foreign exchange (Forex), high-frequency five-minute, intraday EUR/USD currency pair between 12/07/2008 and 04/06/2010.

The simple Moving average strategies, Momentum 1, Momentum 2, and Contrarian were implemented as described in Section 3.A and Table 1.

For ANFIS training and trading phases, sets of 1000 data points were derived from a five-minute mid-price dataset of exchange rate data/figures. The first half of these data points were used in system training/to train the system and the other half to carry out a performance check and to update the network structure using the output error. Next, an out-of-sample group consisting of 400 sets of data (1000 data points per set) was introduced into the system to test it. It is possible to make extremely efficient use of the available data as the 500 points used for testing the system can then be reused in another simulation for retraining purposes. Forex was chosen as the source of input data for the system due to the fact that it is a 24-hour market in which the majority of statistical properties found in financial time series are reflected in the prices [4]. [5] affirms that the stationarity hypothesis also applies to Forex data; past returns do not always reflect future performance. Additionally, gain/loss asymmetry and heavy tails can be identified in Forex return distributions alongside other such stylized facts. Orders are currently matched using automated brokerage terminals, and the price at the Forex market is formed by buying and selling currencies to exporters and importers, traders and institutions, portfolio managers and tourists. According to [31] approximately 85% of all Forex trading takes place among market makers, creating substantial room for speculative gains and losses. Now that currencies are floating, traders can trade against each other as opposed to against central banks. The system has been tested on EUR-USD. The dataset was split into various sets for testing and checking the system performance. The resulting system was able to perform predictions about the next move in the market. This systems aim is to forecast financial time series at high frequency using intraday data. The idea is to utilise previous values of the time series, which will be used as training data, to predict the value at a point in the future . When such a prediction is made, a trade is placed accordingly. For example, we place a buy position is placed if the price is predicted to rise or a sell if the price is predicted to drop.

V. Performance

Comparing the optimized ANFIS with standard strategies utilized in the industry will effectively evaluate the system's performance, such as Buy and Hold [31], or linear forecasting models using trend following or trend reverting signals [26]. Different measures for assessment will be used, such as (a) the wining rate, (b) the profit factor, (c) the return of investment (ROI), (d) the Sharpe ratio and (e) the Sortino ratio. Table 4.2 displays the performance of ANFIS against the other strategies for the EUR/USD FX rats.

With respect to the winning rate, Table 2 demonstrates that in most cases, the ANFIS system outperforms the standard strategies in the overall number of wins. In terms of the profit factor, which indicates the actual profitability of a series of trades on an investment, the results reveal that the ANFIS system also has a profit factor greater than one in most cases. The Sharpe ration can tell investors whether the returns of an asset or a portfolio come from a smart trading strategy or excess risk. The Sharpe ratio is defined as

$$Sharpe \ Ratio = \frac{R_p - R_f}{\sigma_p},$$

where R_p denotes the expected return, R_f the risk-free interest rate and σ_p the portfolio volatility. The Sharpe ratio measures the risk premium per each unit of total risk in an investment asset or a portfolio. Investors often pick investments with high Sharpe ratios because the higher the Sharpe ratio, the better its

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risk-adjusted performance has been. Similarly, the Sortino ratio is defined as

$$Sortino\ Ratio = \frac{R_p - R_f}{\sigma_{neg}},$$

where σ_{neg} denotes the standard deviation of only negative asset returns. The main difference between the Sharpe ratio and the Sortino ratio is that the Sortino ratio only penalizes the downside volatility, while the Sharpe ratio penalizes both upside and downside volatility. Thus, the Sortino ratio measures the risk premium per each unit of downside risk in an investment asset or a portfolio.

This is an initial attempt to test ANFIS against the other strategies using an initial currency, then, the number of epochs were optimised according to the results; this then allowed the optimised ANFIS system to be used with dataset. Positive Sharpe and Sortino ratios in Table 2 indicate that the amount of return achieved was not obtained at comparably high risk. In addition, ROI is 19%, which is a high rate for a two year investment described for this paper. Likewise, a high level of accuracy in the system's prediction rate was demonstrated by the winning rate and profit factor.

As mentioned before, all sub-data sets used for validation of the implemented trading system are considered as the "out-of-sample". Performance measures are computed for each validation data set. Table 2 reports the overall average performance measure for (i) the Buy and Hold, (ii) the momentum (trend following) strategies, (iii) the contrarian (trend reversal) strategy and (iv) the Intraday ANFIS trading strategy.

TABLE II
PERFORMANCE RESULTS

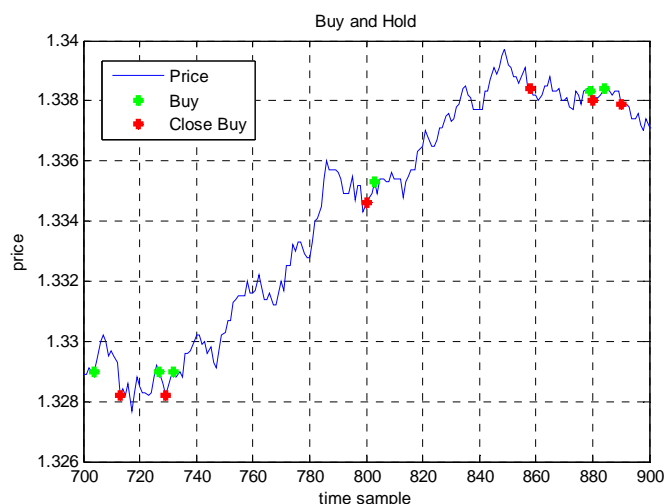| FX Pair	Winning Rate	Return of Investment	Sharpe Ratio	Sortino Ratio
EUR-USD				
Buy and Hold	0.32	0.02	-0.02	-0.01
Momentum 1	0.22	0.01	0.04	-0.10
Momentum 2	0.14	0.02	-0.02	-0.05
Contrarian	0.19	-0.01	0.09	0.02
Optimised Neuro Fuzzy System	0.57	0.19	0.19	0.12

Fig. 5a Sample performance of the buy and hold strategy
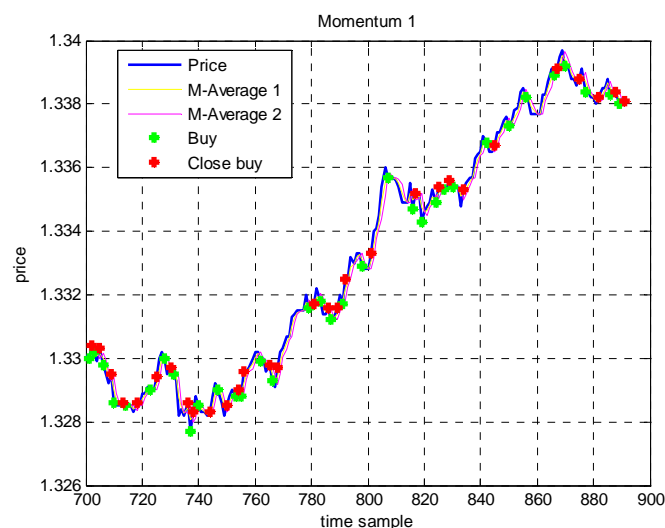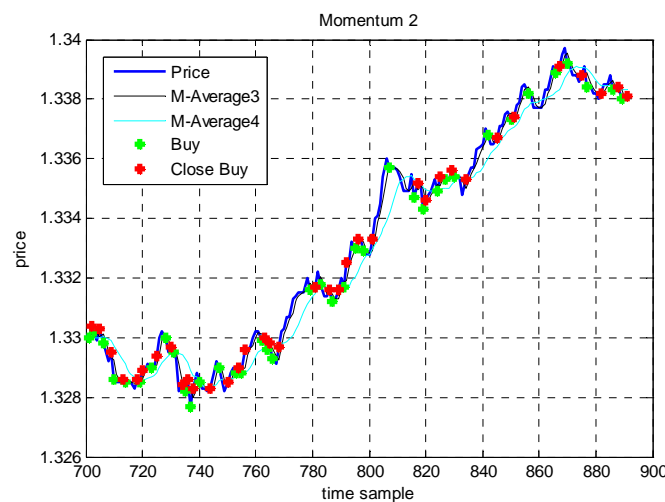

Fig. 5b Sample performance of the Momentum 1 strategy

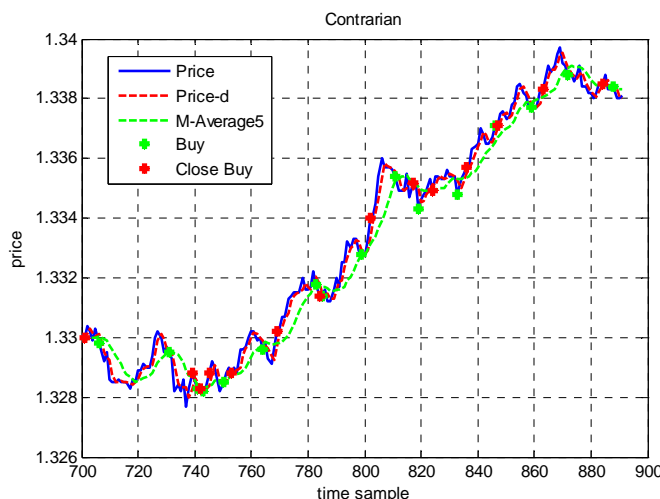Fig. 5c Sample performance of Momentum 2 strategy

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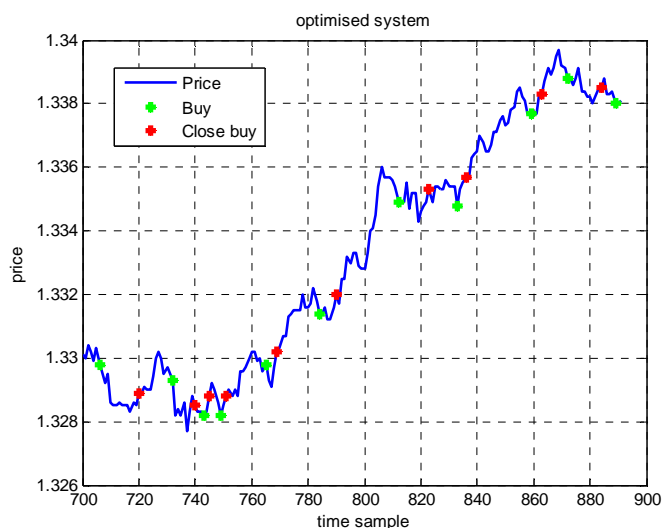Fig. 5d Sample performance of the Contraria strategy

Fig. 5e Sample performance of the Optimised ANFIS

VI. CONCLUSION

There are still relatively few studies comparing artificial intelligence approaches and classical models such as time series theory. However, recent research continues to suggest that adopting artificial intelligence techniques for the technical analysis of financial systems will yield positive results. With several researchers now questioning the Efficient Market Hypothesis particularly in light of the recent global financial crisis, artificial intelligence approaches such as time series that can improve analysis will likely continue to hold the interest of researchers and investors alike.

This paper presented an optimized ANFIS that uses output from previously created Simple Moving Average Strategies to optimize its decision making strategy to enter a trade. The system dynamically switches between three strategies and performs forecasts as to where the next price observation will move and place a trade accordingly. Results presented show promising results that can be expanded in future work. Future work should include testing various periods of moving averages and dynamically changing the periods of the various moving average.

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