World Academy of Science, Engineering and Technology
International Journal of Mathematical and Computational Sciences
Vol:1, No:11, 2007

An Alternative Proof for the NP-completeness of Top Right Access point-Minimum Length Corridor Problem

Priyadarsini P.L.K and Hemalatha T.

Abstract**—**In the Top Right Access point Minimum Length Corridor (TRA-MLC) problem [1], a rectangular boundary partitioned into rectilinear polygons is given and the problem is to find a *corridor* of *least* total length and it must include the top right corner of the outer rectangular boundary. A *corridor* is a tree containing a set of line segments lying along the outer rectangular boundary and/or on the boundary of the rectilinear polygons. The corridor must contain at least one point from the boundaries of the outer rectangle and also the rectilinear polygons. Gutierrez and Gonzalez [1] proved that the MLC problem, along with some of its restricted versions and variants, are NP-complete. In this paper, we give a shorter proof of NP-Completeness of TRA-MLC by findig the reduction in the following way.

Connected vertex cover in 2-connected planar graph with maximum degree 4

$\downarrow$

Top-Right Access Point Minimum Length Corridor (TRA-MLC)

Keywords**—**NP-complete, 2-Connected planar graph, Grid embedding of a plane graph.

I. Introduction

In the Minimum-Length Corridor (MLC) problem [1], a rectangular boundary partitioned into rectilinear polygons is given and the problem is to find a *corridor* of *least* total length. A *corridor* is a tree containing a set of line segments lying along the outer rectangular boundary and/or on the boundary of the rectilinear polygons. The corridor must contain at least one point from the boundaries of the outer rectangle and also the rectilinear polygons. An access point of a cooridor is any point on the rectangular boundary. If this access point is constrained to be at the top right corner of the outer rectangular boundary, then this problem is referred to as TRA-MLC. In the MLC problem, and in its variants, it is assumed that the rectangular boundary and the partitions are orthogonal. In fig. 1 , we can see an instance of TRA-MLC and the thick line refers to an optimal corridor with top right access point included.

This work is a part of the project funded by DST, Govt. of India, under Women Scientist Scheme

Priyadarsini P.L.K. is a Research Scholar in the Dept. of Comp. Applns, NIT, Trichy; Corr. Author;E-mail: darsini@nitt.edu

Hemalatha T. is a Professor in the Dept of Mathematics, NIT,Trichy; E-mail: hema@nitt.edu

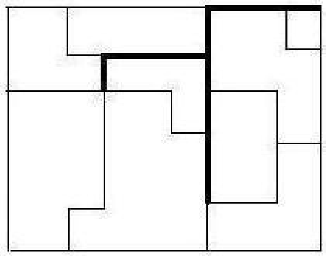

Fig 1: An Optimal corridor for an instance of TRA-MLC

The decision version of TRA-MLC can be defined as follows

Instance: A Pair (F, R) and a real number k, where F is a rectangular boundary and R is a set of rectilinear partitions $R_1, R_2, ...R_p$

Question: Does there exist a set S of line segments which form a tree such that $L(S) \leq k$, where $L(S)$, called the edge length, is the sum of the lengths of the line segments in S.

This problem has many applications in laying optical fibre cables for data communication and electrical wiring in floor plans. We can consider (F, R) as floor plan with the rectilinear partitions representing p rooms. The corridor refers to placement of cables. There are many other applcations which include signal communication in circuit layout design [1].

The Minimum Length Corridor (MLC) problem was first posed by Naoki Katoh [2] as an architectural design problem and its restricted version MLC-R was introduced by Eppstein [3]. An extensive study of these problems and their variants is made by Gutierrez and Gonzalez [1]. They also proved that the decision version of MLC problem, along with some of its restricted versions and variants, are NP-complete. To do this, they reduced the planar 3-SAT problem to TRA-MLC and TRA-MLC-R problems. From these two problems they found polynomial reductions to other variants of MLC. In the next section of this paper, we attempt to give an alternative proof of NP-completeness of the TRA-MLC problem. The proof which we are going to present is shorter and it uses popularly known graph theoretic concepts. Considering the reductions given in [1] , we can say that the varients of MLC and TRA-MLC are NP-Complete. We find the polynomial reduction in the following way.

Connected vertex cover in 2-connected planar graph with maximum degree 4

$\downarrow$

Top-Right Access Point Minimum Length Corridor (TRA-MLC)

Garey and Johnson proved that the problem of *finding connected vertex cover in planar graphs with maximum degree 4* (CVC) is NP-complete [4]. As a first step, we attempted to prove in [5] that a restricted version : *connected vertex cover in 2-connected planar graphs with maximum degree 4* (hereafter referred to as CVC-2) is also NP-complete by finding polynomial reduction from CVC to CVC-2. Now, in this paper, we find a polynomial reduction from CVC-2 to TRA-MLC thereby proving TRA-MLC is NP-complete.

To prove that any problem P to be NP-complete we need to show that

1. $P \in NP$: x is a *yes* instance of P if and only if there exists a *concise certificate* $c(x)$, and it is *verifiable* by a polynomial time algorithm.
2. Some known NP-complete problem P' is *polynomially reducible* to P : For any given instance x of P' , we should be able to construct an instance y of P within polynomial in $|x|$ time, such that x is a *yes* instance of P' if and only if y is a *yes* instance of P.

For more explanation on NP-completeness, reader is referred to [6, 7].

II. The Proof

Theorem: *TRA-MLC is NP-complete.*

Proof: It can be understood, from [1], that TRA-MLC $\in$ NP. Now, we construct an instance of TRA-MLC, from the given instance of the problem of *connected vertex cover in 2-connected planar graph with maximum degree 4* . Let an instance of CVC-2 be given by a 2-connected planar graph G_1 and an integer K. Assume that G_1 has n vertices with maximum degree 4 and m edges and K is the upper bound on the size of the vertex cover.

Our construction begins with finding a planar representaion G' of G_1 [8]. Let us consider a vertex x on the external face of G' and let the degree of x be d. We will now replace x with an edge (u, v) in the following way. We know that d will be equal to 2,3,or 4. If d is 2, we add one edge to each of u, v and the degrees of $u\&v$ will be 2. If d is 3, then u, v will be of degree 2 and 3 respectively as we add one incident edge to u, and two remaining consecutive edges in clockwise order around x to v. In the case of d being 4, we add frist two edges in clockwise order around x to u, and the remaining 2 edges to v making the degrees of u, v to be equal to 3. Let us call the new graph as G having $n + 1$ vertices and $m + 1$ edges and clearly this graph is also a 2-connected planar graph with maximum degree 4. Fig. 2(a), 2(b) show the example of G', G.

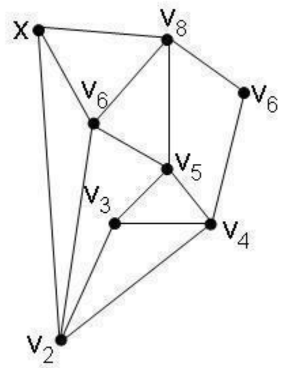

Fig 2(a): Graph G_1

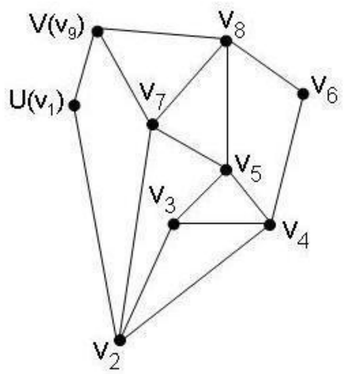

Fig 2(b): Graph G
(After replacing vertex X **with edge** (u, v) **)**

Now, we find the *weak visibility representation* [9] of G by selecting the edge (u, v) for st-numbering by taking $s = u$ and $t = v$ (All the vertices of G will be distinctly numbered from 1 to $n + 1$ making $u = v_1$ and $v = v_{n+1}$ (we refer to u as v_1 and v as v_{n+1} from now onwards). Then find the orthogonal representation for G followed by a grid embedding, as described in [10], on a discrete grid of squares with all points of the form $(6i, 6j)$ where i, j are integers. Let p_i denote the point in the grid corresponding to a vertex v_i in G and all these points will have coordinates of the form $(6i, 6j)$. It is easy to find out the coordinates of the corners of the smallest rectangle which encloses the grid embedding and let us call the four corners, in clockwise order starting from bottom-left, as $(x_1, y_1), (x_1, y_2), (x_2, y_2)$ and (x_2, y_1). Fig. 3 shows the grid embeding of the graph G.

After obtaining the grid embedding, we add some more line segments to it to get an instance of TRA-MLC as follows. Refer to fig. 4, which is an instance R of TRA-MLC for the construction. Let $h = y_2 - y_1$ (height of the rectangle) and let $d = 6n^2 - h - 6$ (as the area of the rectangle is $\mathcal{O}(n^2)$ [12], d will be non-negative). Now, we draw a rectangle with $A = (x_1 - 6, y_1 - d), B = (x_1 - 6, y_2 + 6), C = (x_2 + 6, y_2 + 6)$

and $B = (x_2+6, y_1-d)$ as corners in clockwise order starting from bottom-left corner. This rectangle $ABCD$ completely encloses the grid embedding. The degrees of v_1, v_{n+1} in G are not more than 3, and hence by the way the transformations are done in [10], the points p_1 and p_{n+1} would not have edges at the bottom and on top respectively. Now, we shall draw vertical lines joining p_{n+1} to the horizontal line $\bar{BC}$ and p_1 to the horizontal line $\bar{AD}$ and let us call the intersection points as E and F respectively.

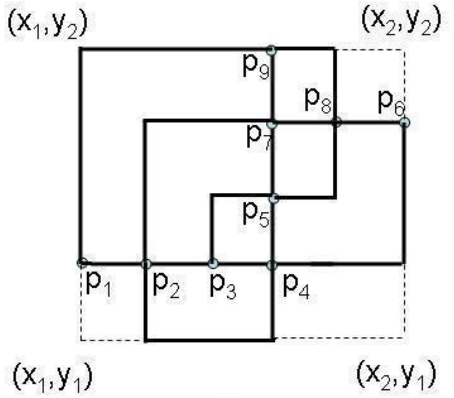

Fig 3: Grid (6×6) embedding of Graph G

For all the line segments (p_i, p_j) where $1 \leq i, j \leq (n+1)$ corresponding to the edges in G, we draw unit squares on both the sides (top and bottom for the horizontal component, left and right for the vertical component), as shown in fig. 4, leaving a line segment of length 2 units at both the end points p_i and p_j. For the line segment (p_{n+1}, E) draw unit squares on either sides leaving two units at p_{n+1}. Let us call a point on the line (p_{n+1}, E), which is at a distance of 3 units from p_{n+1} as H. For the line segment $\bar{EC}$, we draw unit squares at the bottom of the line. The resultant rectangle, $(ABCD)$ is a retangular grid, say R, divided into rectilinear partitions and this forms the instance of TRA-MLC.

For any point p_i corresponding to a vertex v_i in G, there will be d_i (degree of v_i) line segments having one end at p_i, and let us call parts of these line segments, each of 3 units of length from p_i, together with p_i as p_i's *region*. For any line segment (p_i, p_j), corresponding to the edge (v_i, v_j) in G, remove the line segments in p_i & p_j's *regions* and let us call the remaining line segment as *edge component* of (v_i, v_j) (This line segment is sufficient to cover all the squares as it touches the corner points of the two squares in p_i and p_j *regions*). Fig. 5 shows a line segment corresponding to an edge in G.

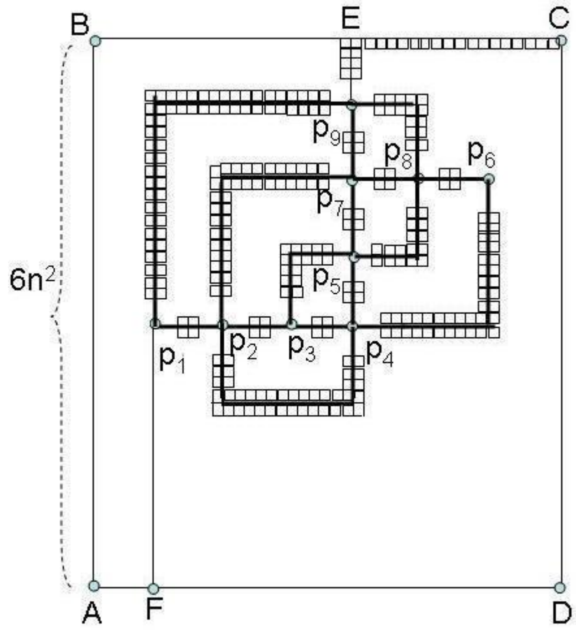

Fig 4: Instance R of TRA-MLC (Constructed from G)

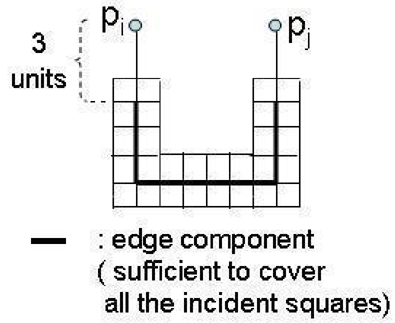

Fig 5: A line segment in R representing an edge in G

Now find the lengths of all the $(m+1)$ *edge components* and let this total length be l. Add length of $\bar{CE}$ and length of $\bar{EH}$ to l. Now let us consider the integer $L = l + 3(m + K + 2)$. We prove that the given 2-connected planar graph G_1 with maximum degree 4 will have a connected vertex cover of size $C_1 \leq K$ if and only if the instance R of TRA-MLC will have a corridor RLT, in which top right access point is included, and with a length $C' \leq L$.

First assume that V_1 is a connected vertex cover of G_1 and with size $C_1 \leq K$. If the vertex x in G_1 belongs to V_1 then we take $V = \{v_1, v_{n+1}\}\ U\ (V_1 - \{x\})$ which is a subset of the vertices of graph G. If $x \notin V_1$ then we take $V = \{v_{n+1}\}\ U\ V_1$. Now V forms a connected vertex cover of size C_1+1 which is less than or equal to $K+1$ for the new graph G. The vertex v_{n+1} will always be present in the vertex cover V. Find a tree T induced by V in G with C_1 edges. In the rectangular grid R corresponding to G, any corridor should include atleast all the *edge components* inorder to cover the unit squares drawn on both the sides of these *edge components*. Also, any corridor must include the line segments $\bar{CE}$ and $\bar{EH}$ to cover the squares incident on them. Let us construct a rectilinear tree (a corridor) RLT starting with all these *edge components* along with the line segments $\bar{CE}$ and $\bar{EH}$. For the line segment corresponding to any edge (v_i, v_j), belonging to the tree T, add length 3 line segments, in both p_i, p_j regions, on the line (p_i, p_j) along with p_i, p_j to RLT. For any edge (v_k, v_h), which is not in T, either v_k or v_h or both must be present in V and without loss of generality let us assume that $v_k \in V$. Now, add length 3 line segment on the line (p_k, p_h), in p_k's*region* along with p_k, to RLT. Finally, add length 3 line segment (p_{n+1}, H) to RLT. This line segment will be along the borders of the two rectilinear partitions formed by the sides of the outer rectangle $\bar{AB}$ and $\bar{CD}$. For any rectilinear region corresponding to the face of the graph G, there will be atleast one point $p_i \in RLT$ which corresponds to a vertex covering the edge incident on it and which is on the border of the face. Now RLT will be a rectilinear tree along the sides of the rectilinear partitions and the outer rectangle. It includes the top right access point C and has a length $C' = l + 3 + 3(m + 1) + 3C_1$ which is less than or equal to L. So RLT becomes the required corridor.

Conversely, suppose the instance R of TRA-MLC has a corridor RLT including the top right access point C, and it is of length $C' \leq L$. As mentioned above, RLT should include all the *edge components* to cover all the unit squares drawn along the line segments representing edges in G. Also it should include the lines CE and EH to cover the squares drawn along these lines. So the length of these line segments together is l and this should be a part of C'. The length of the remaining line segments in the corridor RLT will be at the most ($3(m + k + 2)$) as $C' \leq L$. The line segments $H\bar{p}_{n+1}$, $\bar{Ep}_1$ and (C, p_1) connect the outer rectangle to the inner grid embedding of G. Among these, RLT cannot include $\bar{Ep}_1$ and (C, p_1) because, by way of construction of R, each of their lengths will be greater than $6n^2$ and hence it is greater than $3(m + K + 2)$. So RLT must include the line segment (H, p_{n+1}) to connect the rectangular boundary to the inner rectilinear partitions and hence the length of the remaining part of RLT will be at the most $3(m + 1) + 3K$. There are $m + 1$ *edge components* corresponding to the edges in G and inorder to connect them together into a tree, atleast one length 3 line segment connecting the *edge component* to one of its incident points should be present in RLT. These line segments together will have a length of $3(m + 1)$ and the remaining line segments in RLT will have length at the most $3K$. This extra length comes from the length 3 line segments on the other side of some of the *edge components* which are included in RLT ie. For a maximum of K *edge components* the length 3 line segments joining to both the incident points are present in RLT. Now let us consider a subset V of the vertices of G, containing all the vertices corresponding to the points for which atleast one length 3 line segment in their *region* is included in RLT. The set V will obviously cover all the $(m+1)$ edges in G. If we consider the edges corresponding to the *edge components* for which the length 3 line segments on both the sides along with both the incident points are in RLT, they will be at the most K. As these line segments are part of a tree RLT in R, we can say that the corresponding K edges in G form a tree connecting vertices of V and hence $|V|$ is at the most $K + 1$. To find a corresponding vertex cover in the original graph G_1 let us take a subset V_1 of the vertices of G_1, with the vertices in the set $(V - \{v_1, v_{n+1}\})$. If $P_1 \in RLT$ then corresponding v_1 will also be in V, and hence we add x to V_1. The size of V_1 will be at the most K and it forms a connected vertex cover for G_1. Hence the proof.

III. CONCLUSIONS

The proof given in this paper is shorter and it uses the most commonly known concepts of graph theory. The restricted version TRA-MLC-R, imposes a constraint on TRA-MLC that all the rectilinear partitions should be rectangles. We are hopeful that, in future, a shorter proof of the complexity of this problem can also be given.

ACKNOWLEDGMENTS

We wish to thank Dr. Venkatesh Raman professor of IMS, Chennai for his timely help in getting important references without which this work would have been delayed further.We also would like to thank Dr. A. Ramakalyan, Asst. Professor, NITT for his support.

REFERENCES

[1] A.Gonzalez-Gutierrez and T.F.Gonzalez , " Complexity of the Minimum-Length Corridor Problem ",*J. Comp. Geometry, Theory and Applns.,Els.* vol.37, no.2, 2007, 72-103.
[2] E.D.Domaine and J.O.Rourke, " Open Problems from CCCG 2000 ", *Proc. of the 13th Canadian Conference on Computational Geometry(CCCG 2001)*, 2001, 185-187.
[3] D.Eppstein , " Some Open Problems in Graph Theory and Computational Geometry ", 2001 http:// www.ics.uci.edu/~ eppstein/200-f01.pdf.
[4] M.R.Garey and D.S.Johnson , " The Rectilinear Steiner Tree Problem is NP - complete ", J. Appl. Math.(SIAM), vol. 32, no. 4, 1977, pp. 826-834.
[5] Priyadarsini P.L.K and Hemalatha T, " Connected Vertex Cover in 2-Connected Planar Graph with Maximum Degree 4 is NP-complete ", submitted to International conference AMNA 2007.
[6] M.R. Garey and D.S. Johnson , *Computers and Intractability: A Guide to the Theory of NP-Completeness*, W.H. Freeman & Co, 1979.
[7] C.H. Papadimitriou and K. Steiglitz , *Combinatorial optimization: Algorithms and Complexity*, Prentice Hall India, 1997.
[8] R. Weiskircher, " Drawing Planar Graphs ", in " Drawing Graphs" (M.Kaufmann & D.Wagner, Eds.), pp. 23-45, LNCS 2025,Springer-Verlag, 2001.
[9] R.Tamassia and I.G.Tollis, "A Unified Aproach to Visibility Representation of Planar Graphs",*J. Discrete and Computational Geometry, Springer-Verlag*, vol. 1, 1986, 321-341.
[10] R.Tamassia and I.G.Tollis, "Planar Grid Embedding in Linear Time", IEEE Transactions on circuits and systems, vol. 36 no. 9, 1989, 1230-1234.