Compensation–Based Current Decomposition

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Abstract—This paper deals with the current space-vector decomposition in three-phase, three-wire systems on the basis of some case studies. We propose four components of the current space-vector in terms of DC and AC components of the instantaneous active and reactive powers. The term of supplementary useless current vector is also pointed out. The analysis shows that the current decomposition which respects the definition of the instantaneous apparent power vector is useful for compensation reasons only if the supply voltages are sinusoidal. A modified definition of the components of the current is proposed for the operation under nonsinusoidal voltage conditions.

Keywords—Active current, Active filtering, p–q theory, Reactive current.

I. INTRODUCTION

A wide variety of approaches have been proposed to decompose the current waveform into various components in the general case of nonsinusoidal conditions [1]–[4]. Especially, the decomposition into necessary and useless components is needed for the control of compensators such as active filters.

In 1983, Akagi and his coauthors introduced the so-called "p–q theory" in three-phase, three-wire systems which was expected to be valid for any instantaneous variation of voltage and current [3]. This theory uses the complex space vector theory and introduces the concepts of instantaneous active power (*p*) and instantaneous reactive power (*q*). Then, the definitions of d and q-axis instantaneous active and reactive currents use only the instantaneous powers and the voltages in d-q coordinates [4]. Many extensions of the original p-q theory have been developed [5]–[7].

However, some conceptual limitations of this theory were pointed out by Willems in [8], [9]. Moreover, Professor L.S. Czarnecki from Louisiana State University has investigated how power phenomena and properties of three-phase systems are described and interpreted by the instantaneous p–q theory

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[10], [11]. The argumentation through which Czarnecki disagrees with the p-q theory is principally based on the relativity of the active and reactive character of the currents defined by Akagi and his followers. Czarnecki introduced his own current decomposition in 1988 [12]. These components are referred to as Current's Physical Components (CPC) and used as a tool for study [13]. Still, there are discussions about p–q and CPC theories [14], [15].

The decomposition of the currents proposed by the authors avoids both terminology and interpretation ambiguities by practical examples.

Under nonsinusoidal voltage conditions, the proposed components of the current do not respect the definition of the complex apparent power introduced by Akagi.

II. P-Q THEORY AND CURRENT'S COMPONENTS

The p–q theory introduced by Akagi and his coauthors uses the complex space-vector theory that implements a transformation from a reference system in R-S-T coordinates to a stationary system with only two orthogonal axes d-q [3].

By using a space-vector notations, the complex apparent power ($\underline{s}$) is calculated as [4],

$$\underline{s} = \frac{3}{2}\underline{u}\cdot\underline{i}^{*}. \tag{1}$$

It should be mentioned that the constant used in the space vector definition was selected to be 2/3, which is a non power invariant scaling.

As a result, if the zero-sequence components are absent, the real part of the complex apparent power named as instantaneous active power may be expressed by using the peak-value scaled space-vector representation as

$$p = \mathrm{Re}\{\underline{s}\} = \frac{3}{2}\left(u_d i_d + u_q i_q\right). \tag{2}$$

Moreover, the imaginary part of the complex apparent power named as instantaneous reactive power is

$$q = \mathrm{Im}\{\underline{s}\} = \frac{3}{2}\left(-u_d i_q + u_q i_d\right). \tag{3}$$

We specify that Akagi originally defined *q* as a negation of (3).

The expression (1) allows expressing the current vector

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$$\underline{i} = \frac{2}{3} \cdot \frac{u \cdot s^*}{|u|^2} = \frac{2}{3} \cdot \frac{1}{|u|^2} \left[p u_d + q u_q + j\left(-q u_d + p u_q\right) \right], \qquad (4)$$

where $|u|^2$ is the square of voltage space-vector modulus.

Thus, the active ($\underline{i}_a$) and reactive ($\underline{i}_r$) components of the instantaneous current vector have been introduced [4]:

$$\underline{i}_a = i_{ad} + j i_{aq} = \frac{2}{3} \cdot \frac{u_d}{|u|^2} p + j \frac{2}{3} \cdot \frac{u_q}{|u|^2} p \; ; \qquad (5)$$

$$\underline{i}_r = i_{rd} + j i_{rq} = \frac{2}{3} \cdot \frac{u_q}{|u|^2} q - j \frac{2}{3} \cdot \frac{u_d}{|u|^2} q \; . \qquad (6)$$

The names assigned to the previous currents have been criticized with good reason by Czarnecki [10] who has found examples in which the current defined by (5) has not corresponded to the active power, and the current defined by (6) has not corresponded to the reactive power.

This remark is right because, as he has shown, the instantaneous powers p and q contain both active and reactive power as well as the components which characterize the nonsinusoidal and unbalanced power.

III. CORRECT INTERPRETATION OF P-Q THEORY

In order to obviate the above ambiguity, a possible decomposition of the current space-vector takes into account the DC components (P and Q) and the AC components ($p_\sim$ and $q_\sim$) of the instantaneous powers p and q. Thus, the expression (4) of the current space-vector becomes

$$\underline{i} = \frac{2}{3} \cdot \frac{1}{|u|^2} \left[(P + p_\sim) u_d + (Q + q_\sim) u_q + \right. \\ \left. + j\left(-(Q + q_\sim) u_d + (P + p_\sim) u_q\right) \right] \qquad (7)$$

Starting from expression (7), the following current space-vectors can be defined.

1. The active and reactive current vectors ($\underline{i}_a$ and $\underline{i}_r$), whose components are:

$$i_{ad} = \frac{2}{3} \cdot \frac{u_d}{|u|^2} P \; ; \quad i_{aq} = \frac{2}{3} \cdot \frac{u_q}{|u|^2} P \; ; \qquad (8)$$

$$i_{rd} = \frac{2}{3} \cdot \frac{u_q}{|u|^2} Q \; ; \quad i_{rq} = -\frac{2}{3} \cdot \frac{u_d}{|u|^2} Q. \qquad (9)$$

2. The supplementary useless current vectors on account of $p_\sim$ ($\underline{i}_{sp}$) and $q_\sim$ ($\underline{i}_{sq}$), whose components are:

$$i_{spd} = \frac{2}{3} \cdot \frac{u_d}{|u|^2} p_\sim \; ; \quad i_{spq} = \frac{2}{3} \cdot \frac{u_q}{|u|^2} p_\sim \; ; \qquad (10)$$

$$i_{sqd} = \frac{2}{3} \cdot \frac{u_q}{|u|^2} q_\sim \; ; \quad i_{sqq} = -\frac{2}{3} \cdot \frac{u_d}{|u|^2} q_\sim \; . \qquad (11)$$

It is also possible to define the total supplementary useless current vector ($\underline{i}_s$) as a sum of the two supplementary useless current vectors. Thus, its components are:

$$i_{sd} = \frac{2}{3} \frac{u_d p_\sim + u_q q_\sim}{|u|^2} \; ; \quad i_{sq} = \frac{2}{3} \frac{u_q p_\sim - u_d q_\sim}{|u|^2} \; . \qquad (12)$$

It is easy to see that the moduli of above vectors comply with the next orthogonality condition

$$\left| \underline{i}_a + \underline{i}_{sp} \right|^2 + \left| \underline{i}_r + \underline{i}_{sq} \right|^2 = \left| \underline{i} \right|^2 . \qquad (13)$$

As far as sum of $\underline{i}_a$, $\underline{i}_r$ and $\underline{i}_s$ moduli are concerned, it has been found that

$$\left| \underline{i}_a \right|^2 + \left| \underline{i}_r \right|^2 + \left| \underline{i}_s \right|^2 = \left| \underline{i} \right|^2 - \frac{8}{9} \cdot \frac{1}{|u|^2} \left(P p_\sim + Q q_\sim \right). \qquad (14)$$

By integrating (14), the next expression has been obtained

$$I_a^2 + I_r^2 + I_s^2 = I^2 - \frac{8}{9} \cdot \frac{1}{2\pi} \int_0^{2\pi} \frac{P p_\sim + Q q_\sim}{|u|^2} d(\omega t), \qquad (15)$$

where I_a, I_r, I_s and I denote the rms values of $\left| \underline{i}_a \right|$, $\left| \underline{i}_r \right|$, $\left| \underline{i}_s \right|$ and $\left| \underline{i} \right|$.

It can be seen that the second term at the right side of (15) is zero only if $|u|$ is constant. It means that the rms values of the current components moduli are mutually ortogonal, i.e.

$$I_a^2 + I_r^2 + I_s^2 = I^2, \qquad (16)$$

only under sinusoidal voltage conditions.

If the voltages waveform is not sinusoidal, then $|u|$ is not constant and, consequently,

$$\frac{1}{2\pi} \int_0^{2\pi} \frac{P p_\sim + Q q_\sim}{|u|^2} d(\omega t) \neq 0 . \qquad (17)$$

The proposed decomposition of the current gives us a new point of view on the reference current calculation for active power filters. Thus, if Akagi has focused his attention on powers to be compensated, we think that the concern has to be

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on the supply current waveform. In this way, as mathematical solutions should not ignore practical implementation issues, it will result a diminution of the amount of calculation.

Taking into account that the main goal is to obtain a supply current that has to provide only the required active power, the active filter has to provide the current vector

$$\underline{i}_F = \underline{i}_L - \underline{i}_a \,, \tag{18}$$

where $\underline{i}_L$ is the load current vector.

Therefore, in the reference current calculation, only one integral is to be made in order to calculate the active power.

As it will be shown in next section, the above current decomposition is not useful for reference current calculation under nonsinusoidal voltages. It is pointed out that a simple replacement of $|\underline{u}|$ with its rms value in expressions from (8) to (12) of the current components makes possible the use of this theory even if the voltages are not sinusoidal in shape.

IV. CASE STUDIES

Three case studies have been taken into consideration in order to validate the proposed current decomposition for compensation reasons.

The simulations were carried out under Matlab-Simulink environment.

A. Sinusoidal Voltages and Nonsinusoidal Currents

Let us consider the thee-phase system with sinusoidal voltages and nonsinusoidal currents in the primary of a D/Y transformer which supplies a DC motor via a full controlled rectifier. The waveforms of phase voltage and distorted current for a control angle of 30° are shown in Fig. 1.
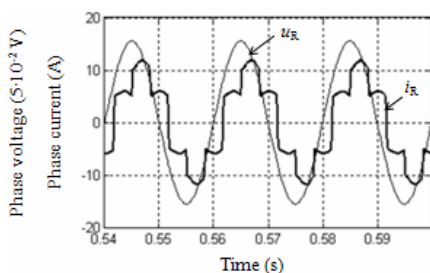

Fig. 1 Phase voltage and current in the primary of the transformer

As it can be seen in Fig. 2, the proposed active current waveform is sinusoidal, unlike the active current defined by Akagi in (5) (Fig. 3), although the both currents are in phase with the phase voltage. This happens because the Akagi's active current contains both the proper active component and the distortion component.
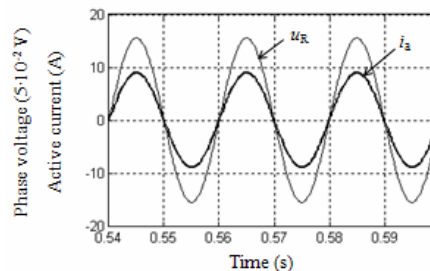

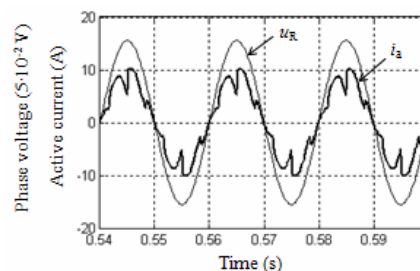Fig. 2 Phase voltage and proposed active current waveforms

Fig. 3 Phase voltage and active current defined by (5)

As regards the proposed reactive component of the current, it has a sinusoidal shape like the active component, but shifted by 90° behind the voltage, as expected (Fig. 4).
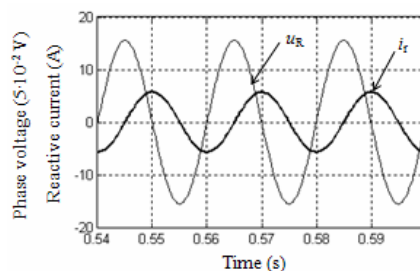

Fig. 4 Phase voltage and proposed reactive current waveforms

Although the Akagi's reactive current expressed by (6) lags the voltage by 90°, it is much distorted owing to its components which characterize the nonsinusoidal conditions (Fig. 5).
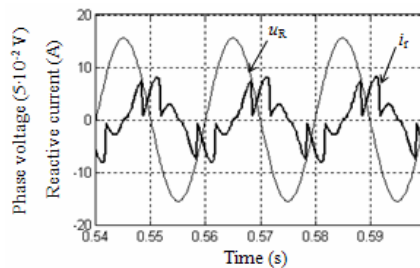

Fig. 5 Phase voltage and reactive current defined by (6)

As the active power transfer is achieved only on the fundamental frequency in the case of sinusoidal voltage conditions, the components of the current introduced by (5) and (6) have nothing in common with the meaning of the

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active and reactive currents as used in electrical engineering [1], [8], [10], [11].

Obviously, the trajectories of the active and reactive current space-vectors are circles only with the proposed definitions (Fig. 6).
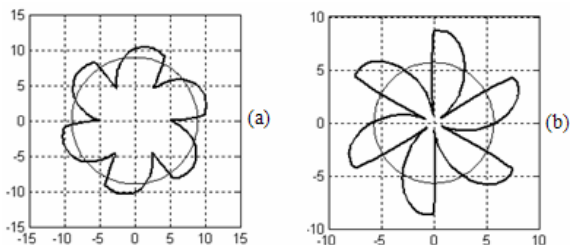

Fig. 6 Space-vector trajectories of the active current (a) and reactive current (b) defined by Akagi (thin line) and proposed (thick line) by (8) and (9)

The total supplementary useless current according to (12) and its vector locus are shown in Fig. 7 and Fig. 8.
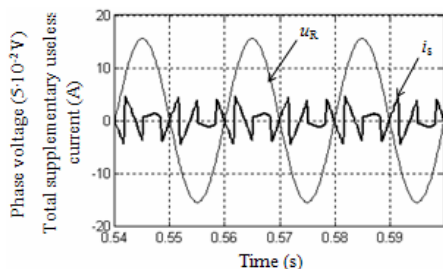
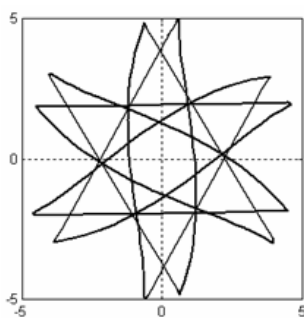Fig. 7 Phase voltage and total supplementary useless current

Fig. 8 Space-vector trajectory of the supplementary useless current

B. Nonsinusoidal Voltages and Balanced Resistive Load

As voltages in the secondary of the transformer have low distortion level, we have chosen another case study to serve as a model to current decomposition. A three-phase balanced resistive load of $R = 2$ Ω is supplied by a three-phase nonsinusoidal voltage system as follows:

$$u_R = \sqrt{2}\left(100\sin\omega t + 50\sin 5\omega t\right);$$
$$u_S = \sqrt{2}\left(100\sin\left(\omega t - 2\pi/3\right) + 50\sin 5\left(\omega t - 2\pi/3\right)\right); \qquad (19)$$
$$u_T = \sqrt{2}\left(100\sin\left(\omega t + 2\pi/3\right) + 50\sin 5\left(\omega t + 2\pi/3\right)\right).$$

The waveforms of phase voltage and supply current are both nonsinusoidal but they are in-phase (Fig. 9).
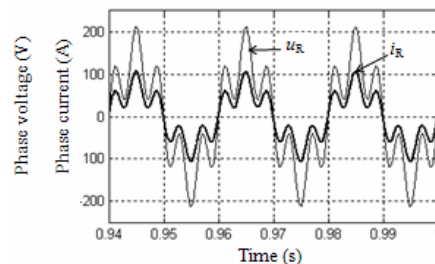

Fig. 9 Nonsinusoidal supply voltage and current waveforms in the case of purely resistive load

As it can be seen, the active current, as defined by (8), is substantially different in shape compared to the supply voltage even in the case of linear load (Fig. 10).
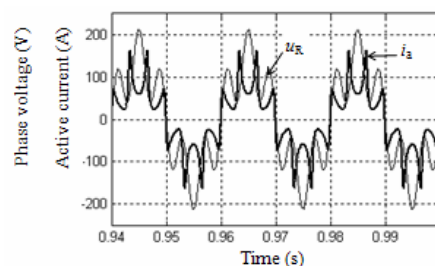

Fig. 10 Nonsinusoidal supply voltage and active current, as defined by (8), in the case of linear balanced load

This situation is generated by the fact that the square of voltage vector modulus in active current component definition is time-dependent (Fig. 11).
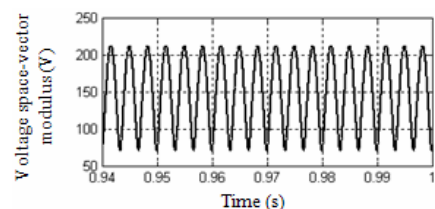

Fig. 11 Evolution of the voltage vector modulus

As expected, the reactive component of the current does not exist and the supplementary useless current is shown in Fig. 12.

On the other hand, the linear character of the balanced load makes the Akagi's current defined by (5) have the same waveform as the supply voltage in this particular situation (Fig. 13).

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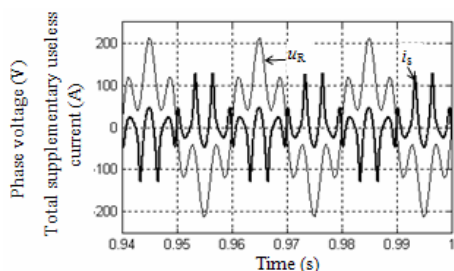International Journal of Physical and Mathematical Sciences
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Fig. 12 Total supplementary useless current, as defined by (12), related to supply voltage in the case of linear balanced load
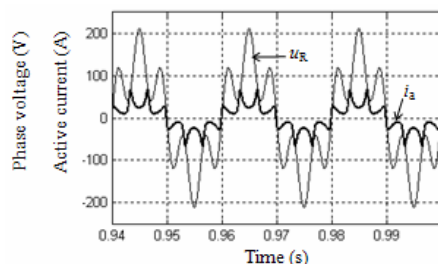

Fig. 15 Nonsinusoidal supply voltage and active current, as defined by (8), in the case of nonlinear balanced load
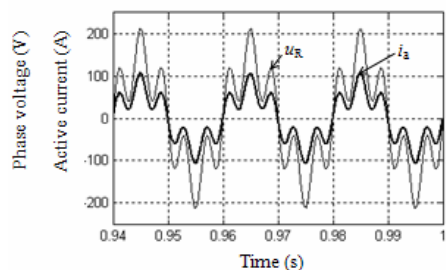

Fig. 13 Akagi's active current related to supply voltage in the case of linear balanced load
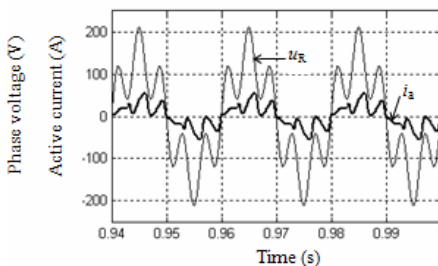

Fig. 16 Akagi's active current related to supply voltage in the case of nonlinear balanced load

C. Nonsinusoidal Voltages and Balanced Nonlinear Load

In this example, a series RL load of $R = X_L = 2\ \Omega$ is supplied by the three-phase nonsinusoidal voltage system specified by (19). This time, the distorted current and voltage have different waveforms. Moreover, a delay of the supply current with respect to the supply voltage occurs in such a circuit (Fig. 14).

Fig. 17 Nonsinusoidal supply voltage and reactive current, as defined by (9), in the case of nonlinear balanced load

D. Results Interpretation

Taking into account the results obtained by analyzing the previous typical examples, some concluding remarks can be made evident with reference to decomposition of the nonsinusoidal current in three-phase, three-wire systems.

1. The Akagi's component of the current, as introduced by (5), can be an active one only if the load is linear and balanced.

2. The component of the current, as proposed by (8), can be the active one only under sinusoidal voltage conditions for both linear and nonlinear balanced load.

3. If the supply voltage system is not sinusoidal, the current proposed by (8) cannot be an active component. This result can be explained by the fact that the voltage vector modulus in the denominator has a time variation (Fig. 11). As a result, the harmonics spectrum of this component of the current is not the same with the voltage harmonics spectrum.

The results in these simple case studies allow us to conclude that the current components expressed by (8)–(12) are not useful for reference current calculation in active
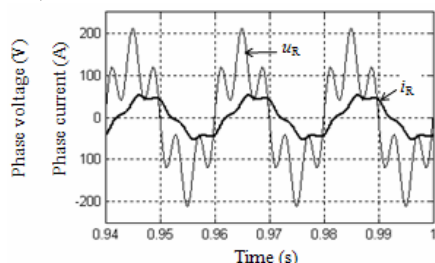
Fig. 14 Nonsinusoidal supply voltage and current waveforms in the case of RL load

The distorted active component of the current (Fig. 15), as defined by (8), has the following properties: its zero-passing coincide with the voltage zero-passing; it leads to an active power of 7.6 kW which is equal to the power consumed by the resistive component of the load; its rms value is of 29.4 A.

The nonlinear character of the load makes the Akagi's active current be much distorted with respect to the supply voltage (Fig. 16), unlike the purely resistive load situation shown in Fig. 13.

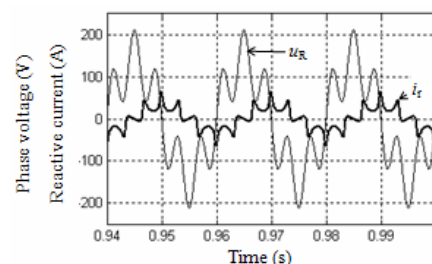As it can be seen in Fig. 17, the reactive current, as proposed by (9), lags the voltage by 90°.

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filtering if the voltages have not a sinusoidal shape.

Indeed, for the second case study, if the compensation is achieved by a parallel active filter and its reference current is distorted related to the supply voltage, the rms value of the supply current is higher than the initial load current even if this new current provides the necessary active power, removes the AC component of the instantaneous active power ($p_\sim$) and has the same phase with the voltage. For example, in this case study, the rms initial load current is exceeded by about 30% after compensation. Consequently, it is not a better solution.

In the last case study, the component of the current defined by (8) contains harmonics whose order is $6k+1$. Clearly, such a current generates active power only on fundamental frequency, which explains the rms value of 29.4 A of this current.

4. In order to solve this aspect of the problem, the replacement of $|\underline{u}|$ in (8)–(12) with its rms value is proposed, i.e.

$$U^2 = \frac{1}{T}\int_{t-T}^{t} |\underline{u}|^2 \, dt \, . \tag{20}$$

After this replacement, the new active, reactive and supplementary useless components of the current are:

$$i_{ad} = \frac{2}{3} \cdot \frac{u_d}{U^2} P; \quad i_{aq} = \frac{2}{3} \cdot \frac{u_q}{U} P \, ; \tag{21}$$

$$i_{rd} = \frac{2}{3} \cdot \frac{u_q}{U^2} Q; \quad i_{rq} = -\frac{2}{3} \cdot \frac{u_d}{U^2} Q. \, ; \tag{22}$$

$$i_{sd} = \frac{2}{3}\frac{u_d \, p_\sim + u_q q_\sim}{U^2}; \quad i_{sq} = \frac{2}{3}\frac{u_q \, p_\sim - u_d q_\sim}{U^2} \, . \tag{23}$$

It is obvious that the use of expression (21) for the active current calculation makes this current keep the voltage waveform. In the case of last case study, the active current calculated with (21) provides the required active power with only 22.8 A rms value of this current (Fig. 18).
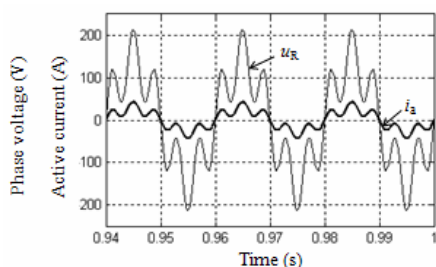

Fig. 18 Nonsinusoidal supply voltage and active current, as defined by (21), in the case of nonlinear balanced load

4. Undoubtedly, the proposed current decomposition based on complex apparent power vector is useful in the calculation of the reference current for active power filters. Thus, when total compensation is expected, the reference current requires only the load current and its active component.

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