Main Bearing Stiffness Investigation

B. Bellakhdhar, A. Dogui and J.L. Ligier

Abstract—Simplified coupled engine block-crankshaft models based on beam theory provide an efficient substitute to engine simulation in the design process. These models require accurate definition of the main bearing stiffness. In this paper, an investigation of this stiffness is presented. The clearance effect is studied using a smooth bearing model. It is manifested for low shaft displacement. The hydrodynamic assessment model shows that the oil film has no stiffness for low loads and it is infinitely rigid for important loads. The deformation stiffness is determined using a suitable finite elements model based on real CADs. As a result, a main bearing behaviour law is suggested. Several models are used to investigate the main bearing stiffness: a full smooth bearing model to explore the clearance effect, the hydrodynamic lubrication model to study the oil film stiffness and finite elements model to determine the bearing deformation stiffness. Consequently, the established behaviour law takes into account, in the same time, the clearance, the hydrodynamic assessment and the deformation stiffness.

II. MAIN BEARING STIFFNESS

The main objective of the analysis of reaction stiffness of crankshaft bearing carried out in this study is to determine a behaviour law of the bearing to use in a simplified coupled crankshaft-engine block beam model. This behaviour law connects the bearing reaction to the shaft motion compared to the bearing centre.

The considered bearing consists of a bearing cap assembled on the crank case. It has a hydrodynamic lubrication. The relative movement of “the axis” of the shaft compared to “the axis” of the bearing is mainly due to three factors:

- The clearance between the shaft and the journal bearing: the clearance affects first of all the value of the eccentricity due to the movement of hydrodynamic lubrication. It also influences the deformations stiffness of the bearing.

- The oil film stiffness: The stiffness of the oil film under hydrodynamic pressure must be analyzed to know if it is necessary to take it into account or consider that this film is infinitely rigid.

- The deformation of the bearing and the shaft: This deformation is not essentially localized in the contact zone. Thus, it does not depend only on dimensions of contact surfaces; the external geometry of the bearing can have a considerable influence (function of the applied efforts).

These analyses are made using analytical contact models to investigate the influence of the clearance and oil film on the stiffness of the bearing and finite elements models to determine the deformation stiffness.

A. The Clearance Effect

Classically, a simplified smooth bearing model is used to determine the distribution of the contact pressure in smooth bearings. Here a full smooth bearing model, without approximations, is used to analyze the influence of the clearance on the total rigidity of the contact shaft/bearing.

The shaft is supposed to be stiff. Therefore all the
deformation is supposed to be undergone by the main bearing. Imposing a displacement to the shaft, it’s supposed that the shaft imposes its form on the deformation of the bearing. The reaction force is obtained by the integration of the contact pressure on the entire contact surface. The reaction force has the same direction as displacement (Fig.1).

Fig. 1 Smooth bearing model

The contact pressure \( p \) in a point \( M \) of the surface of contact is supposed to be proportional to the “deformation” \( \partial \varepsilon \): PM:

\[
p(\theta) = k R \partial \varepsilon(\theta)
\]

Using the geometrical closings \((O_e, O_i, M_0)\) and \((O_e, O_i, M)\) it is possible to determine the angle \( \theta_0 \) and \( \partial \varepsilon(\theta) \), thus \( p(\theta) \), for a given displacement \( U \), the integration of the contact pressure results in the reaction force \( R_f \). It has the same direction as the displacement: \( R_f = k U \).

Using the following dimensionless variables:
- The relative displacement \( u = U/R \);
- The relative clearance \( j = J/R \);
- The dimensionless reaction force \( f = |R_f|/2kLR \).

We obtain:

\[
\cos(\theta_0) = \frac{2 j^2 + 2 u + u^2 + 2 j}{2(1 + j)(u + j)}
\]

\[
f = u \sin(\theta_0) + j(\sin(\theta_0) - \theta_0)(j + u)^2 \frac{1}{8} \sin(2\theta_0 - 2\theta_0)
\]

(1)

If the clearance is null \((j = 0)\), the previous equation becomes:

\[
f = \frac{u}{2} \sqrt{4 - u^2} \left(1 + \frac{u^2}{8} - \frac{u}{2} \arccos\left(\frac{u}{2}\right)\right)
\]

(2)

The curves of the figure 2 show the evolution of the dimensionless reaction force \( f \) versus the relative displacement \( u \) for different values of relative clearance \( j \).

We note an important variation of the initial slope between the case without clearance and that with clearance. But, for important values of relative displacement \( u \), the variation becomes smaller. The curves of the figure 3 illustrate the evolution of the slope \( df/du \).

A fundamental difference is immediately noted between the existence or not of the clearance: at the origin \((u=0)\), for a null clearance, initial stiffness is equal to the unit \((df/du=1)\). For a non null clearance \((j \neq 0)\), even with low value, initial stiffness is null \((df/du=0)\).

When the displacement increases, the influence of the clearance decreases. The lower is the clearance, the quicker the corresponding curve merges with the null clearance curve.

So the clearance influences the deformation stiffness of the bearing especially under the action of weak efforts. It affects also the value of the eccentricity due to the movement of hydrodynamic lubrication.

B. Hydrodynamic Film Stiffness

The hydrodynamic bearing theory is used to investigate the oil film stiffness. The shaft and the bearing are supposed to be perfectly rigid. Only the oil film undergoes the deformation. The bearing is supposed to be infinitely short account of the main bearing dimensions.

\( U \) is the motion of the bearing center, \( J \) is the radial clearance and \( \varepsilon = U/J \) is the relative eccentricity.

For a given rotation speed, the load on the shaft generates a relative eccentricity \( \varepsilon \) thus a shaft displacement \( U = J\varepsilon \).

The curves of the figures 4 represent the evolution of the dimensionless bearing load \( f \) and it’s slope \( df/du \) according to \( \varepsilon \) for different values of relative speed \( r = \Omega_0/\Omega \) (\( \Omega \) is the rotation speed and \( \Omega_0 \) the reference rotation speed).

\[
f(\varepsilon; r) = \frac{r\varepsilon}{\pi(1-\varepsilon^2)^2} \sqrt{16\varepsilon^2 + \pi^2(1-\varepsilon^2)}
\]

(3)
The slope \( df/d\varepsilon \) remains very weak for \( \varepsilon \) lower than 0.75 and increases very quickly for \( \varepsilon \) higher than 0.9 whatever the rotation speed value. The oil film rigidity is very low for little loads and becomes very important for strong loads.

So it appears that the oil film has no stiffness for the weak loads and it is infinitely rigid for strong loads.

\[ f(\varepsilon, r) \]

\[ df/d\varepsilon(\varepsilon, r) \]

\[ 0 \leq \varepsilon \leq 0.75 \]

\[ 0.75 < \varepsilon \leq 0.9 \]

\[ r \leq 1/2 \]

\[ r = 2/2 \]

\[ (a) \]

\[ (b) \]

\[ \text{Fig. 4(a) Dimensionless bearing load } f(\varepsilon, r) \]

\[ \text{(b) Dimensionless stiffness } df/d\varepsilon(\varepsilon, r) \]

C. The Deformation Stiffness

The deformation stiffness is caused both by the main bearing deformation and the shaft deformation. In addition, the main bearing deformation is not mainly localized in the contact zone. So, it does not depend only on dimensions of contact surfaces. The external geometry of the bearing and the boundary conditions can have a considerable influence.

To determine the deformation stiffness of the bearing, a finite element model was created. This model is based on real CADs of the crankcase and bearing caps, of a four-cylinder in line engine (Fig.5).

\[ 30 \mu \text{m} \]

\[ E = 210 \text{GPa}, \nu = 0.27 \]

\[ E = 120 \text{GPa}, \nu = 0.27 \]

\[ \text{Fig. 5 Burst finite element model} \]

The crankcase was embedded at the bottom surface (cylinder head side). The bearing caps were fixed to the crankcase. A deformable beam, owing the same diameter as the crankshaft pivot, was considered. More than 20000 elements divided on two types are used to describe the model:

- tetrahedral elements with 4 nodes (C3D4) adapted well for the mesh of the complex geometries.
- hexahedral elements with 8 nodes (C3D8R) adapted well to solve the contact problems.

The considered clearance is about 30\( \mu \text{m} \).

The crankcase and the bearing caps are made of cast iron (Young’s modulus \( E = 210 \text{GPa} \), Poisson’s ratio \( \nu = 0.27 \)). The shaft and the bearings liners are made of steel (\( E = 120 \text{GPa} \), \( \nu = 0.27 \)).

The average line nodes of the deformable beam were coupled to a reference point. A vertical displacement was imposed to this reference point. So, the beam crushes the bearing liner.

The figure 6 shows the bearing 2 deformation stiffness versus low vertical motion of the shaft.

\[ \text{Fig. 6 The bearing 2 deformation stiffness} \]

The same shape of the deformation rigidity as the figure 4 is found. Initially the stiffness is null. The effect of the clearance disappears gradually as the movement of the shaft increases.

III. SUGGESTED MAIN BEARING STIFFNESS ANALYTICAL MODEL

The smooth bearing model highlights the influence of the clearance on the bearing stiffness: the influence of the clearance intervenes only for low shaft motion, therefore for low efforts (Fig.3). For great efforts, stiffness is relatively independent of shaft motion.

The hydrodynamic lubrication model highlights the influence of the oil film rigidity on the overall rigidity of the bearing: the oil film stiffness is almost null for low effort. The shaft moves of 75% of the value of the clearance for a very weak effort (Fig.4). This influence becomes ever weaker (very important oil film rigidity) as the load increases.

The finite element model results approve smooth bearing model results (Fig.6) and it allows us to determine the deformation stiffness \( K_d \).

To illustrate the different findings, the following data corresponding to a real case are considered:

- the maximum crankshaft bearing effort is about 50kN;
- the bearing radius is equal to 24 mm;
- the bearing width is equal to 22 mm;
- the bearing clearance is equal to 30 µm;
- the oil viscosity $\mu$ is equal to 0.005 Ns/m²;
- the rotation speed $\Omega_0$ is equal to 3000 tr/mn;
- the deformation stiffness $K_d$ is about 7.5 $10^8$ N/m.

For a given load $F$ applied to the shaft, his displacement will be the sum of hydrodynamic displacement $U_h=J\frac{U}{d}$ obtained by the hydrodynamic lubrication model and the displacement $U_d=F/K_d$ due to the elastic deformation of the bearing.

The maximum relative eccentricity due to hydrodynamic deformation is about 0.952 which correspond to a shaft displacement $U$ of about 28.6 µm.

Figure 7 shows the effort curves:
- $F(U_h)$ corresponding to the bearing effort versus the shaft centre displacement due to elastic deformation.
- $F(U_h)$ corresponding to the bearing effort versus the shaft centre displacement due to oil film deformation.
- $F(U=U_d+U_h)$ corresponding to the bearing effort versus the total shaft centre displacement.

We suggest regularizing the function stiffness $K=dF/dU$ through the consideration of the oil film effect and eventually the clearance effect as follows:

$$K = K_d\left(1 + \left(1 + e^{-\frac{U}{d}}\right) e^{-\frac{J}{U-d}}\right)$$

(4)

Where: $dx$ is the regulation parameter.

The integration of the regularized function $K(U)$ compared to the displacement $U$ gives the bearing load $F(U)$:

$$F(U)=K_d\int\left(1 + e^{-\frac{dx}{d}}\right)\left[\ln\left(1 + e^{-\frac{dx}{d}}\right) - U e^{-\frac{dx}{d}}\right]$$

(5)

To illustrate the quality of this regularization, the model (4) is identified starting from the curve $F(U_h+U_d)$ of figure 7. The identified values of $K_d$, $dx$ and $J$ are presented in the table below.

The identified deformation stiffness decreases of about 9%. We note that the suggested analytical model envisages rather precisely the behaviour of the main bearing and ensures properly the transition from the configuration low rigidity to the configuration high rigidity (Fig.8).

**TABLE I**

<table>
<thead>
<tr>
<th>Initial</th>
<th>Identified</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_d(10^5N/m)$</td>
<td>7.67 6.9</td>
</tr>
<tr>
<td>$J(\mu m)$</td>
<td>30 30</td>
</tr>
<tr>
<td>$dx(\mu m)$</td>
<td>7.5</td>
</tr>
</tbody>
</table>

The merit of the behaviour law proposed is that it can be advantageously useful in beam crankshaft-engine block combined models.

**IV. CONCLUSION**

This work presents an investigation of main bearing stiffness. Due to the use of smooth bearing model it was shown that the clearance effect occurs for low loads. Due to the hydrodynamic model it was shown that the oil film has a very low stiffness for low loads and becomes very rigid for important loads.

The main bearing deformation stiffness was determined through an appropriate finite element model based on real CADs.

Subsequently, a main bearing behaviour law has been proposed. This behaviour law takes into account at the same time clearance effect, hydrodynamic sustention and deformation stiffness. This behaviour law is useful in simplified beam engine models.

**REFERENCES**


